Test Generation – Predicate Analysis

Andrea Polini

Fundamentals of Software Testing
MSc in Computer Science
University of Camerino

Dependency modeling

CEG aka Dependency Modeling

The very general idea is to make explicit, also through a graphical representation, the relation among input conditions (causes) and output conditions (effects) and to exploit such relations for testing purposes.
In any case the relation can be fruitfully represented by a boolean expression

Cause and effects

A cause is any condition in the requirements that may effect the program output.
An effect is the response of the program to some combination of input conditions. An effect is not necessarily visible to the external user, while it can be retrieved introducing suitable probes (test points)

Exercise

The LED close to the product description should be switched on when the credit becomes greater then the price of the snack

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Test generation from CEG

CEG and test generation

- Identify cause and effects reading the requirements. Assign a unique identifier
- Express the relationship between causes and effects using a CEG
- Tranform the CEG into a decision table
- Generate tests from the decision table

Are we checking iff relations?

The strategy reported above checks that "if the conditions happen the effect should be visible". In general effects could be produced as consequence of additional behaviour so it may possible that you are not interested in checking the other way around i.e. an effect is present only if those conditions holds

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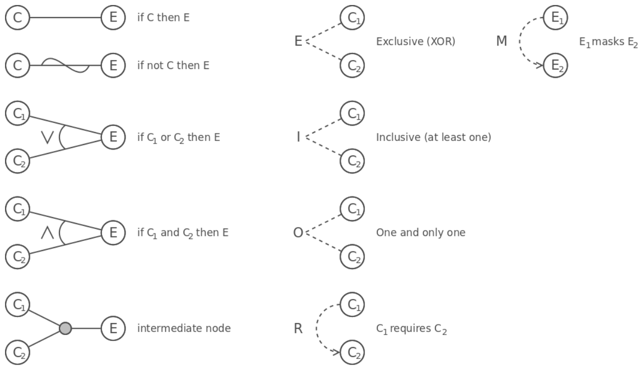
CEG Notation



- The mask relation can be expressed as additional relations on the causes
- What if I would like to check iff relations among causes and effects?

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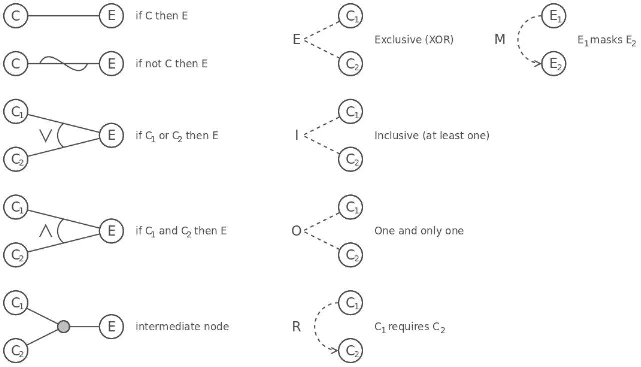

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Creating a CFG

Process

To create a CFG follow the process below:

- carefully identify causes and effects from a thoughtful analysis of the requirements.
- assign to each cause and each effect a unique identifier
- represent the identified relations in a CFG

Example

Computer purchase system

A web based company sells computers (CPU), printers (PR), monitors (M), and additional memories (RAM). The Web GUI will include 4 windows for driving the buyer through the selection process. A final window with the free giveaway items is then displayed. For simplicity only one item per category can be purchased.
Conditions: For each order the buyer may select from 3 CPU, 2 PR, 3M. RAM is available only as a "free" upgrade. M20 and M23 any CPU or as stand alone. M30 only with CPU 3. PR 1 is available free with CPU 2 or 3. M and PR can be purchased as stand alone. Not M30. CPU 1 gets RAM 256 upgrade. CPU 2 o 3 gets RAM 512 upgrade. RAM 1G upgrade and free PR2 available if CPU 3 purchased with M30. There is a window to make selection with menus in particular a widget displaying the free item available and a "Price" widget reports the calculation related to prices.
The strategyy asks you to:

- Read carefully the requirements
- Identify causes and effects
- Identify relations among them

Example - The CFG for the Computer Purchase System

Construct the CFG for the Computer Purchase System

- causes?
- effects?
- relations?
- Do we need to consider relations as iff?

