Model Checking I
alias
Reactive Systems Verification

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Topics

- Parallelism
- Interleaving operator for Transition Systems
- Examples

Material

Reading:

Chapter 2 of the book, pages 35–39.

More:

The slides in the following pages are taken from the material of the course "Introduction to Model Checking" held by Prof. Dr. Ir. Joost-Pieter Katoen at Aachen University.

Overview

Parallelism and communication

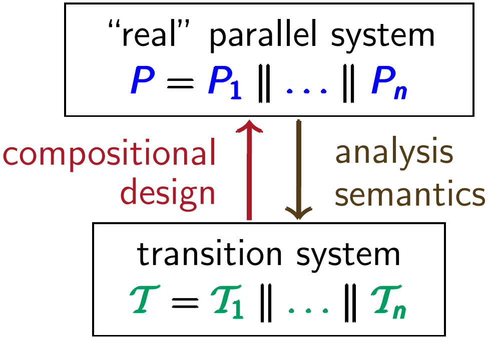
"real" parallel system

$P = P_1 \parallel \ldots \parallel P_n$

transition system

$\mathcal{T} = \mathcal{T}_1 \parallel \ldots \parallel \mathcal{T}_n$

Parallelism and communication



Parallelism and communication

"real" parallel system

$$P = P_1 \parallel \ldots \parallel P_n$$

compositional design ↑ ↓ analysis semantics

transition system

$$\mathcal{T} = \mathcal{T}_1 \parallel \ldots \parallel \mathcal{T}_n$$

goal: define semantic parallel operators on transition systems or program graphs that model "real" parallel operators

Interleaving operator $|||$ for TS

- interleaving of concurrent, independent actions of parallel processes (modelled by TS)
- representation by nondeterministic choice:
  "which subprocess performs the next step?"

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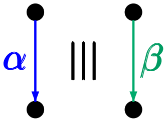

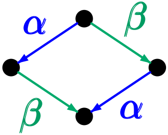

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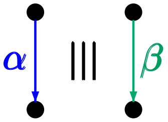

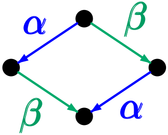

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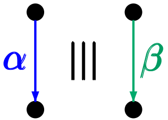

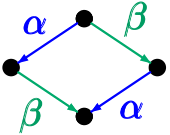

parallel execution of α and β on *two processors* $\widehat{=}$ serial execution on a *single processor* in arbitrary order

Interleaving operator $|||$ for TS

$$\mathcal{T}_1 = (S_1, Act_1, \longrightarrow_1, S_{0,1}, AP_1, L_1)$$
$$\mathcal{T}_2 = (S_2, Act_2, \longrightarrow_2, S_{0,2}, AP_2, L_2)$$

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The transition system $\mathcal{T}_1 \,|||\, \mathcal{T}_2$ is defined by:

$$\mathcal{T}_1 \,|||\, \mathcal{T}_2 = (S_1 \times S_2, Act_1 \cup Act_2, \longrightarrow, S_{0,1} \times S_{0,2}, AP, L)$$

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$$\frac{s_1 \xrightarrow{\alpha}_1 s_1'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1', s_2 \rangle} \qquad \frac{s_2 \xrightarrow{\alpha}_2 s_2'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1, s_2' \rangle}$$

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atomic propositions: $AP = AP_1 \uplus AP_2$

labeling function: $L(\langle s_1, s_2 \rangle) = L_1(s_1) \cup L_2(s_2)$

SOS-notation (structured operational semantics)

just a simple notation for operational semantics

$$\frac{\textbf{premise}}{\textbf{conclusion}}$$

just a simple notation for operational semantics

$$\frac{\textbf{premise}}{\textbf{conclusion}}$$

E.g., *"the relation $\longrightarrow$ is given by ..."*

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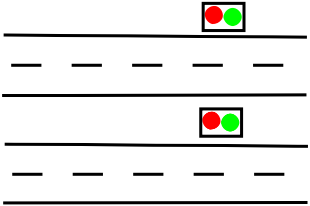
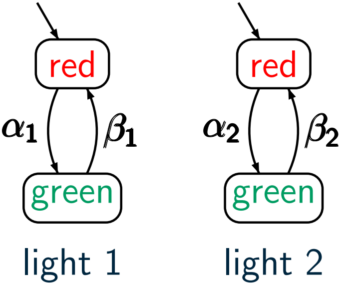
means that $\longrightarrow$ is the smallest relation such that:

