Model Checking I
alias
Reactive Systems Verification

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MSc in Computer Science, University of Camerino

Topics

- Channel Systems
- Synchronous and Asynchronous Communication
- SOS Semantics of Channel Systems as Transition Systems
- Examples

Material

Reading:

Chapter 2 of the book, pages 53–63.

More:

The slides in the following pages are taken from the material of the course “Introduction to Model Checking” held by Prof. Dr. Ir. Joost-Pieter Katoen at Aachen University.

Overview

Introduction

Modelling parallel systems

- Transition systems
- Modeling hard- and software systems
- Parallelism and communication ←

Linear Time Properties

Regular Properties

Linear Temporal Logic

Computation-Tree Logic

Equivalences and Abstraction

Channel systems

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representation of data-dependent parallel systems with

- communication over shared variables
- synchronous message passing
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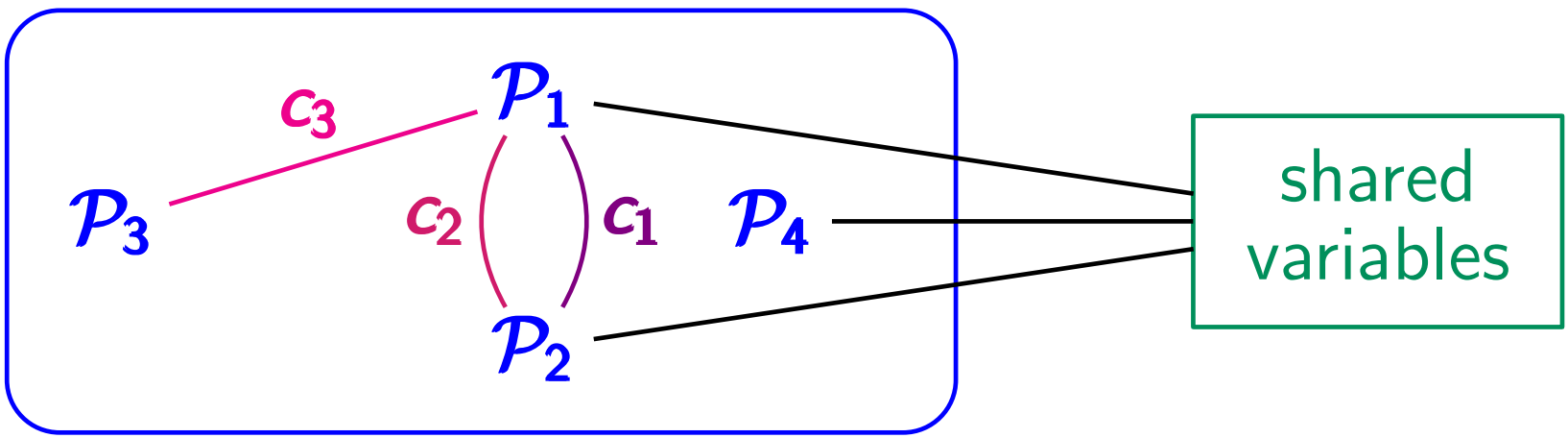

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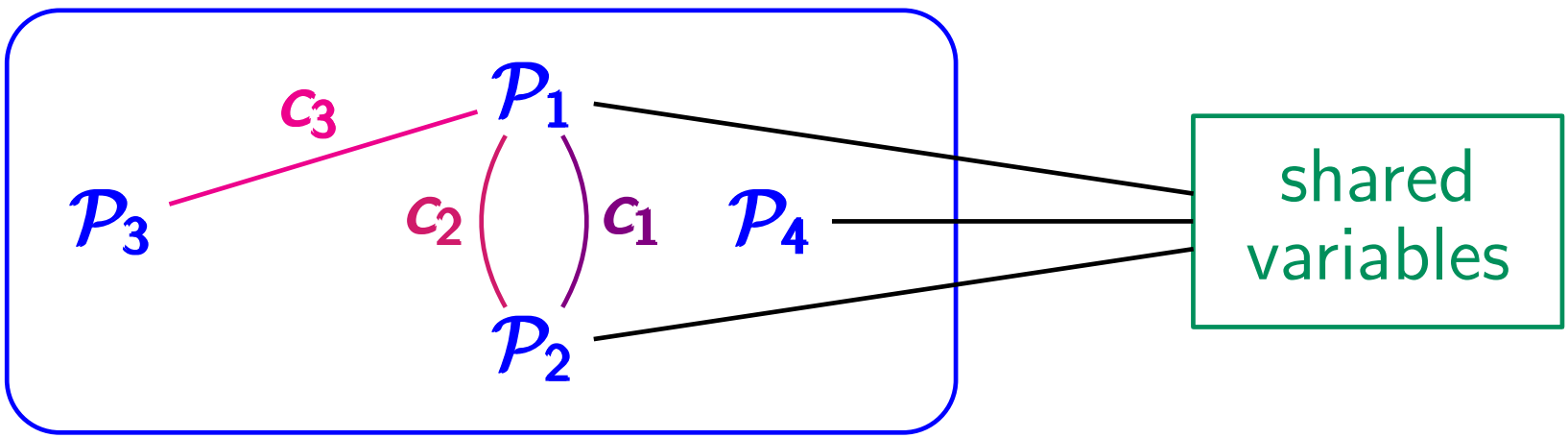

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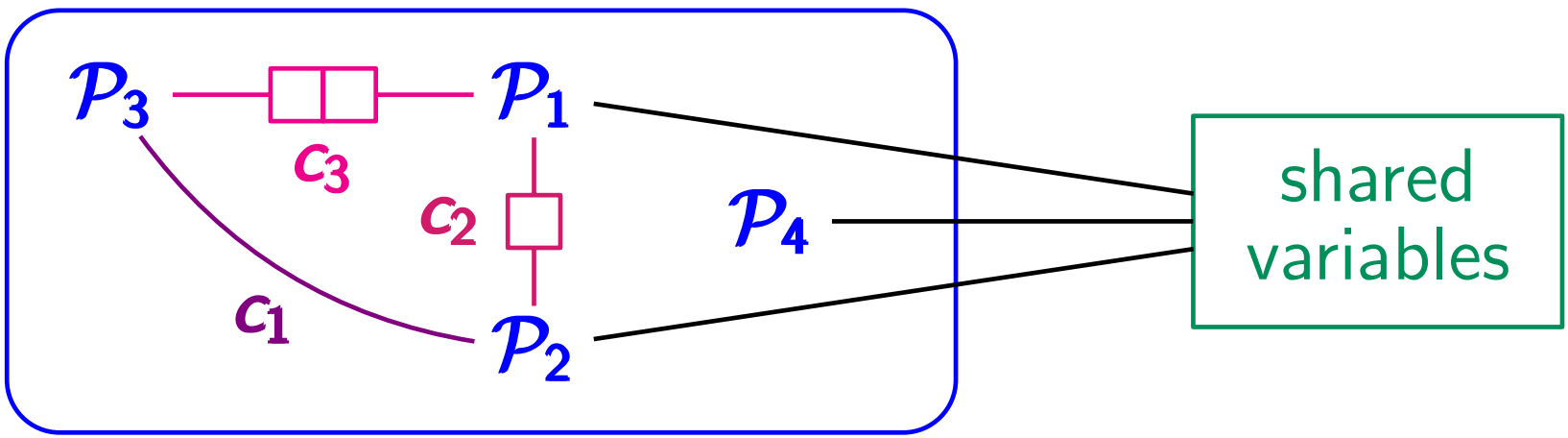

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capacity $\widehat{=}$ number of buffer cells

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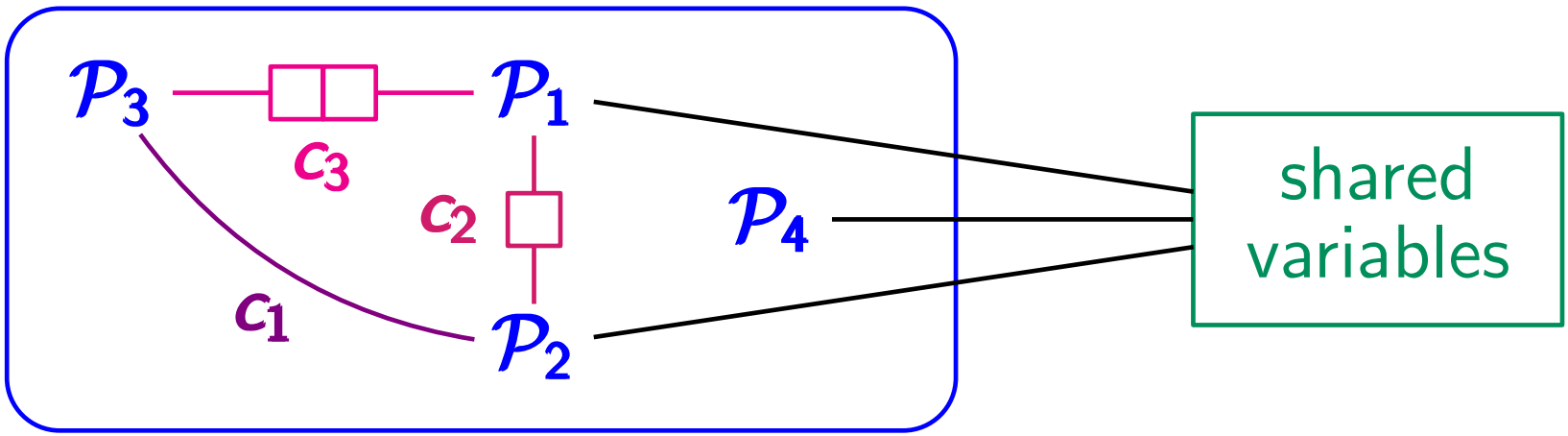
- synchronous message passing ⟵ capacity **0**
- asynchronous message passing ⟵ capacity $\geq$ **1**



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- with conditional transitions $\ell_i \xrightarrow{g:\alpha} \ell_i'$ (as before)
- and communication actions

$\ell_i \xrightarrow{c!v} \ell_i'$	sending value v via channel c
$\ell_i \xrightarrow{c?x} \ell_i'$	receiving a value for variable x via channel c

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s.t. $\xi(c)$ is a word over $Dom(c)$ of length $\leq cap(c)$

Channel system (CS)

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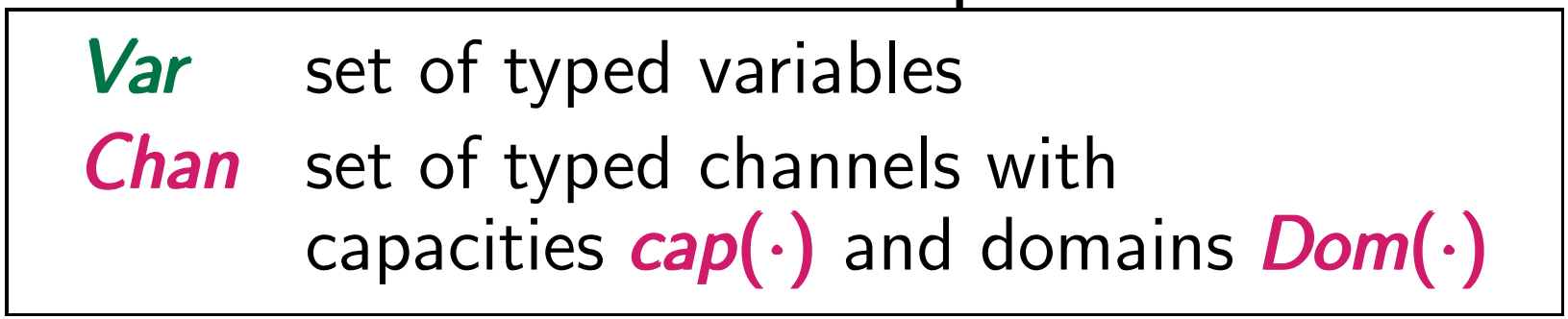

