

Modelling Hardware Circuits

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Reactive Systems Verification

MSc in Computer Science

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Topics

- Modelling Hardware Circuits as Transition Systems

Material

Reading:

Chapter 2 of the book, Section 2.1.2.

More:

The slides in the following pages are taken from the material of the course “Introduction to Model Checking” held by Prof. Dr. Ir. Joost-Pieter Katoen at Aachen University.

Introduction

Modelling parallel systems

Transition systems

Modeling hard- and software systems ←

Parallelism and communication

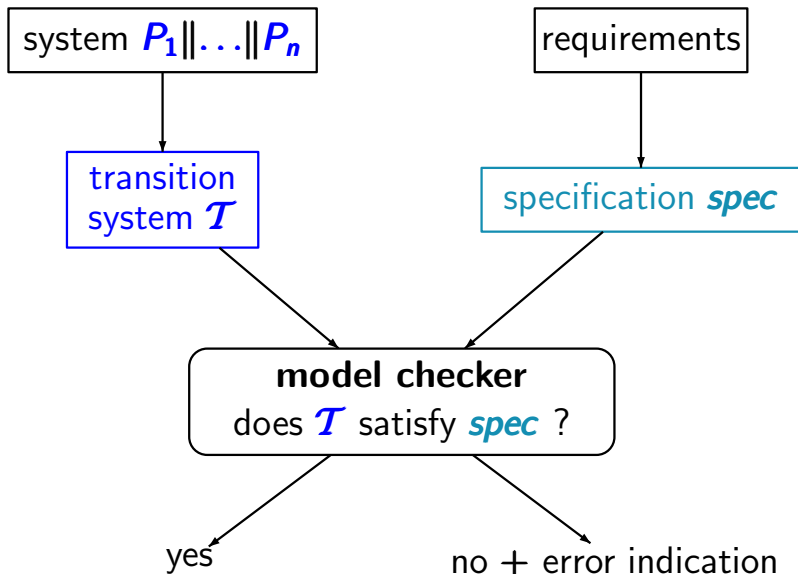
Linear Time Properties

Regular Properties

Linear Temporal Logic

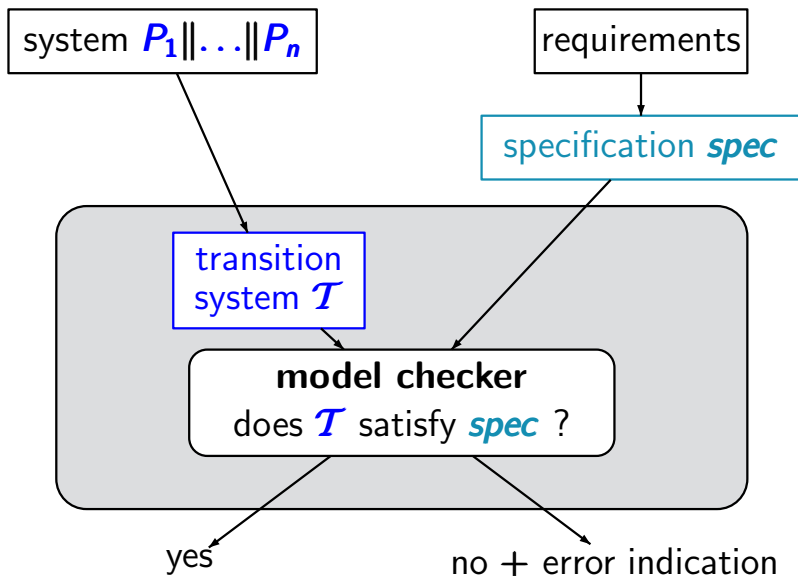
Computation-Tree Logic

Equivalences and Abstraction



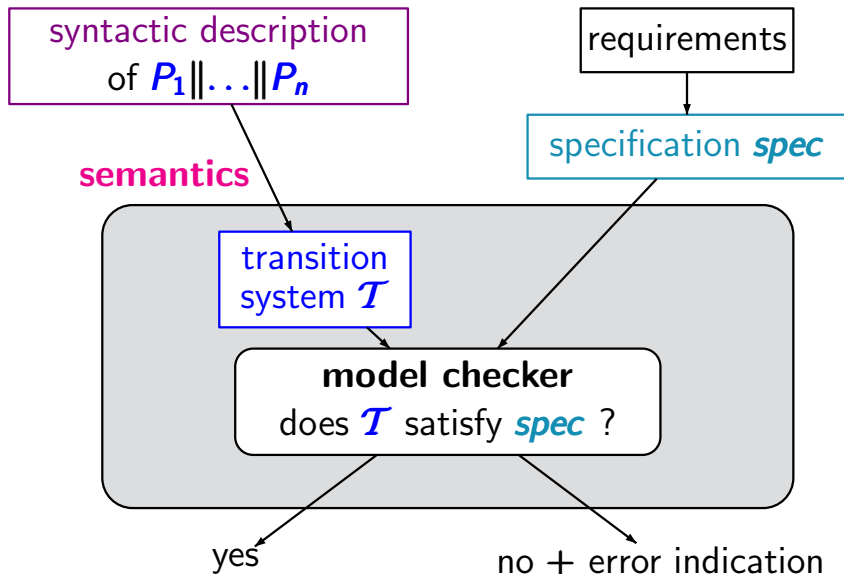
Model checking

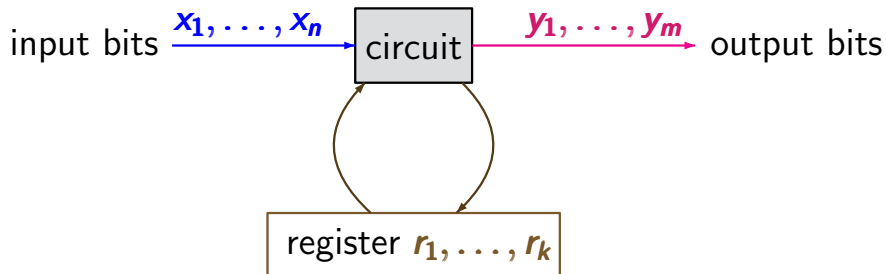
TS1.4-9



Model checking

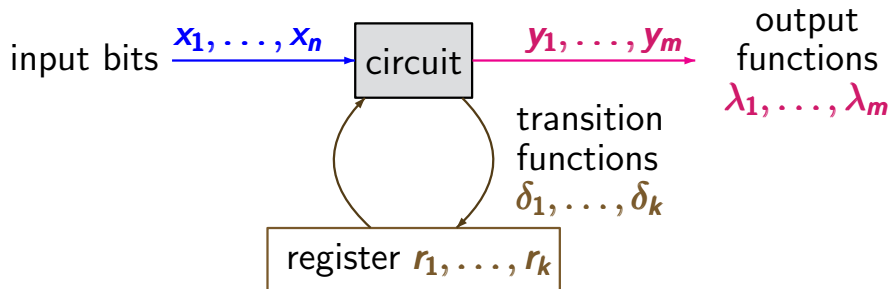
ts1.4-9





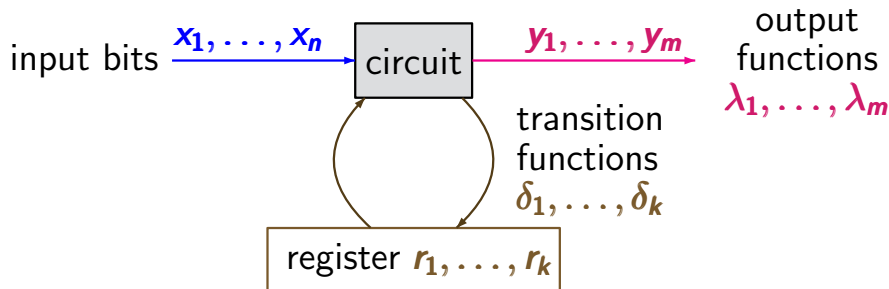
Modelling of sequential circuits by TS

TS1.4-10



Modelling of sequential circuits by TS

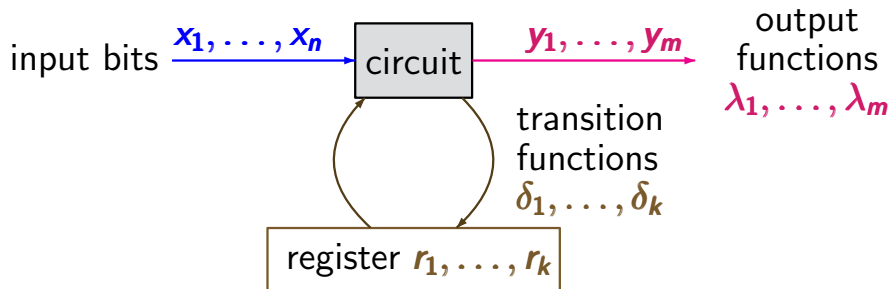
TS1.4-10



$\delta_j, \lambda_i \hat{=} \text{switching functions } \{0, 1\}^n \times \{0, 1\}^k \longrightarrow \{0, 1\}$