(1) If $s_1 \xrightarrow{\alpha}_1 s_1'$, then $\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1', s_2 \rangle$

(2) If $s_2 \xrightarrow{\alpha}_2 s_2'$, then $\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1, s_2' \rangle$

Useless lights for non-crossing streets





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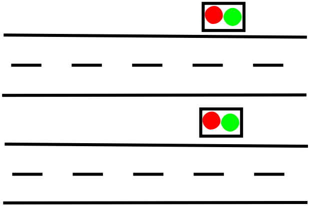
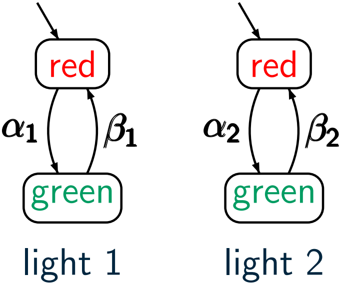
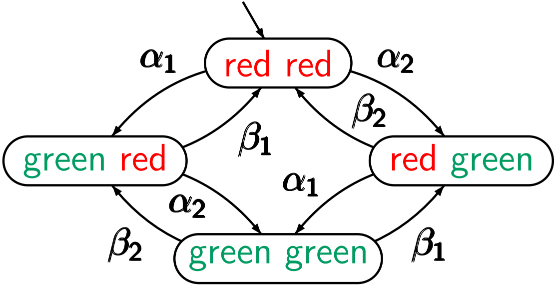




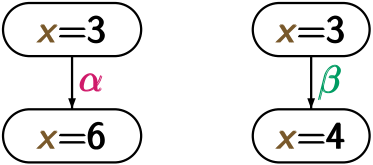
light 1 ||| light 2



Dependent actions

dependent actions $\alpha \mathrel{\widehat{=}} x{:=}2x$ and $\beta \mathrel{\widehat{=}} x{:=}x+1$

representations in transition systems



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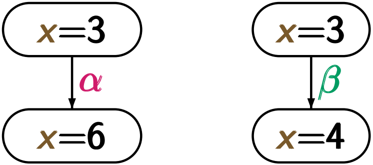
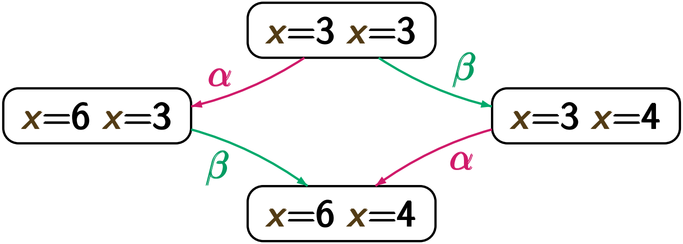


interleaving operator $|||$



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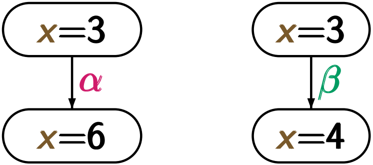
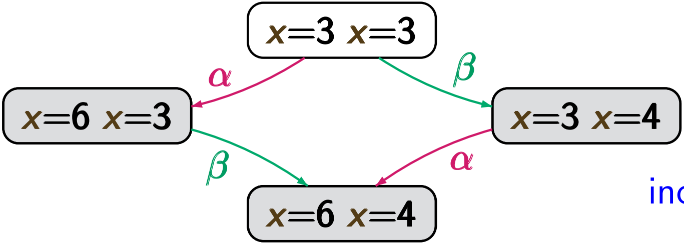


interleaving operator ||| for transition systems "fails"



inconsistent states