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program graphs $\mathcal{P}_i = (\mathit{Loc}_i, \mathit{Act}_i, \mathit{Effect}_i, \hookrightarrow_i, \mathit{Loc}_{0,i}, g_0)$ with conditional transitions

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Effect of communication actions in CS

asynchronous message passing via channels of capacity ≥ 1

	enabled if ...	effect
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receiving $c?x$		

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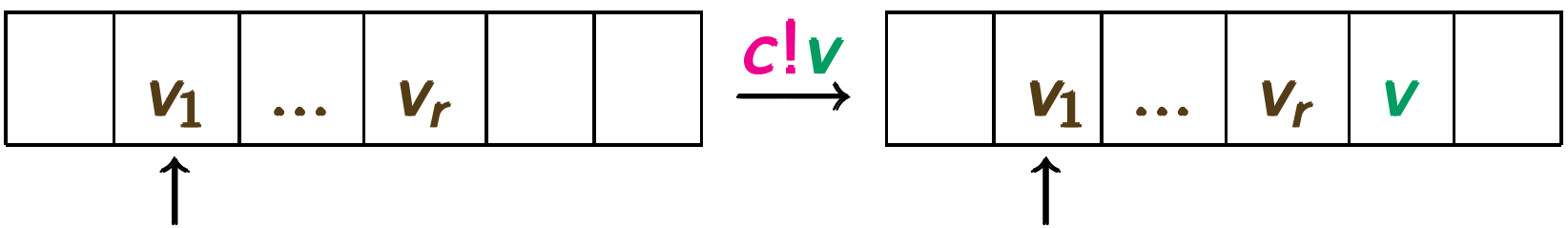

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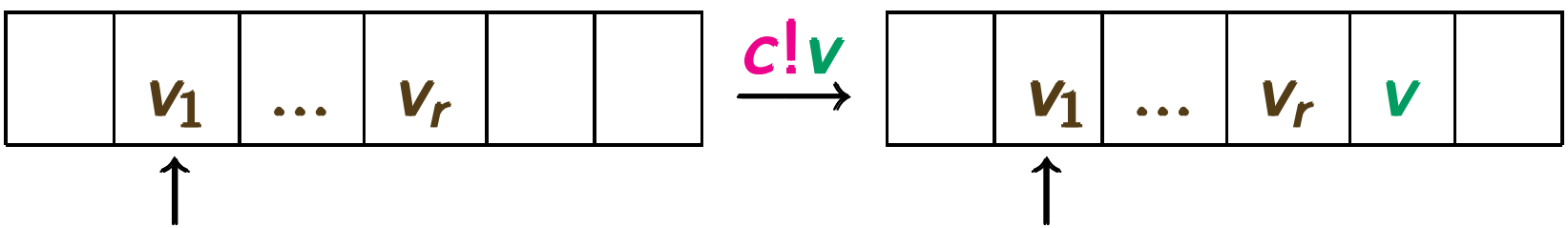

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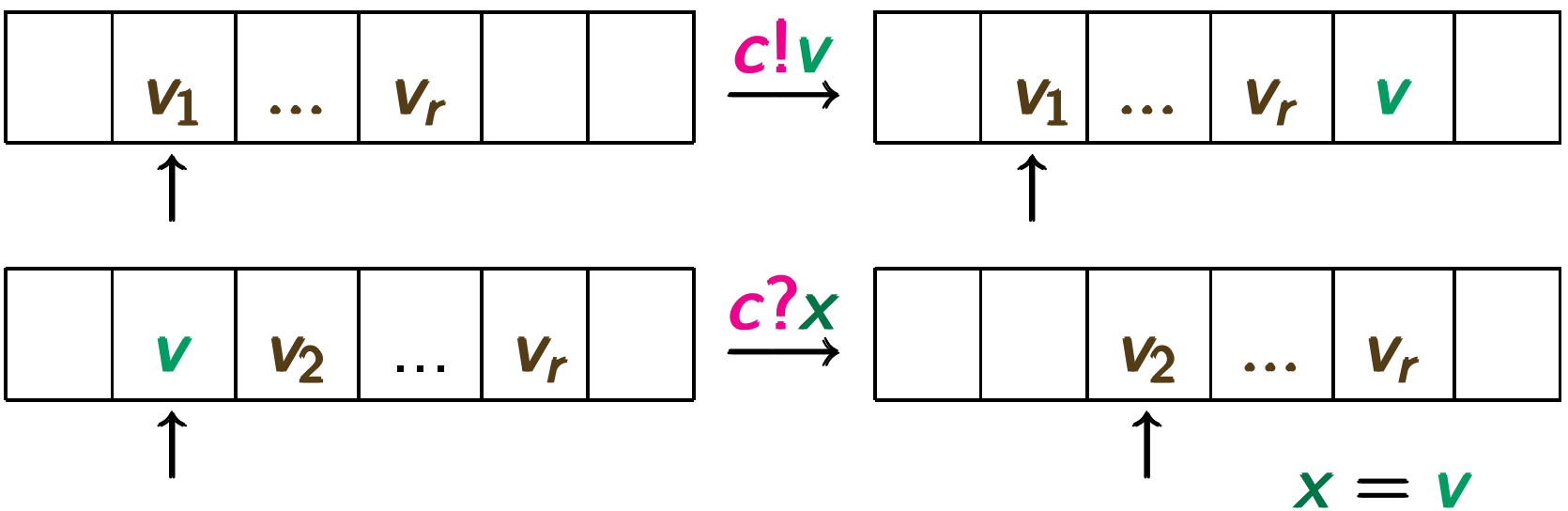

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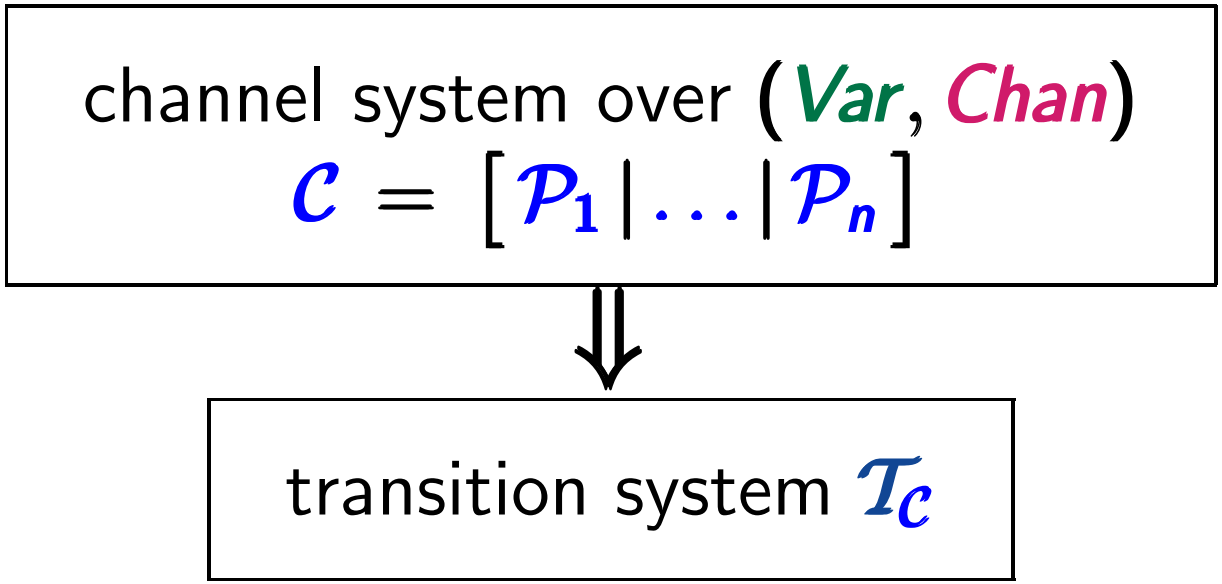
synchronous message passing via channels of capacity 0

- $c!v$ and $c?x$ are executed at the same time
- effect $x := v$

TS-semantics of channel systems



TS-semantics of channel systems

channel system over $(Var, Chan)$

$$\mathcal{C} = [\mathcal{P}_1 | \ldots | \mathcal{P}_n]$$

$\Downarrow$

transition system $\mathcal{T}_{\mathcal{C}}$

states of $\mathcal{T}_{\mathcal{C}}$ have the form

$$\langle \ell_1, \ldots, \ell_n, \eta, \xi \rangle$$

- $\ell_1, \ldots, \ell_n$: locations of $\mathcal{P}_1, \ldots, \mathcal{P}_n$
- η: variable valuation
- ξ: channel evaluation

TS-semantics of channel systems

states $\langle \ell_1, \ldots, \ell_n, \eta, \xi \rangle$ where

- ℓ_i location of program graph $\mathcal{P}_i$,
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variable evaluation:

$$\eta : \mathit{Var} \longrightarrow \bigcup_{x \in \mathit{Var}} \mathit{Dom}(x) \quad \text{with } \eta(x) \in \mathit{Dom}(x)$$

channel evaluation:

$$\xi : \mathit{Chan} \rightarrow \bigcup_{c \in \mathit{Chan}} \mathit{Dom}(c)^* \quad \text{with } \xi(c) \in \mathit{Dom}(c)^* \text{ and } |\xi(c)| \leq \mathit{cap}(c)$$

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only channels c with $\mathit{cap}(c) \geq 1$ are relevant

Transition relation of channel systems

states $\langle \ell_1, \ldots, \ell_n, \eta, \xi \rangle$ where $\ell_i \in \mathit{Loc}_i$, $\eta \in \mathit{Eval}(\mathit{Var})$, $\xi \in \mathit{Eval}(\mathit{Chan})$

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interleaving rule for actions $\alpha \in \mathit{Act}_i$:

$$\frac{\ell_i \xhookrightarrow{g:\alpha}_i \ell_i' \ \wedge \ \eta \models g}{\langle \ell_1, .., \ell_i, \ldots, \ell_n, \eta, \xi \rangle \xrightarrow{\alpha} \langle \ell_1, .., \ell_i', \ldots, \ell_n, \mathit{Effect}_i(\alpha, \eta), \xi \rangle}$$

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$\uparrow$

does not affect the channel evaluation ξ

SOS-rules for asynchronous message passing

for channel c with $cap(c) \geq 1$

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$$\frac{\ell_i \xrightarrow{c?x}_i \ell_i' \ \wedge \ \xi(c) = v_1 v_2 \ldots v_k \ \wedge \ k \geq 1}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_n, \eta, \xi \rangle \xrightarrow{\tau} \langle \ell_1, \ldots, \ell_i', \ldots, \ell_n, \eta', \xi' \rangle}$$

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where $\eta' = \eta[x{:=}v_1]$

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sending a message:

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has a transition system for a channel system with ... ?