Decision Tables from a CEG

CEG models relations among different aspects of the system. The derivation of test requires the definition of the corresponding decision table

Decision tables

For each cause and effect use a row and put test as columns of the matrix. Each entry in the decision table is a 0 or a 1 depending on whether or not the corresponding condition is false or true, respectively.

How to derive a DT

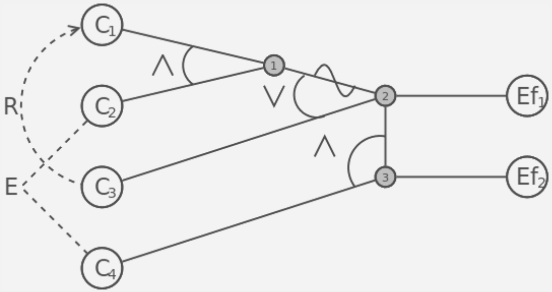
Input: A CEG containing causes $C_1, C_2, ..., C_p$ and effects $Ef_1, Ef_2, ..., Ef_q$
Output: A decision table containing $p + q$ rows and M columns where M depends on relationship between causes and effects.

Procedure CFG2DT
Step1: Initialize DT to an empty DT
Step2: Execute the following steps for i=1 to q

```
    2.1 Select the next effect $e$
    2.2 Find combinations of conditions that cause $e$ to be present and store
         the $m$ generated vector. Avoid combinatorial explosion.
    2.3 update the decision table adding the generated vectors
```

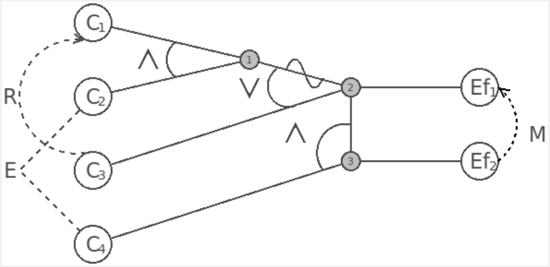
DT derivation

Consider the following CEG and derive the corresponding decision table:



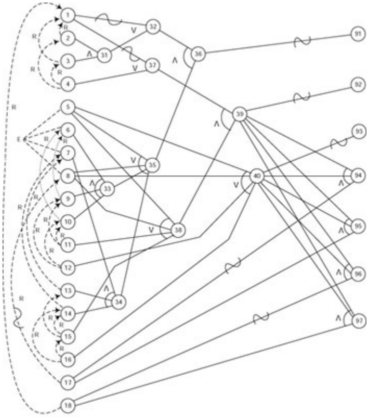
DT derivation

How can we deal with Mask relations?



Example

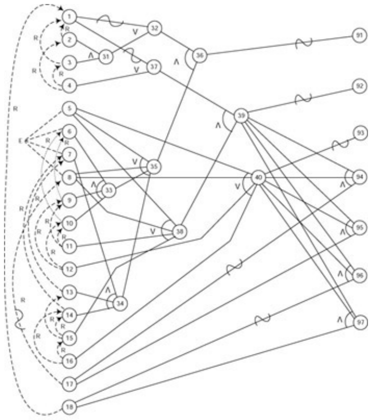
Apply the procedure to the following CEG:



You need to automatize the process.

Example

Apply the procedure to the following CEG:



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Heuristic to avoid combinatorial explosion

The described approach could lead to exponential generation on the number of tests with respect to causes. Indeed having an effect depending on *n* causes can lead to the generation of a number of vectors in the order of 2^n

Reduction strategies

- For or relations: enumerate just those situations in which two causes are both false (0) or one of them true (1)
- For and relations: enumerate those situations for which causes assume different values (0), and those in which all of them are true (1)

Test generation

Tests from a decision table

Each column of the decision table constitutes the source for generating tests. Consider that each condition could be satisfied by more assignement to the variable leading to the generation of more than one test for each column.
To derive a test case you need then to identify the inputs correlated to the cause and the value that makes the cause valid. On such sets you can apply previously considered approaches. Let's go back to the LED example.

Test generation from predicates

Techniques aiming at finding bugs in the coding of conditions

Predicate testing

if the printer is ON and has paper then send the document for printing

```
pr:(printer_status=ON) ∧ printer_tray=!empty
```

We are interested in generating test cases from predicates such that any fault belonging to a class is detected.