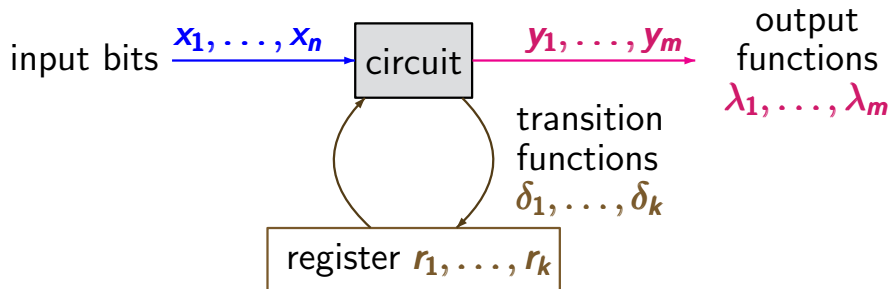
Modelling of sequential circuits by TS

TS1.4-10

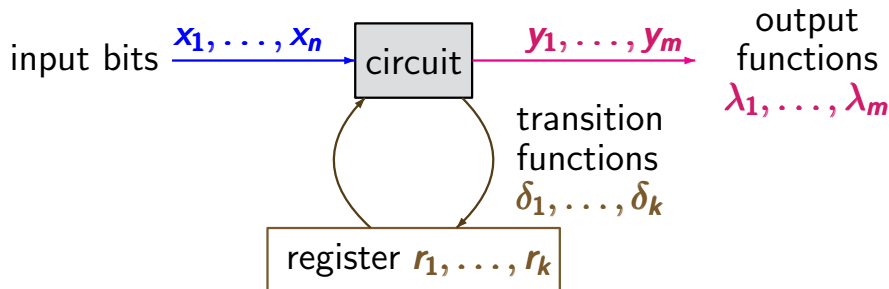


$\delta_j, \lambda_i \hat{=} \text{switching functions } \{0, 1\}^n \times \{0, 1\}^k \longrightarrow \{0, 1\}$

input values a_1, \dots, a_n for the input variables + current values c_1, \dots, c_k of the registers	↦	output value $\lambda_i(\dots)$ for output variable y_i next value $\delta_j(\dots)$ for register r_j
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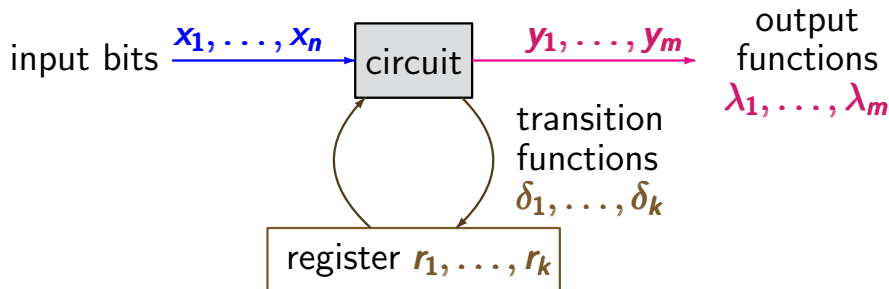
initial register evaluation $[r_1=c_{01}, \dots, r_k=c_{0k}]$