- 2 processes with 2 locations each
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note: $2^{11}-1 = 1+2+2^2+...+2^{10}$

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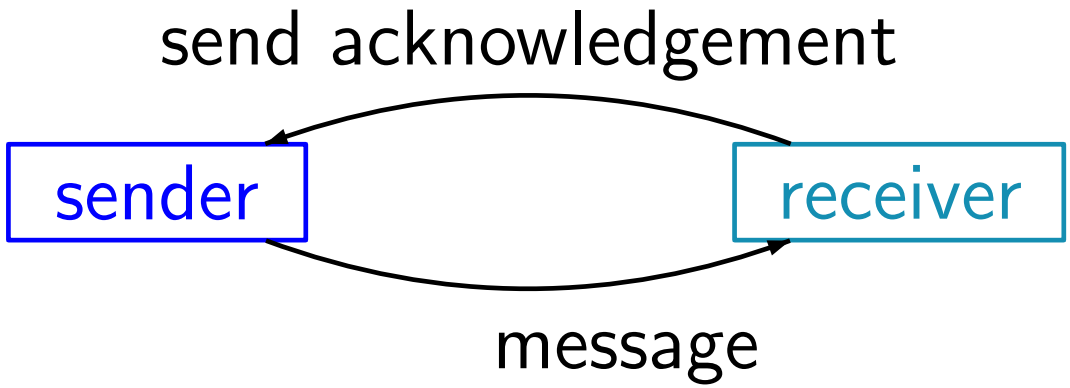
Alternating bit protocol (ABP)



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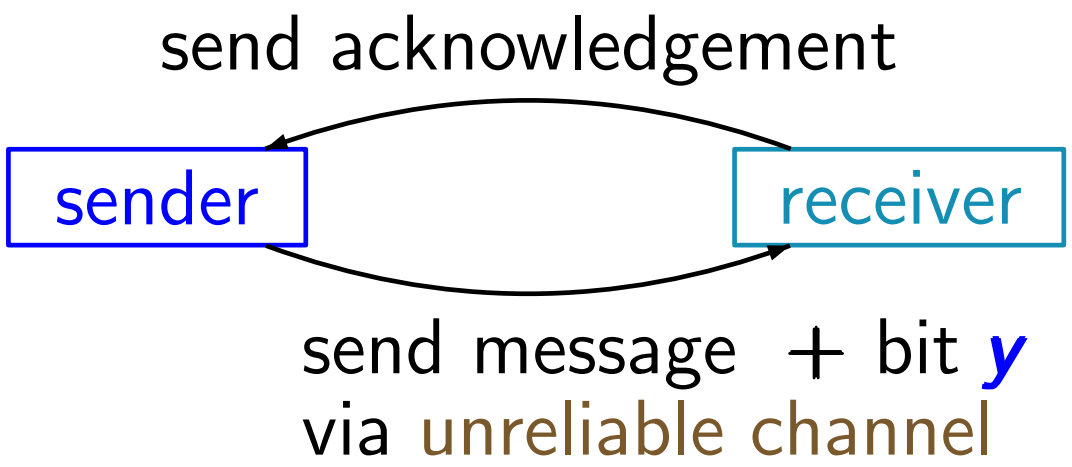

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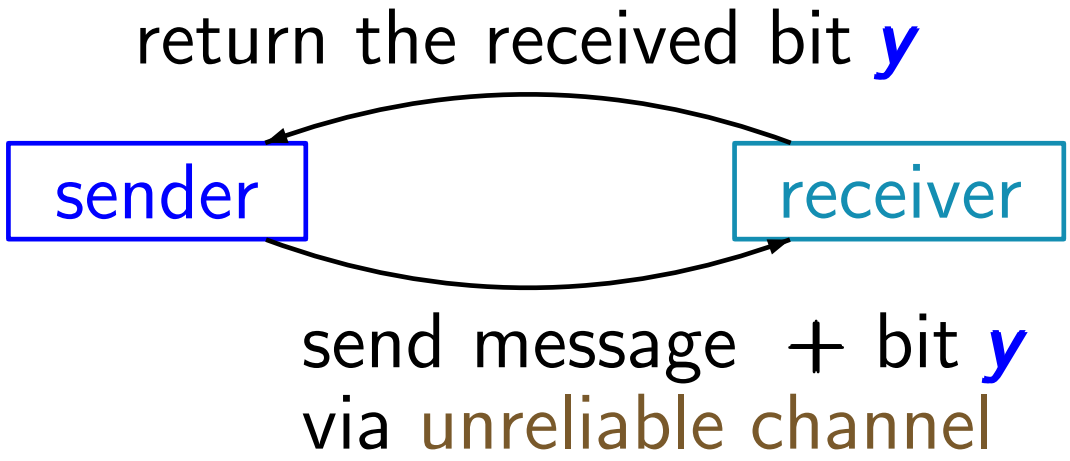

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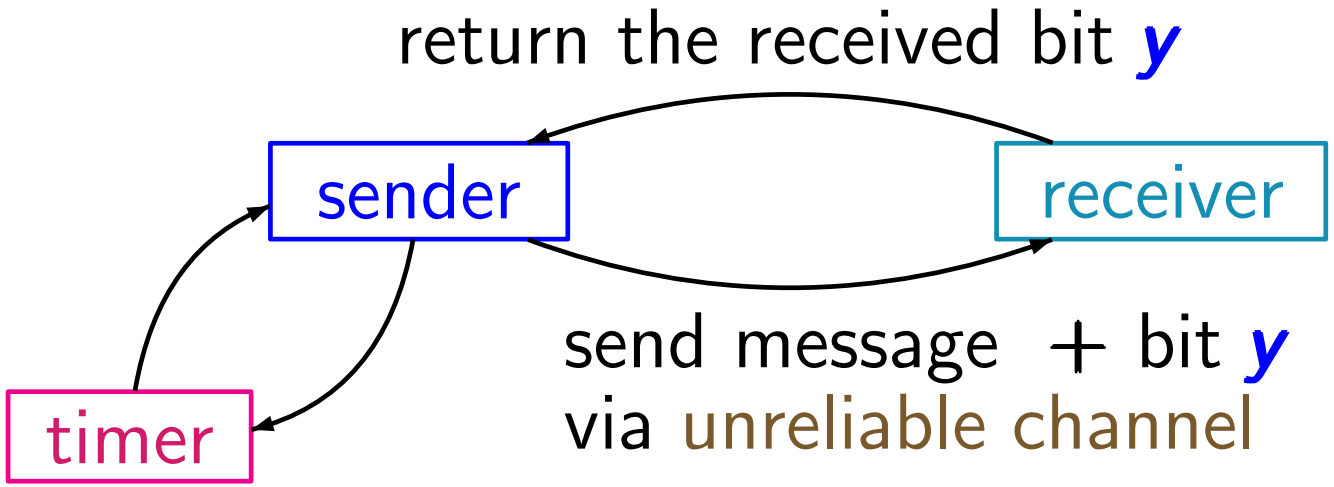
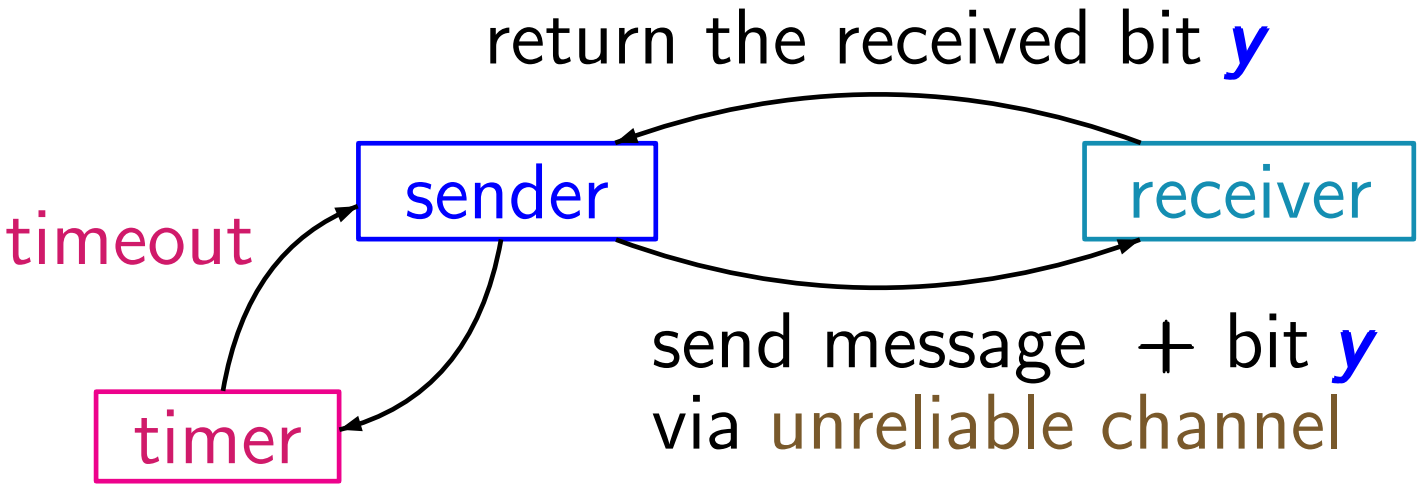


Protocol for the sender




```
LOOP FOREVER
(1)  send message + bit y and activate timer
(2)  AWAIT timeout or acknowledgement DO
          IF timeout THEN goto (1)
                     ELSE turn off timer; y:=¬y
          FI
     OD
```

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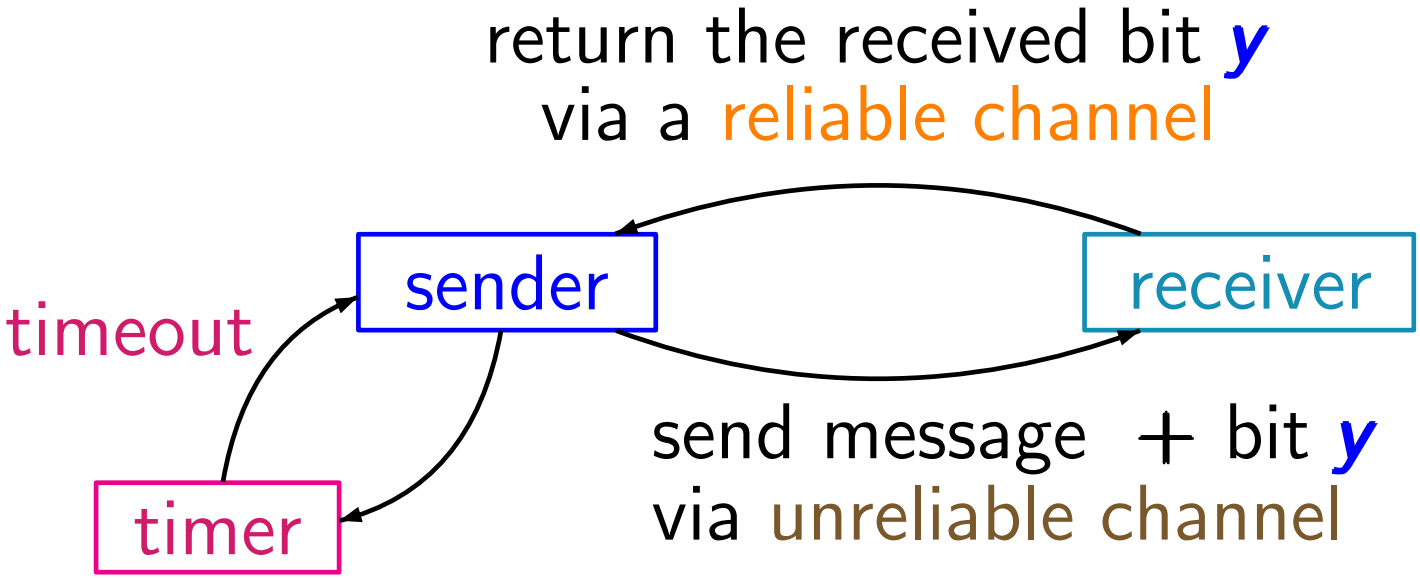