Consider the following predicate:

$$(a < b) \vee (c > d) \wedge e$$

The following test:

$$t = (a = 1, b = 2, c = 4, d = 2, e = true)$$

results in

$$p(t) = true$$

Fault model

Which kind of faults are generally targetted:

- Boolean operator fault
  - incorrect boolean operator used
  - negation missing or placed incorrectly
  - parentheses are incorrect
  - incorrect Boolean variable used
- relational operator fault
  - incorrect relational operator is used
- arithmetic expression fault
  - arithmetic expression is off by an amount equal to ϵ (off-by-ϵ,off-by-ϵ^+,off-by-ϵ^*)

Objective of predicate testing

To generate a test set $\mathscr{T}$ such that there is at least one test cast $t \in \mathscr{T}$ for which p_c and its faulty version p_i are distinguishable

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Missing or extra boolean faults

Missing or extra boolean faults corresponds to situation in which the programmer forgot to include a variable in a condition or added a not needed variable

The following approaches do not provide any guarantee on the possible identification of such kind of faults

Predicate constraints

Let *BR* denote the following set of symbols $\{\mathbf{t}, \mathbf{f}, <, =, >, +\epsilon, -\epsilon\}$. A *BR* specifies a constraint on a Boolean variable of a relational expression.
In particular $+\epsilon$ and $-\epsilon$ are constraints on an expression $e_1 < e_2$ that can be respectively satisfied by tests such that $0 < e_1 - e_2 \leq \epsilon$ and $-\epsilon \leq e_1 - e_2 < 0$
A constraint is considered infeasible if there exist no input values for the variable in the predicate that can satisfy the constraint.

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Predicate testing criteria

Three common criteria:

- BOR (Boolean Operator): A test set $\mathscr{T}$ that satisfied the BOR-testing criterion for a compound predicate p_r, guarantees the detection of single or multiple Boolean operator faults in the implementation of p_r. $\mathscr{T}$ is referred to as a BOR-adequate test set and sometimes written as $\mathscr{T}_{BOR}$.
- BRO (Boolean and relational Operator): A test set $\mathscr{T}$ that satisfied the BRO-testing criterion for a compound predicate p_r, guarantees the detection of single or multiple Boolean operator and relational operator faults in the implementation of p_r. $\mathscr{T}$ is referred to as a BRO-adequate test set and sometimes written as $\mathscr{T}_{BRO}$.
- BRE (Boolean and relational expression): A test set $\mathscr{T}$ that satisfied the BRE-testing criterion for a compound predicate p_r, guarantees the detection of single or multiple Boolean operator, relational operator and arithmetic expression faults in the implementation of p_r. $\mathscr{T}$ is referred to as a BRO-adequate test set and sometimes written as $\mathscr{T}_{BRE}$.

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BOR example

Let $p_r : a < b \wedge c > d$ and $\mathscr{S}$ constraints on p_r where $\mathscr{S} = \{(\mathbf{t}, \mathbf{t}), (\mathbf{t}, \mathbf{f}), (\mathbf{f}, \mathbf{t})\}$ the following test set $\mathscr{T}$ satisfies constraint set $\mathscr{S}$ and the BOR-testing criterion:

$$
\begin{aligned}
\mathscr{T} = \{ & t_1 :< a = 1, b = 2, c = 1, d = 0 >; \\
& t_2 :< a = 1, b = 2, c = 1, d = 2 >; \\
& t_3 :< a = 1, b = 0, c = 1, d = 0 >; \\
& \}
\end{aligned}
$$

BOR example

Covered faults

To discover the covered faults lets modify the proposition introducing one or more operational fault

	Predicate	t1	t2	t3
Correct	$a < b \land c > d$	`true`	`false`	`false`
Single Fault	$a < b \lor c > d$	`true`	**true**	**true**
	$a < b \land \neg c > d$	**false**	**true**	`false`
	$\neg a < b \land c > d$	**false**	`false`	**true**
Multiple Faults	$a < b \lor \neg c > d$	`true`	**true**	`false`
	$\neg a < b \lor c > d$	`true`	`false`	**true**
	$\neg a < b \land \neg c > d$	**false**	`false`	`false`
	$\neg a < b \lor \neg c > d$	`true`	**true**	**true**