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transition system:

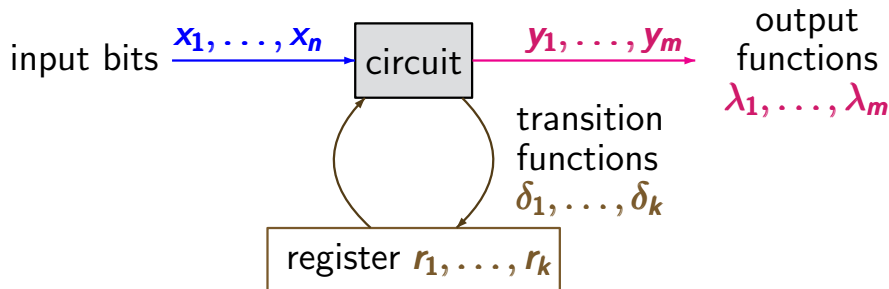
- states: evaluations of $x_1, \dots, x_n, r_1, \dots, r_k$



initial register evaluation $[r_1=c_{01}, \dots, r_k=c_{0k}]$

transition system:

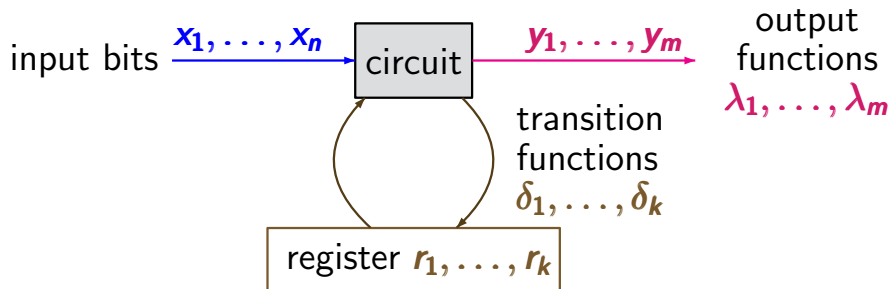
- states: evaluations of $x_1, \dots, x_n, r_1, \dots, r_k$
- transitions represent the stepwise behavior



initial register evaluation $[r_1=c_{01}, \dots, r_k=c_{0k}]$

transition system:

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- transitions represent the stepwise behavior
- values of input bits change nondeterministically



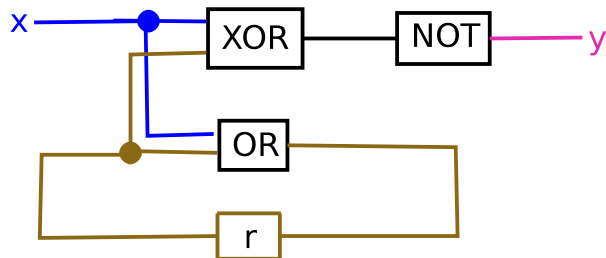
initial register evaluation $[r_1=c_{01}, \dots, r_k=c_{0k}]$

transition system:

- states: evaluations of $x_1, \dots, x_n, r_1, \dots, r_k$
- transitions represent the stepwise behavior
- values of input bits change nondeterministically
- atomic propositions: $x_1, \dots, x_n, y_1, \dots, y_m, r_1, \dots, r_k$