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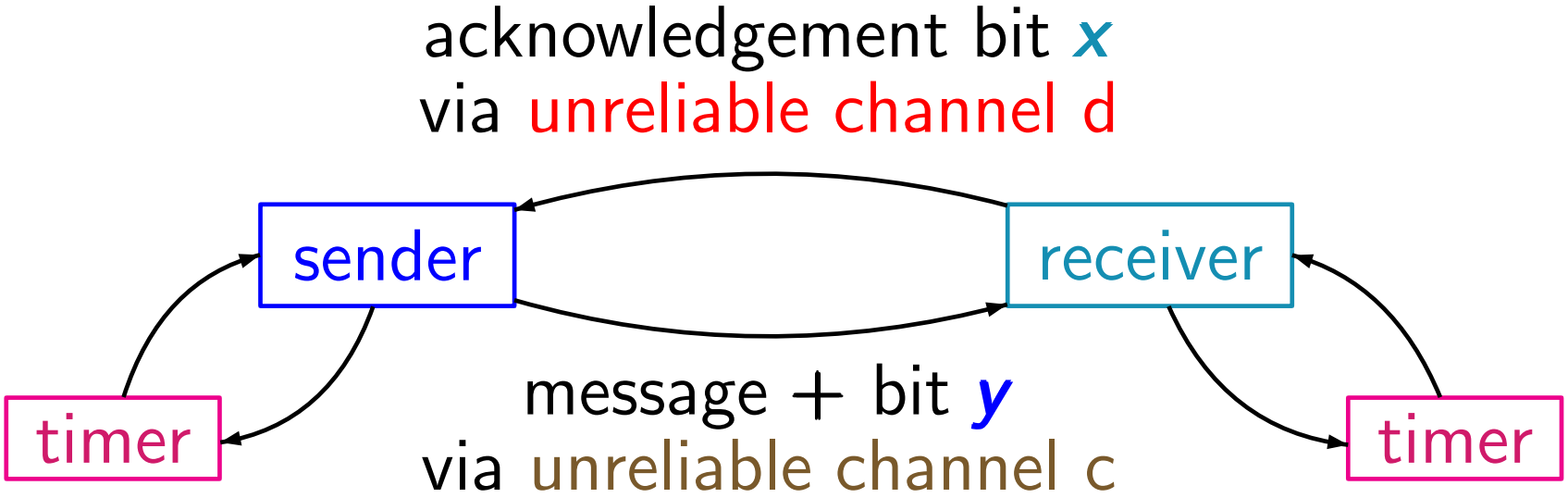
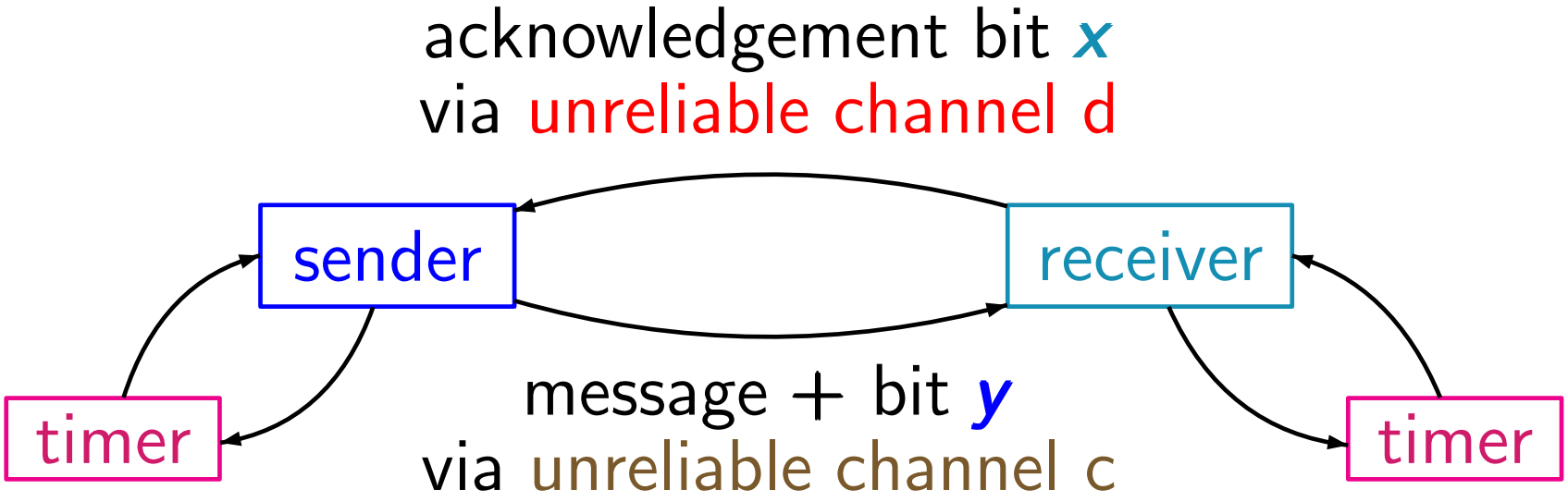
```

If both channels are unreliable ...

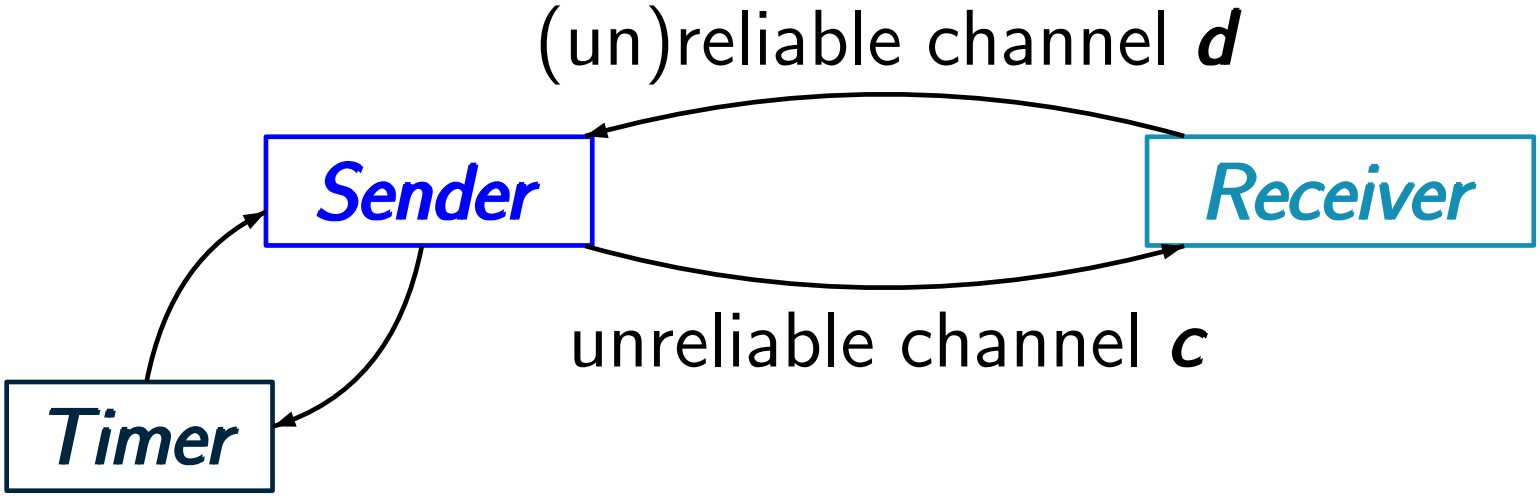


If both channels are unreliable ...



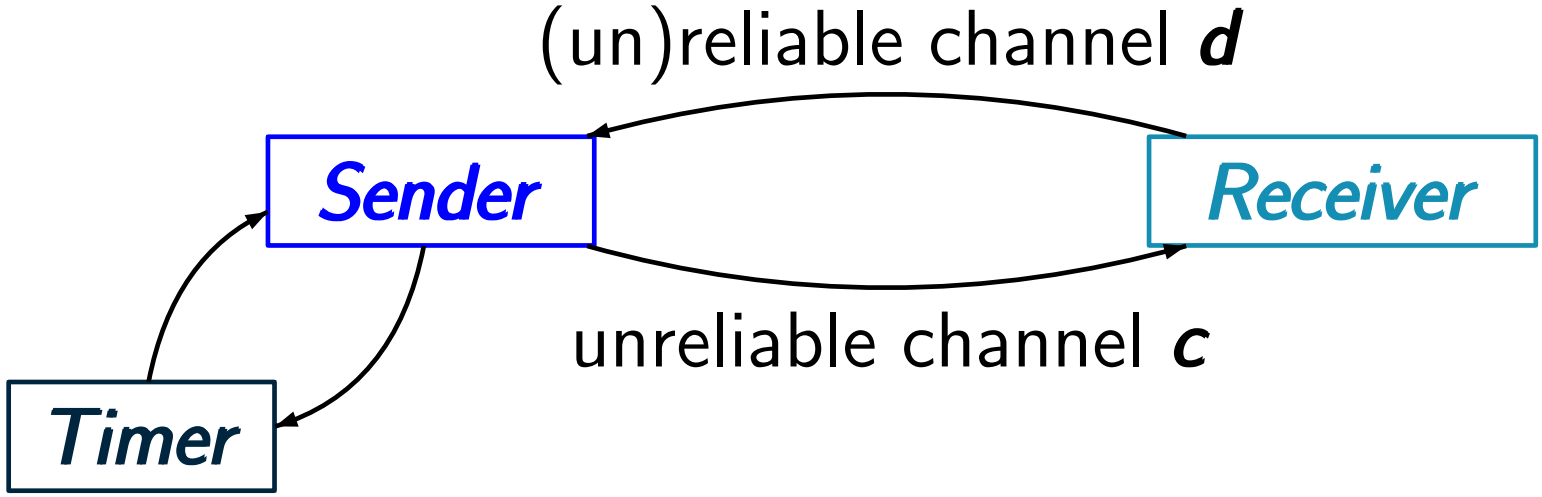