Generating BOR, BRO, BRE adequate tests

A predicate constraint C for predicate p_r is a sequence of $n + 1$ boolean and relational symbols.
A test case t satisfies C for predicate p_r, if each component of p_r satisfies the corresponding constraint in C when evaluted against t.
e.g.: given $p_r = b \wedge r < s \vee u \geq v$ and $C : (t, =, >)$ the following test case satisfies C: <$b = true, r = 1, s = 1, u = 1, v = 0$>

There exist algorithms able to generate adequate tests given a set of constraints on the predicate. They are based on the definition of:

- Cartesian product of sets
- *onto* set product operator
- Abstract Syntax Tree of a predicate- $AST(p_r)$

Onto Operator

Onto Operator

Given two sets A and B the onto operator constructs the minimal set of pairs $\langle a, b \rangle$ where $a \in A$ and $b \in B$ and each element of the two sets is used in at least one of the pairs in the onto set $A \otimes B$.
Which is the cardinality of the onto set?

Let $A = \{t, 0, >\}$ and $B = \{f, <\}$ lets derive the cartesian product and some examples of onto product sets

Abstract Syntax Tree for a Predicate p

AST

The abstract sintax tree provides a tree based representation of a predicate that captures the syntactic relations among the predicate constituents, and that is tipically useful for associating meaning to the predicate itself.
Leaves of the tree are atomic propositions while nodes are boolean operators

AST

Let's build the AST for the proposition:
$a < b \vee q \wedge \neg p \vee (a == c \wedge p)$

Generating the BOR-constraint set

Let p_r be a predicate and $AST(P_r)$ its abstract syntax tree, S_N the constraint set attached to a node N (where $S^t{}_N$ and $S^f{}_N$ are the true and false constraints associated with the node). The following alg. generates the BOR-constraint set for p_r

Input: $AST(p_r)$ (only singular expressions)

Output: BOR-Constraint set attached to the root node

1. Label each leaf node N of $AST(p_r)$ with its constraint set $S_N = \{t, f\}$
2. Visit the AST bottom-up. Let N_1 and N_2 direct descendants of node N and S_{N1} and S_{N2} the corresponding BOR-constraint set. S_N is computed as follows:

   2.1 N is an OR-node:
   - $S^f{}_N = S^f{}_{N1} \otimes S^f{}_{N2}$
   - $S^t{}_N = (S^t{}_{N1} \times \{f_2\}) \cup (\{f_1\} \times S^t{}_{N2})$ where $f_1 \in S^f{}_{N1}$ and $f_2 \in S^f{}_{N2}$

   2.2 N is an AND-node:
   - $S^t{}_N = S^t{}_{N1} \otimes S^t{}_{N2}$
   - $S^f{}_N = (S^f{}_{N1} \times \{t_2\}) \cup (\{t_1\} \times S^f{}_{N2})$ where $t_1 \in S^t{}_{N1}$ and $t_2 \in S^t{}_{N2}$

   2.3 N is NOT-node:
   - $S^t{}_N = S^f{}_{N1}$
   - $S^f{}_N = S^t{}_{N1}$

BOR-constraint set example

Let's apply the BOR-constraint procedure to:

- $(a + b < c) \wedge \neg p \vee (r > s)$

Generating the BRO-constraint set

Input: $AST(p_r)$ (only singular expressions)
Output: BRO-Constraint set attached to the root node

1. Label each leaf node N of $AST(p_r)$ with its constraint set S_N. For each leaf node that represents a Boolean variable $S_N = \{t, f\}$. For each leaf node that is a relational expression $S_N = \{(>), (=), (<)\}$.
2. Visit the AST bottom-up. Let N_1 and N_2 direct descendants of node N and S_{N1} and S_{N2} the corresponding BRO-constraint set. S_N is computed as done for the BOR procedure.