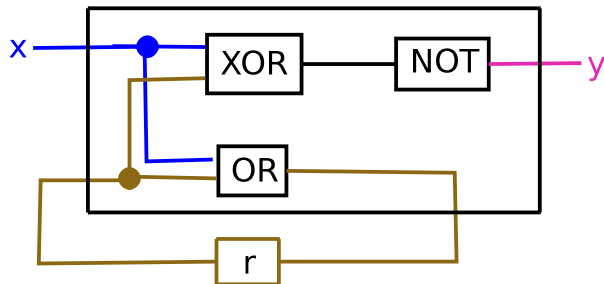
Example: sequential circuit

TS1.4-11A



Example: sequential circuit

TS1.4-11A

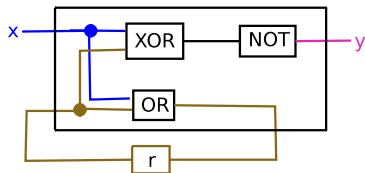


output function: $\lambda_y = \neg(x \oplus r)$

transition function: $\delta_r = x \vee r$

Example: TS for sequential circuit

TS1.4-11



output function

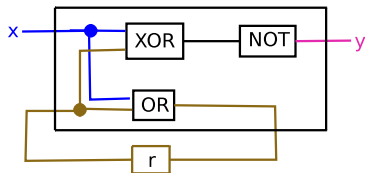
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Example: TS for sequential circuit

TS1.4-11



transition system

output function

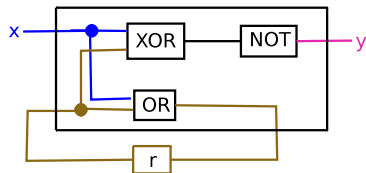
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Example: TS for sequential circuit

TS1.4-11



transition system

$$x=0 \quad r=0$$

$$x=1 \quad r=0$$

$$x=0 \quad r=1$$

$$x=1 \quad r=1$$

output function

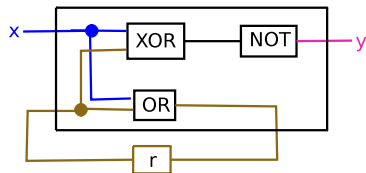
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TS1.4-11



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↙

$$x=0 \ r=0$$

↙

$$x=1 \ r=0$$

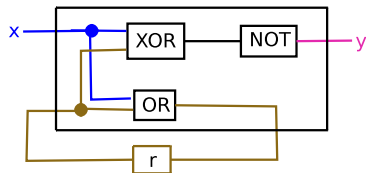
$$x=0 \ r=1$$

$$x=1 \ r=1$$

initial register evaluation: $r=0$

Example: TS for sequential circuit

TS1.4-11



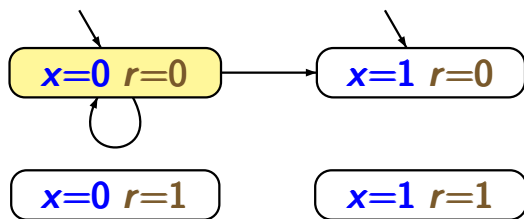
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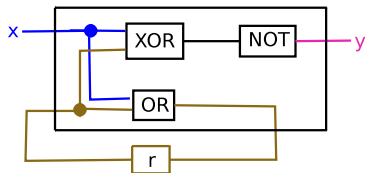
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Example: TS for sequential circuit

TS1.4-11



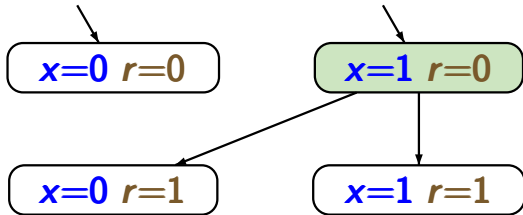
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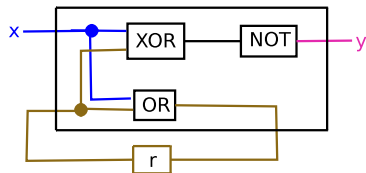
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Example: TS for sequential circuit

TS1.4-11



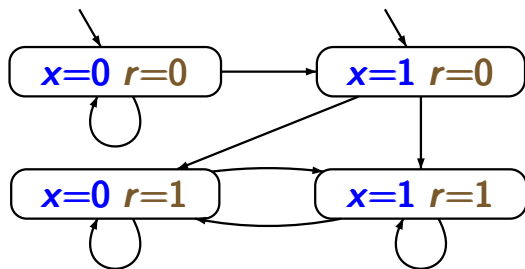
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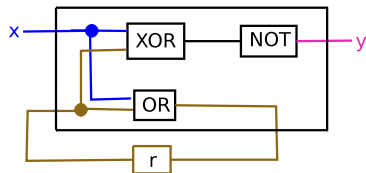
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Example: TS for sequential circuit