```
LOOP FOREVER
(1)  send message + bit y and activate timer
(2)  AWAIT  timeout or acknowledgement x DO
          IF  timeout THEN  goto (1)
           ELSE  IF  x=y THEN  turn off timer; y:=¬y
                             ELSE  ignore x
                 FI
          FI
     OD
```

Alternating bit protocol (ABP)



channel system: $[\,\textit{Sender} \mid \textit{Timer} \mid \textit{Receiver}\,]$

Alternating bit protocol (ABP)

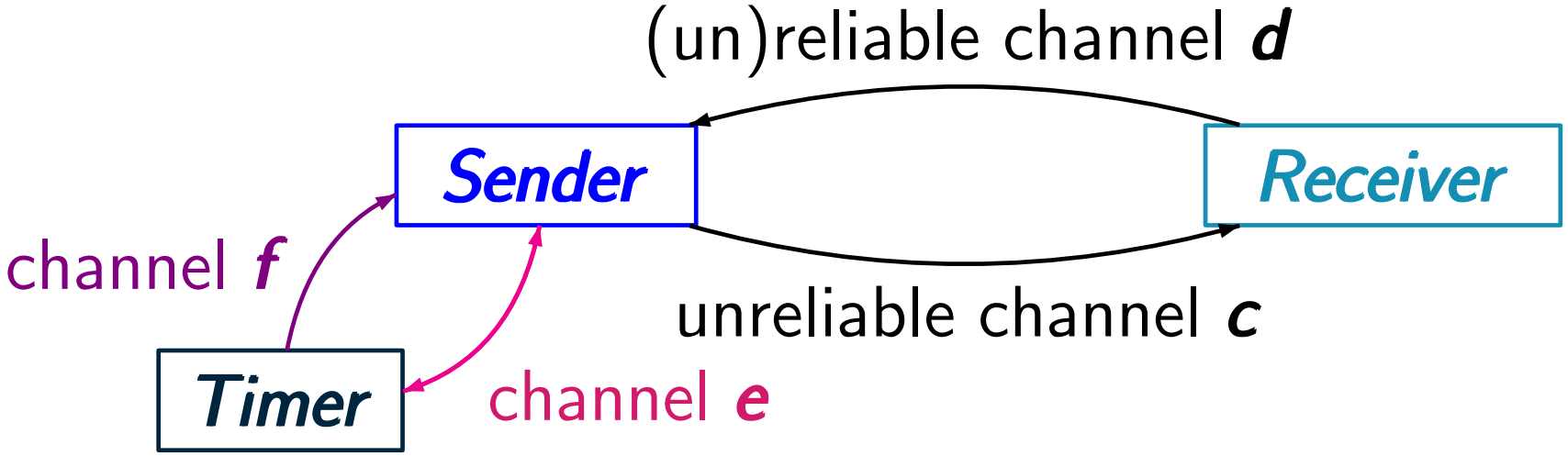


channel system: $[\ \textit{Sender} \mid \textit{Timer} \mid \textit{Receiver}\ ]$

synchronous message passing between
Timer and *Sender*

asynchronous message passing between
Receiver and *Sender*

Alternating bit protocol (ABP)

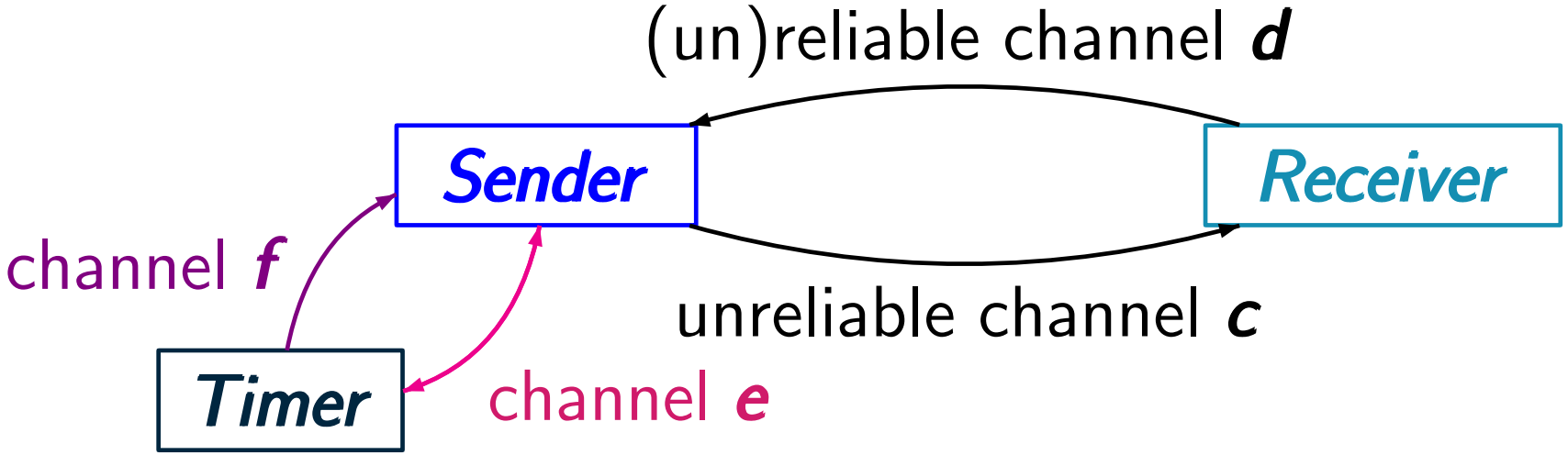


channel system: $[\ \textit{Sender} \mid \textit{Timer} \mid \textit{Receiver}\ ]$