Generating the BRO-constraint set

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BRE constraint sets

Constraint	**Satisfying condition**
$+\epsilon$	$0 < e_1 - e_2 \leq +\epsilon$
$-\epsilon$	$-\epsilon \leq e_1 - e_2 < 0$

True and false components for a relational operator are defined as follows:

relop	S^t	S^f
$>$	$\{(+\epsilon)\}$	$\{(=), (-\epsilon)\}$
$\geq$	$\{(+\epsilon), (=)\}$	$\{(-\epsilon)\}$
$=$	$\{(=)\}$	$\{(+\epsilon), (-\epsilon)\}$
$<$	$\{(-\epsilon)\}$	$\{(=), (+\epsilon)\}$
$\leq$	$\{(-\epsilon), (=)\}$	$\{(+\epsilon)\}$

Generating the BRE-constraint set

Input: $AST(p_r)$ (only singular expressions)
Output: BRE-Constraint set attached to the root node

1. Label each leaf node N of $AST(p_r)$ with its constraint set S_N. For each leaf node that represents a Boolean variable $S_N = \{t, f\}$. For each leaf node that is a relational expression $S_N = \{(+\epsilon), (=), (-\epsilon)\}$.
2. Visit the *AST* bottom-up. Let N_1 and N_2 direct descendants of node N and S_{N1} and S_{N2} the corresponding BRE-constraint set. S_N is computed as done for the BOR procedure.

Which are the relations among the test suites generated by the different methods?

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Generating the BRE-constraint set

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Cardinality of generated Test set

Reflection point

How the cardinality of the test sets generated by the reported algorithms, compares with respect to a brute force approach?
i.e. to the enumeration of all the possible cases and their combination.

Generating test sets for non singular expressions

MI-CSET Procedure

Input: A boolean expression $E = e_1 + e_2 + ... + e_n$ in minimal DNF containing n terms. Term e_i contains $l_i > 0$ literals.
Output: A set of constraints S_E that guarantees the detection of missing or extra NOT operator fault in a faulty version of E.

1. For each term e_i, $1 \le i \le n$ construct T_{e_i} as the set of constraints making e_i true
2. Let $TS_{e_i} = T_{e_i} - \bigcup_{j=1, j \neq i}^{n} T_{e_j}$. Note that for $i \neq j$, $TS_{e_i} \cap TS_{e_j} = \emptyset$
3. Contruct S_E^t by including one constraint from each TS_{e_i}
4. Let e_i^j denotes the term obtained by complementing the j^{th} literal in e_i, for $1 \le i \le n$ and $1 \le j \le l_i$. Construct $F_{e_i^j}$ as the set of contraints that make e_i^j true
5. Let $FS_{e_i^j} = F_{e_i^j} - \bigcup_{k=1}^{n} T_{e_k}$. Thus for any constraint $c \in FS_{e_i^j}, E(c) = \texttt{false}$
6. Construct S_E^f that is minimal and covers each $FS_{e_i^j}$ a least once.
7. Construct the desired constraint set for E as $S_E = S_E^t \cup S_E^f$.

MI-CSET

Let's apply the procedure to: $a \wedge ((b \wedge c) \vee (\neg b \wedge d))$

Generating test sets for non singular expressions

BOR-MI-CSET Procedure

Input: A boolean expression E
Output: A set of constraints S_E that guarantees the detection of boolean operator faults in E

1. Partition E in a set of n mutually singular components $E = \{E_1, E_2, \cdots, E_n\}$
2. Generate the BOR constraint set for each singluar component in E using the BOR-CSET procedure.
3. Generate the MI-constraint set for each non-singular component in E using the MI-CSET procedure
4. Combine the constraints generated in the previous two steps as indicated in step 2 of the BOR-CSET procedure. The result of the combination is the constraint set for E

BOR-MI-CSET procedure - an example

Let's apply the procedure to: $a \wedge ((b \wedge c) \vee (\neg b \wedge d))$

CEG and Predicate testing

CEG

- CEG strategy to define relations among causes and effects ("oracles")
- Decision table technique to identify test cases

Predicate testing

- Stategies for deriving test from predicates, fault coverage guarantees

"Better together"

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Usage of predicate testing techniques

Approaches to test set derivation from predicates can be applied considering different starting points:

- Specification based testing
- Program based testing

The different settings have different consequences

Exercise

Consider the BOR, BRO, BRE criteria for testing predicates including expressions and relational operator, and shortly introduce their objectives and differences. Use the most appropriate criteria for singular expressions to generate a test set, able to discover logical, and relational faults, for the following compound predicates (possibly transforming them):

$$\neg((z \cdot y) \geq (x + y) \wedge \neg p) \wedge ((y = w) \vee p)$$

$$((x\dot{(}y^2 + 1) \geq x) \wedge a \wedge (x^2\dot{y} = 7)) \vee ((x^2\dot{y} - 7 = 0) \wedge xy^2 < 0 \wedge d)$$