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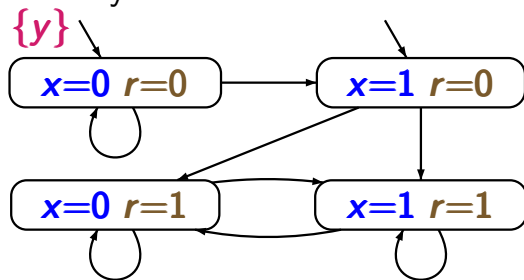
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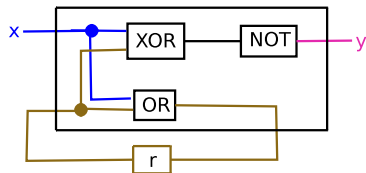
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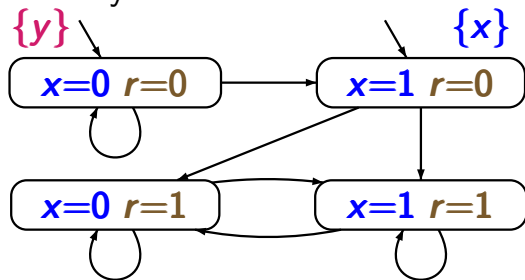
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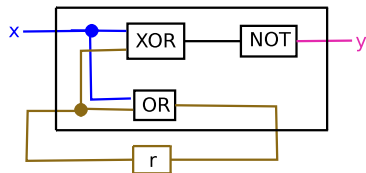
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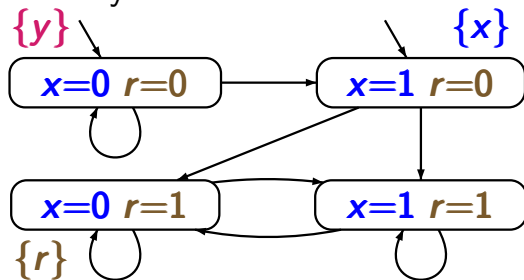
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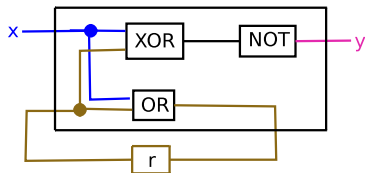
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TS1.4-11



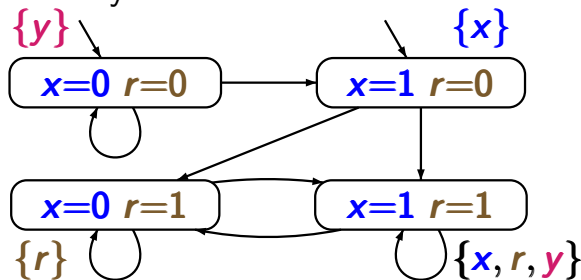
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transition system



initial register evaluation: $r=0$

How many states ...

TS1.4-12

... has the transition system for a circuit of the form?



1 output bit
no input
100 registers

How many states ...

TS1.4-12

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answer: 2^{100}

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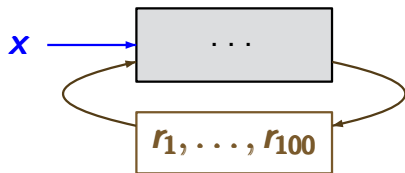
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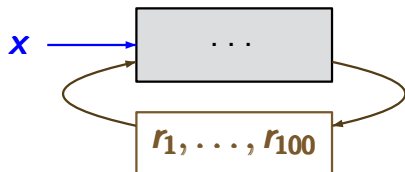
ts1.4-12

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1 output bit
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answer: 2^{100}



no output
1 input bit
100 registers

answer: $2^{100} * 2^1 = 2^{101}$