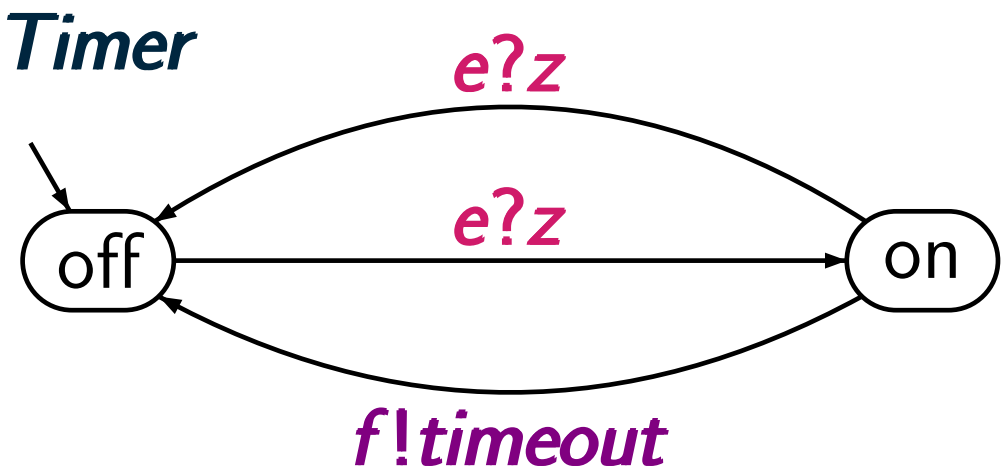
synchronous message passing between
Timer and *Sender* ← channels ***e*** and ***f***

asynchronous message passing between
Receiver and *Sender* ← channels ***c***, ***d***

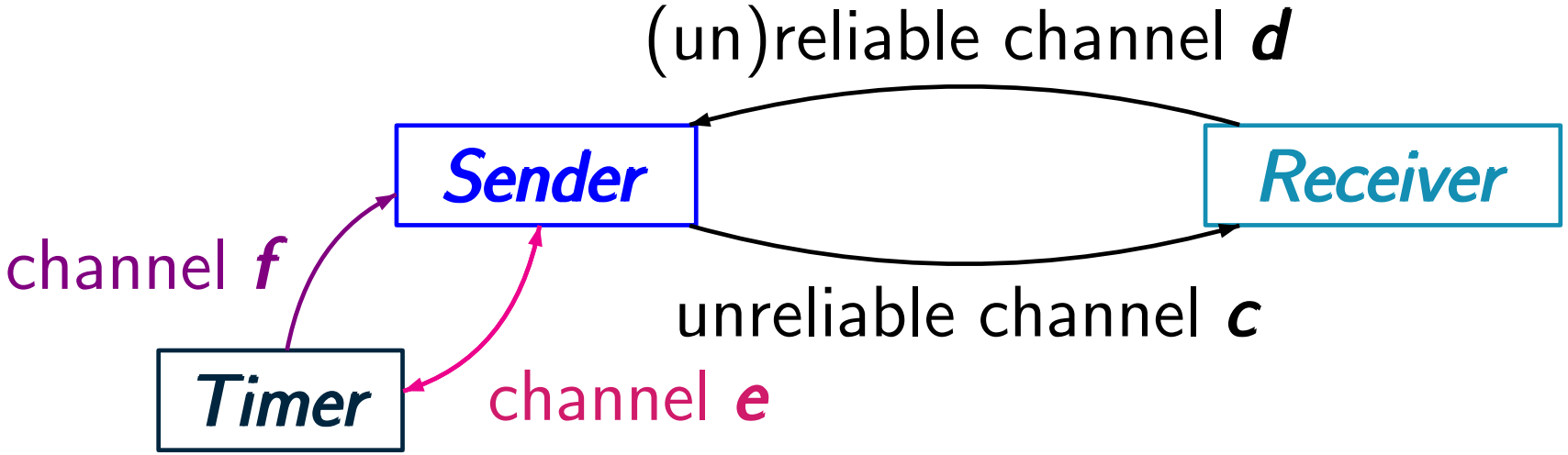
Alternating bit protocol (ABP)



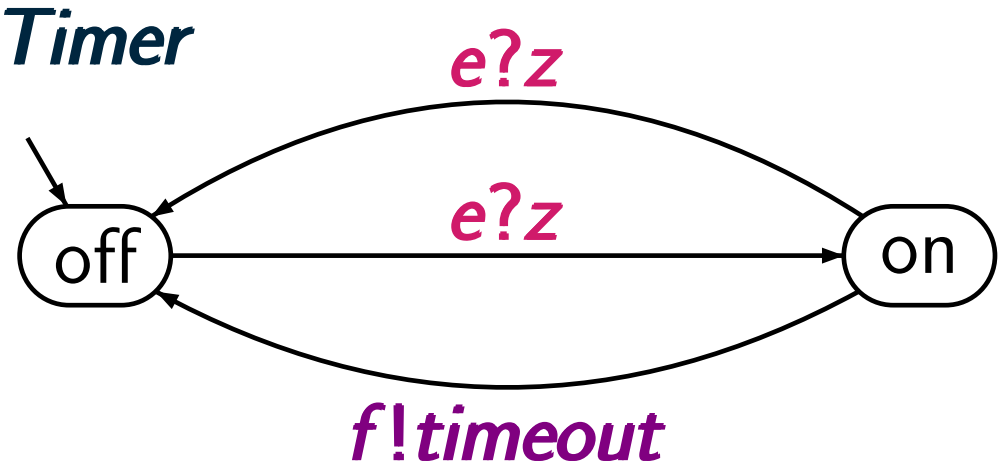
channel system: [*Sender* | *Timer* | *Receiver*]



Alternating bit protocol (ABP)



channel system: $[\ Sender \mid Timer \mid Receiver\ ]$



actions of *Sender*:

$\vdots$

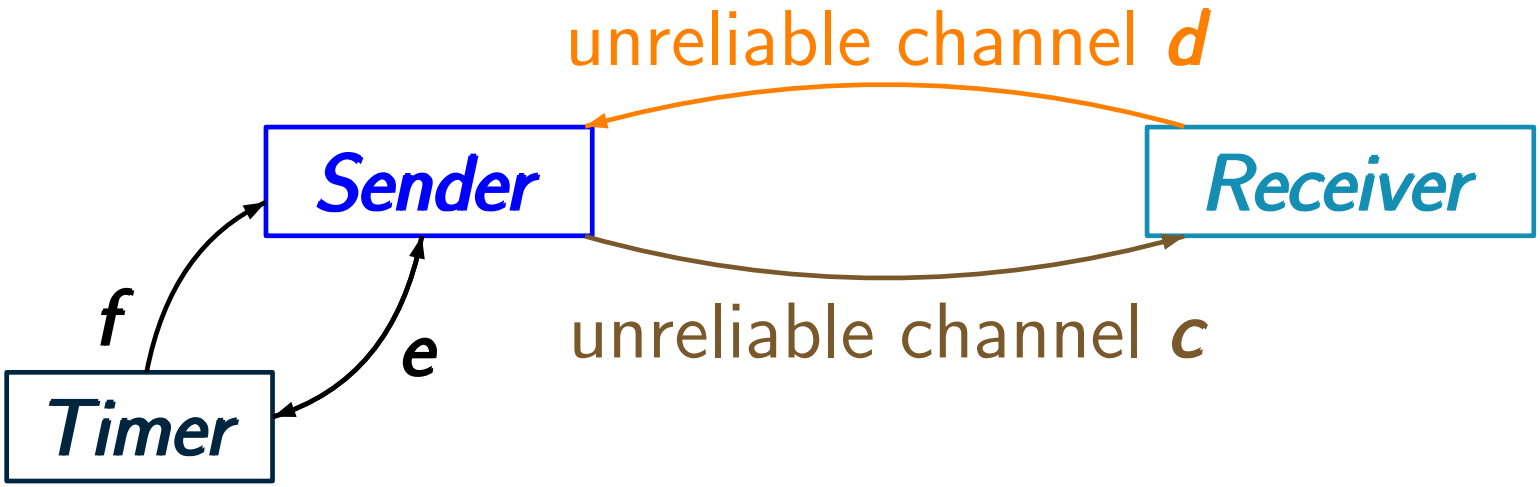
$e!timer\_on$

$e!timer\_off$

$f?z'$

$\vdots$

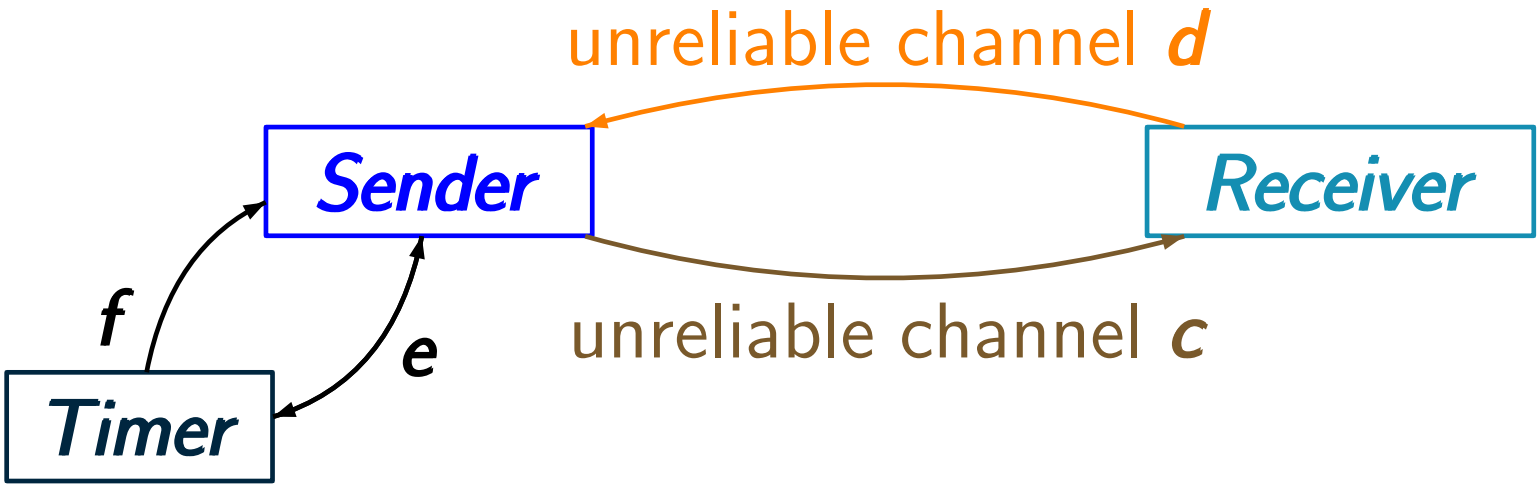
Program graph for the sender



specify the sender by a program graph using

- asynchronous channels ***c*** and ***d***
- synchronous channels ***e*** and ***f***

Program graph for the sender

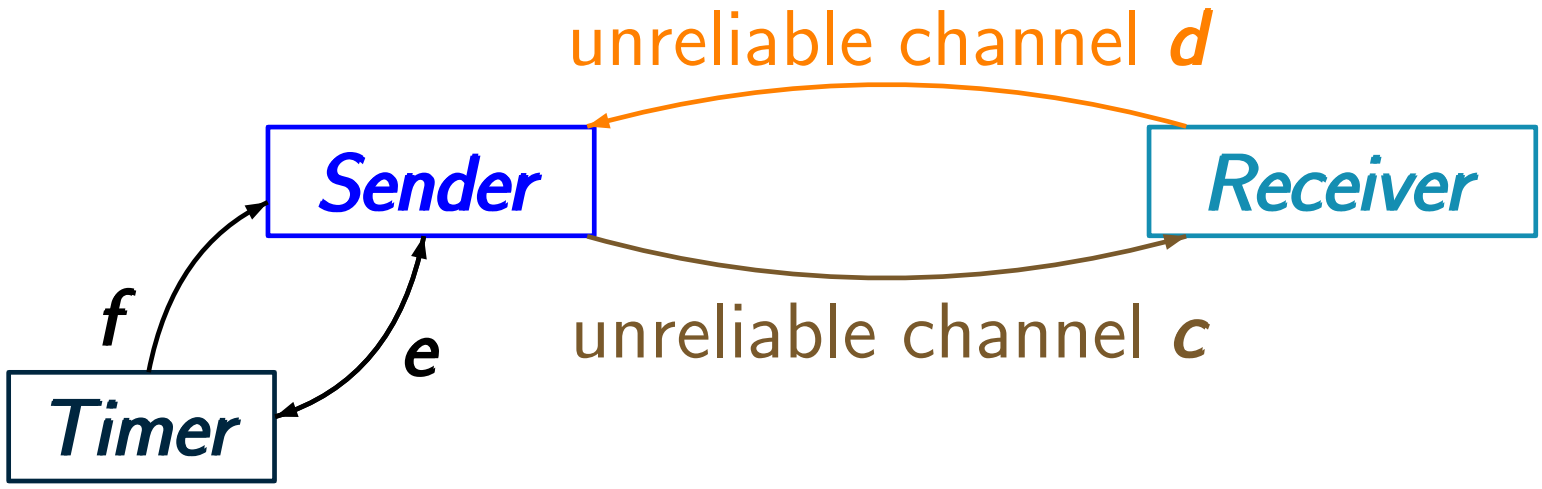


specify the sender by a program graph using

- asynchronous channels *c* and *d*
- synchronous channels *e* and *f*

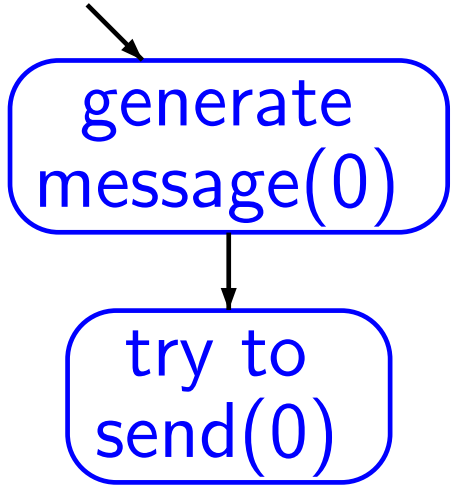
simply write *!timeout*
?timer_on
?timer_off

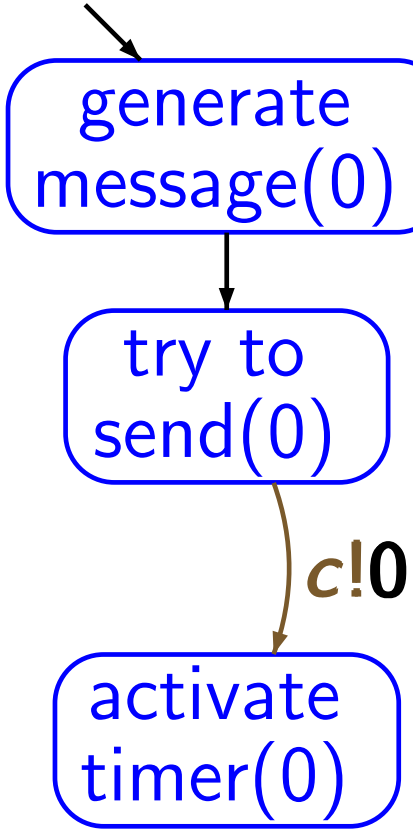
Program graph for the sender

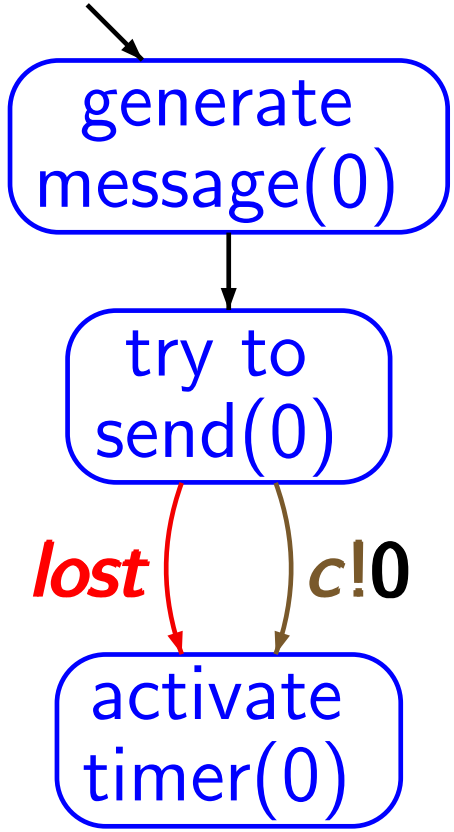


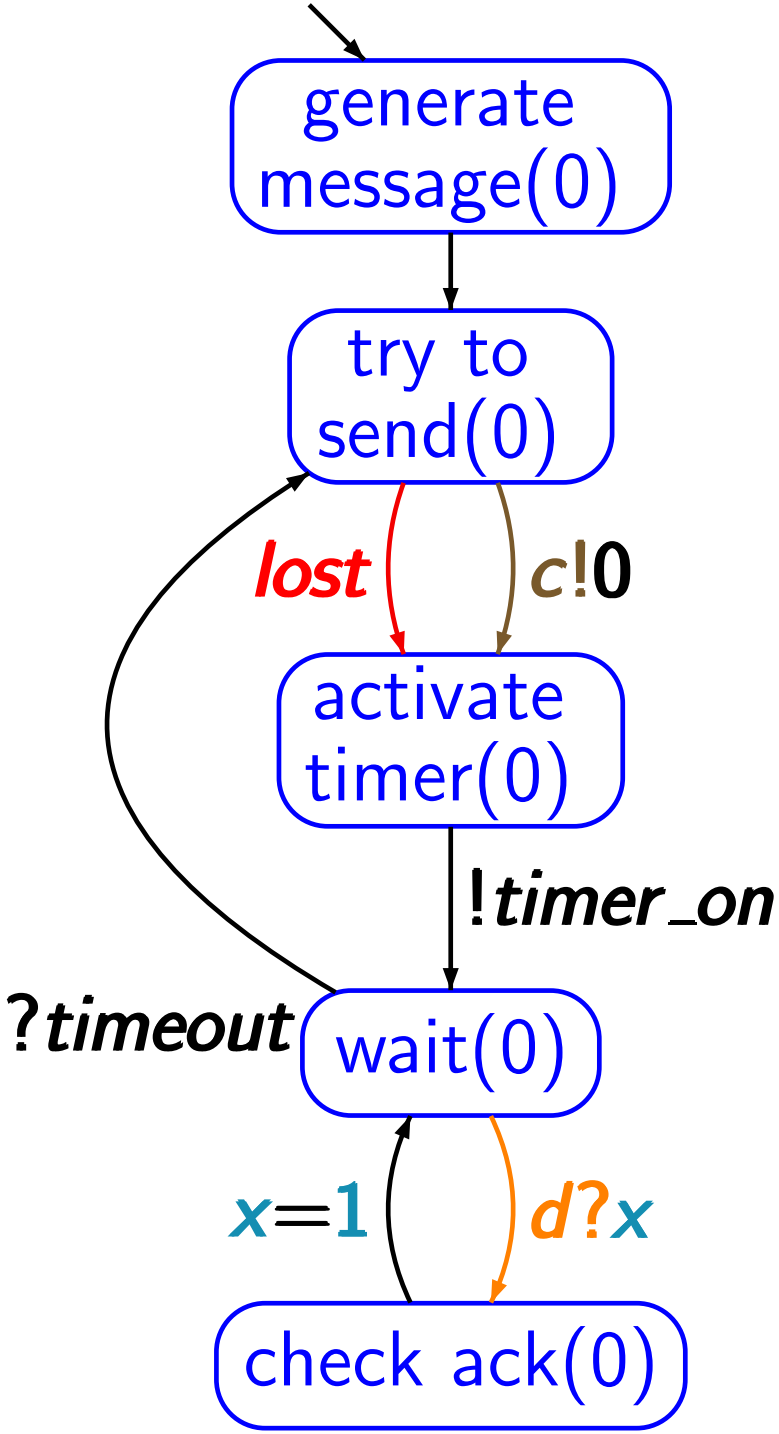
specify the sender by a program graph using

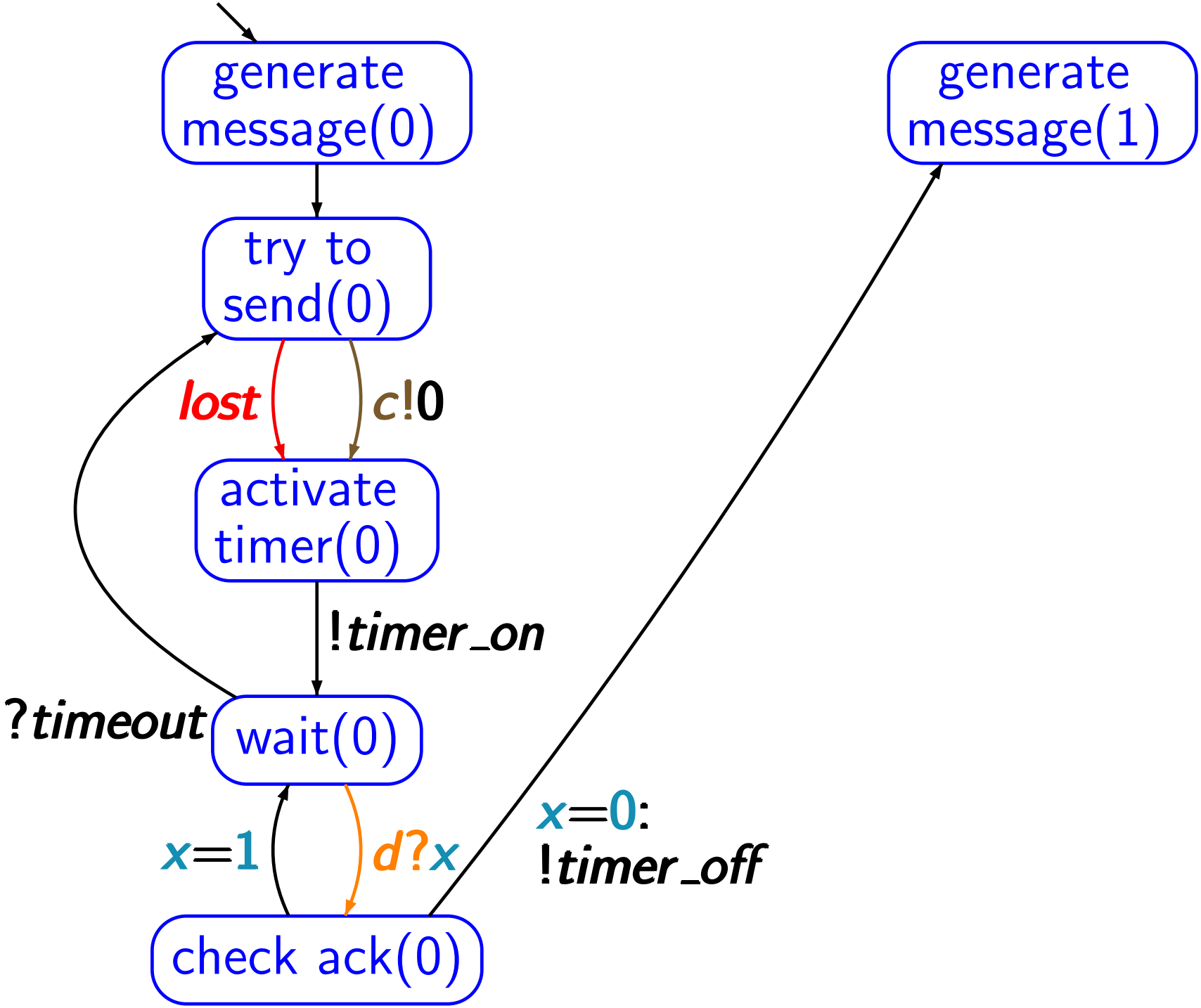
- asynchronous channels ***c*** and ***d***
- synchronous channels ***e*** and ***f***
- Boolean variable ***x*** for the acknowledgement bit sent by the receiver

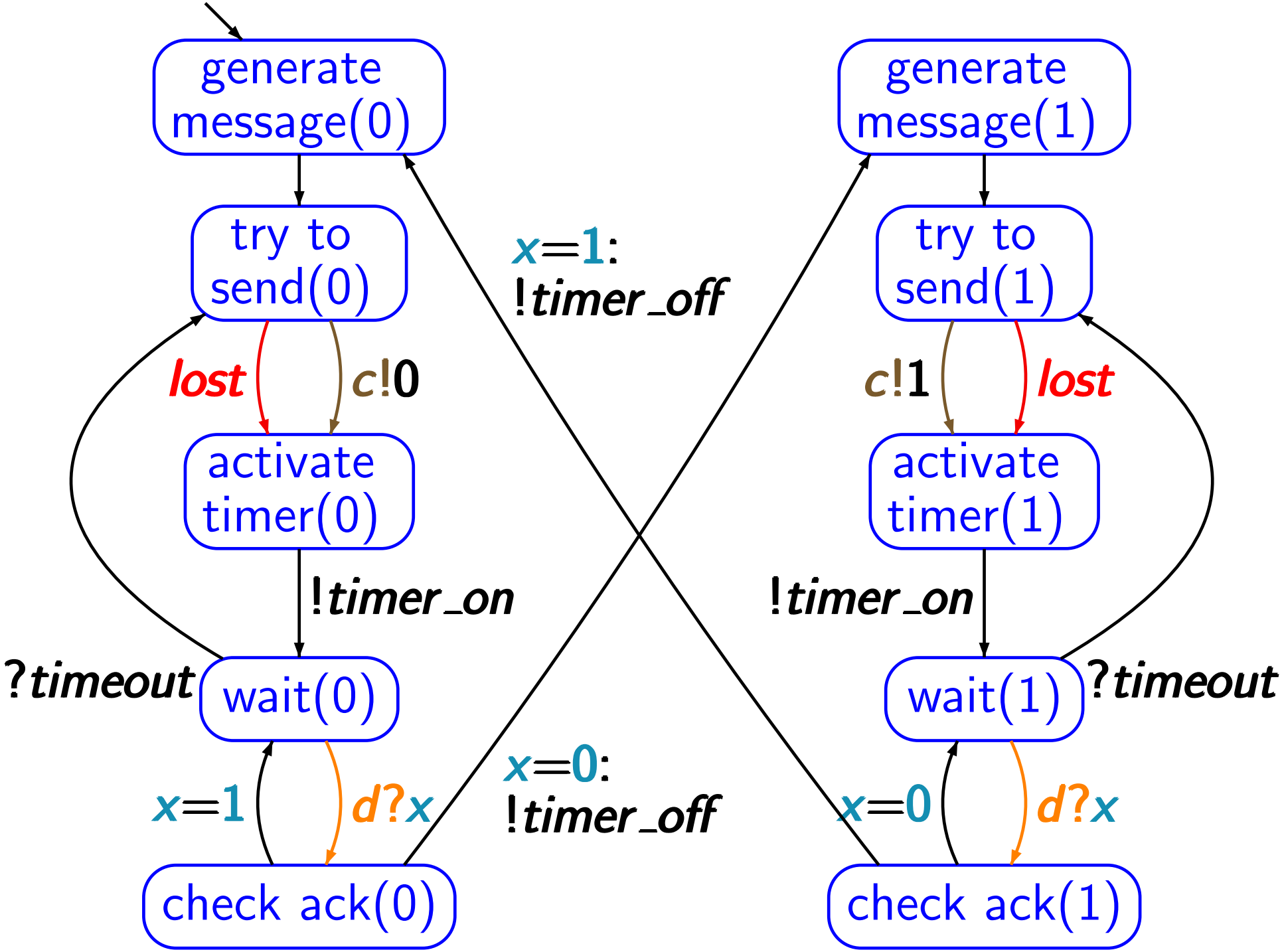

generate
message(0)
try to
send(0)


generate
message(0)
try to
send(0)
c!0
activate
timer(0)

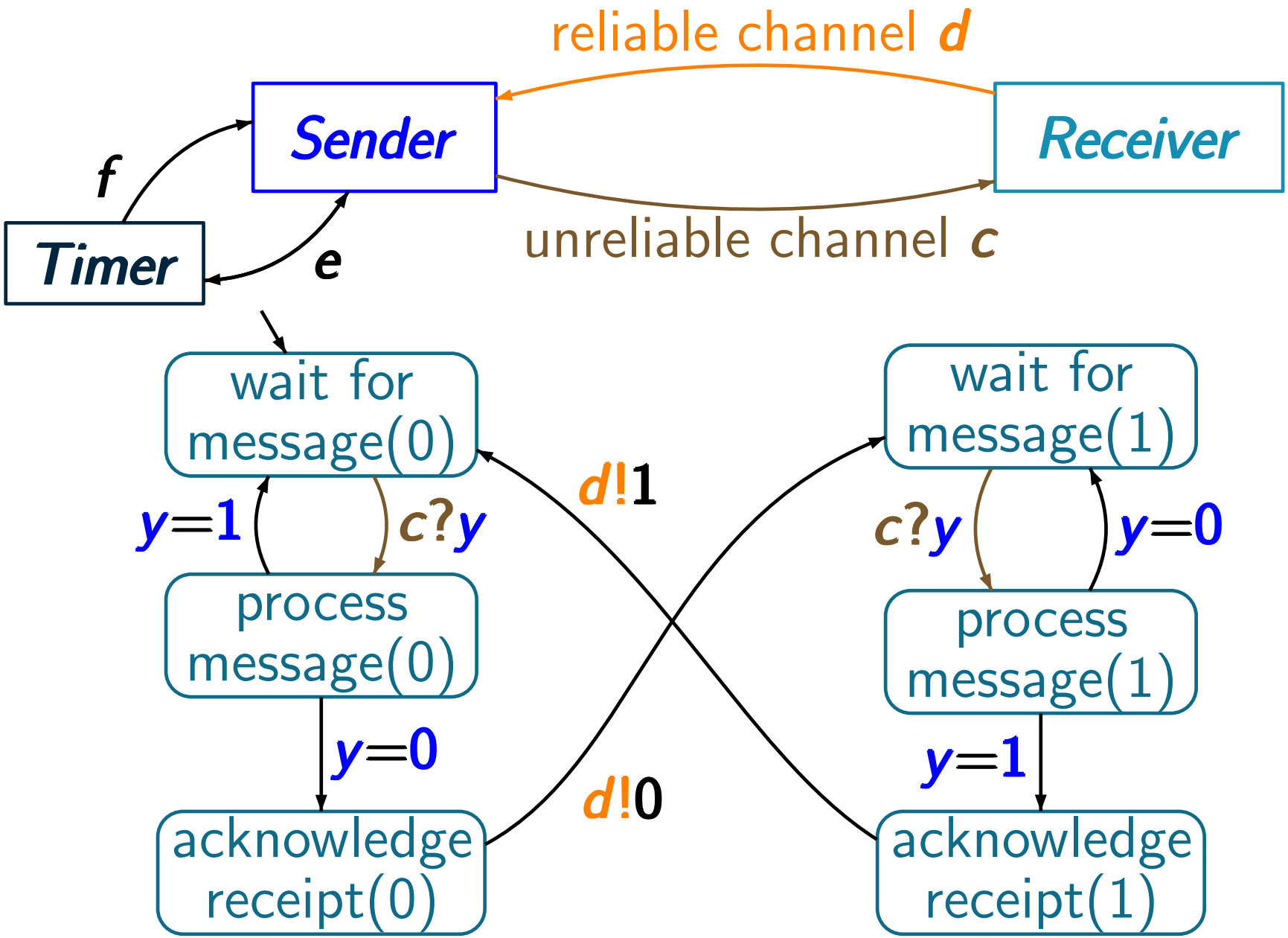

generate
message(0)
try to
send(0)
lost
c!0
activate
timer(0)

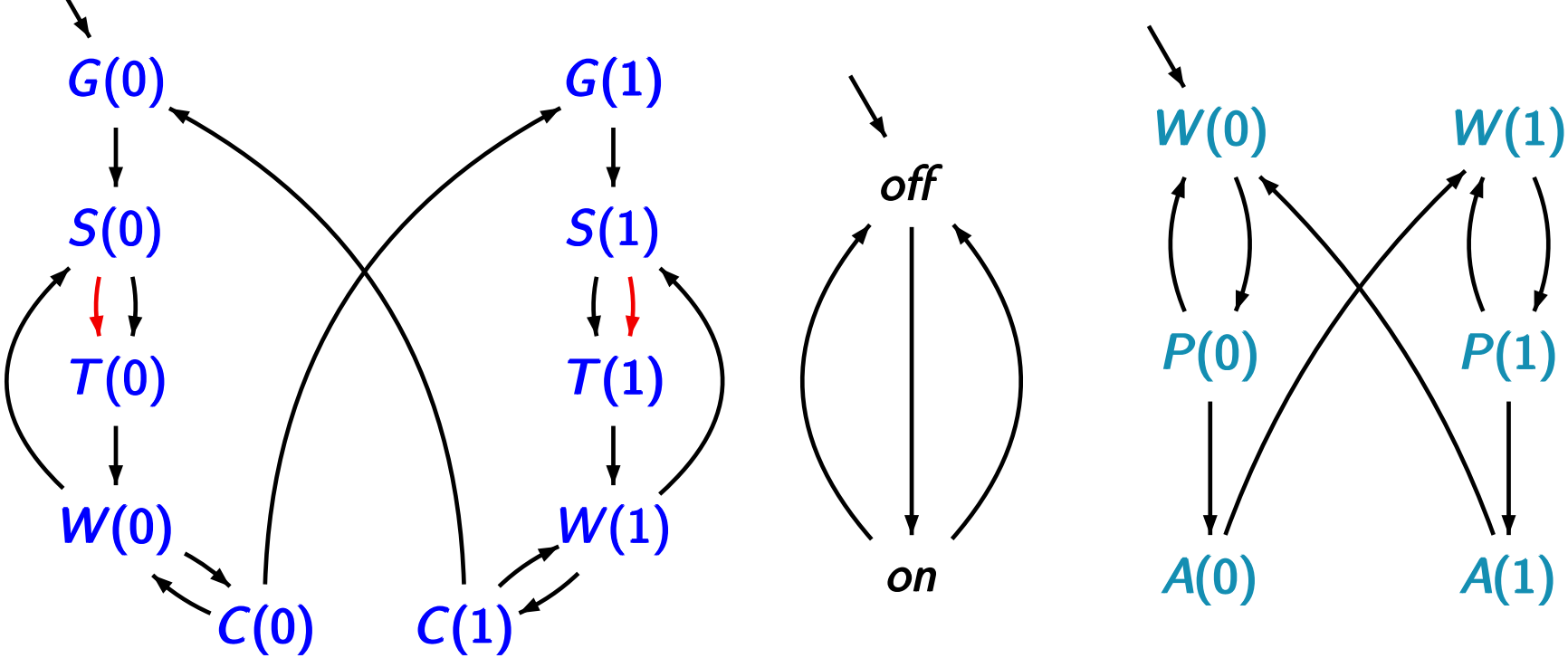

generate message(0)
try to send(0)
lost
c!0
activate timer(0)
!timer_on
?timeout
wait(0)
x=1
d?x
check ack(0)

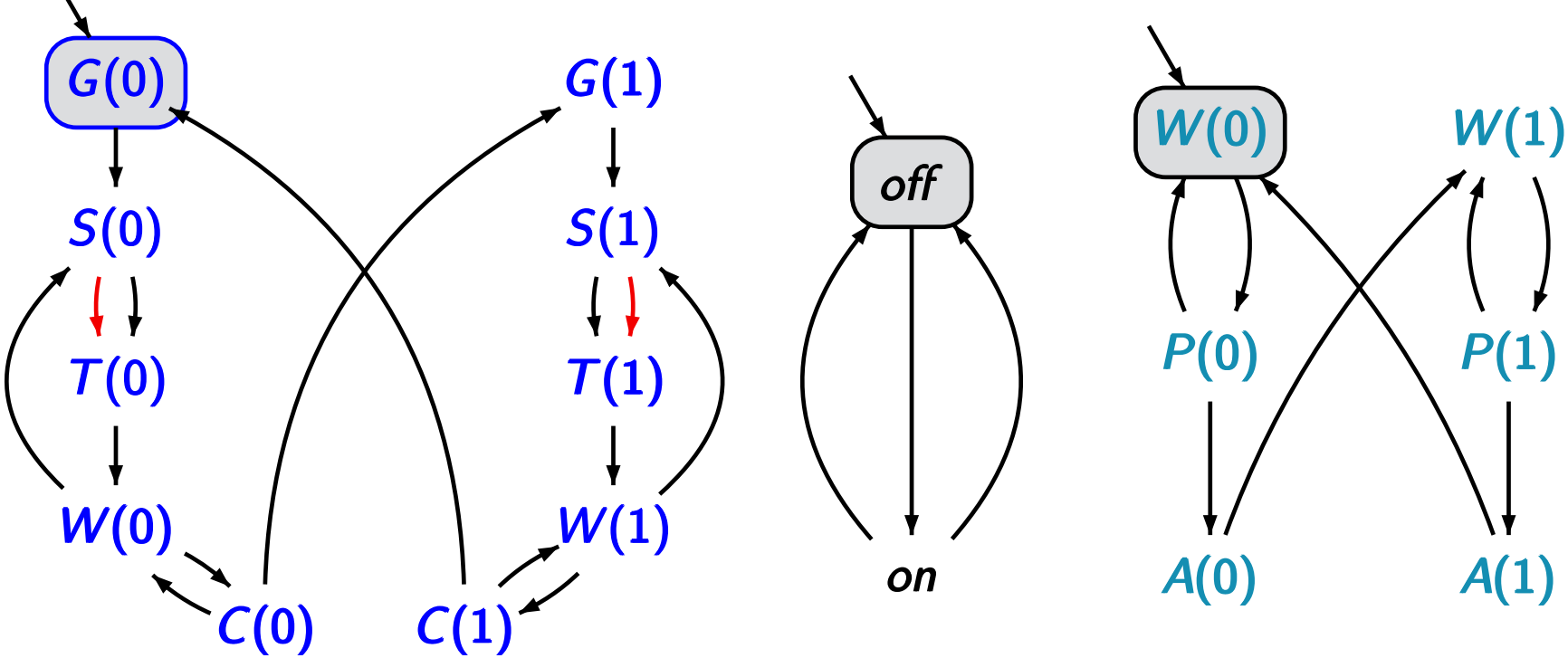

generate message(0)
try to send(0)
lost
c!0
activate timer(0)
!timer_on
?timeout
wait(0)
x=1
d?x
check ack(0)
x=0:
!timer_off
generate message(1)


generate message(0)
try to send(0)
lost
c!0
activate timer(0)
!timer_on
?timeout
wait(0)
x=1
d?x
check ack(0)
x=1:
!timer_off
x=0:
!timer_off
generate message(1)
try to send(1)
c!1
lost
activate timer(1)
!timer_on
wait(1)
?timeout
x=0
d?x
check ack(1)

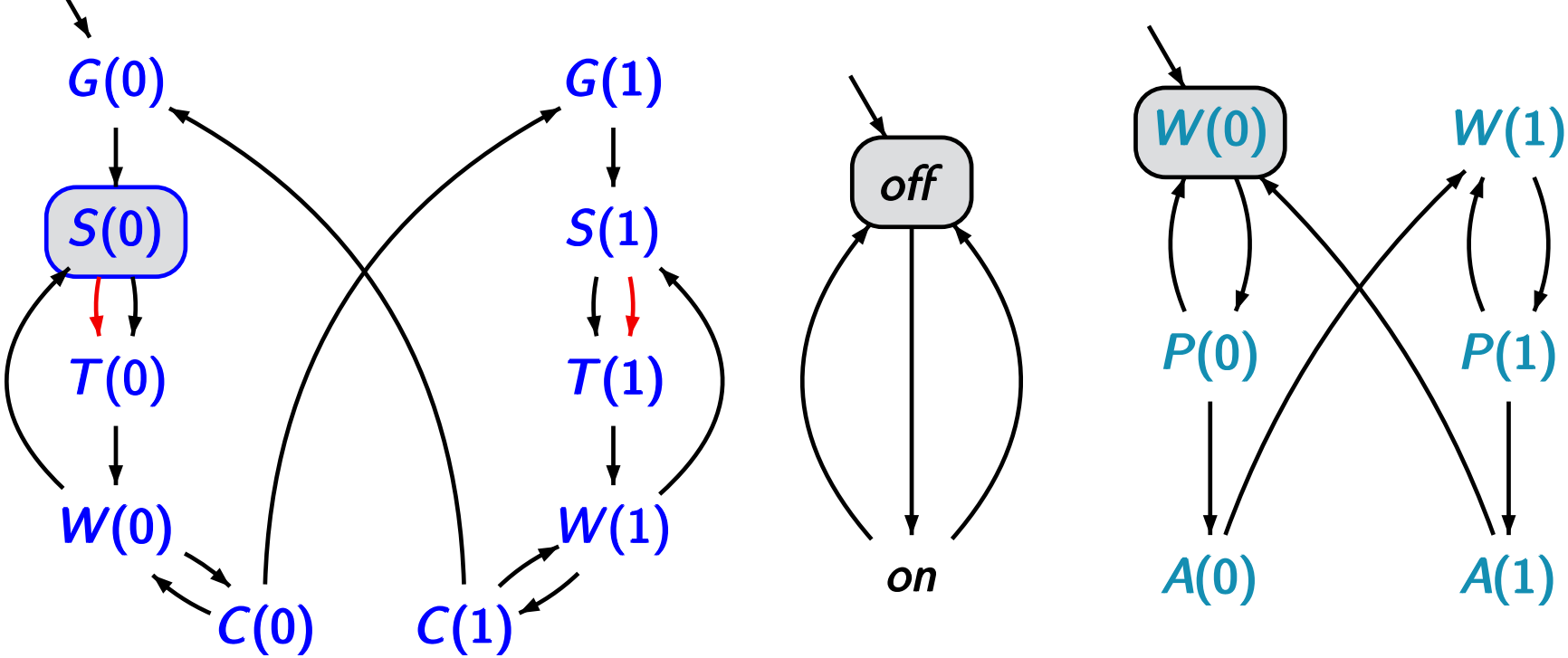
Program graph for the receiver




G(0)
S(0)
T(0)
W(0)
C(0)
G(1)
S(1)
T(1)
W(1)
C(1)
off
on
W(0)
W(1)
P(0)
P(1)
A(0)
A(1)

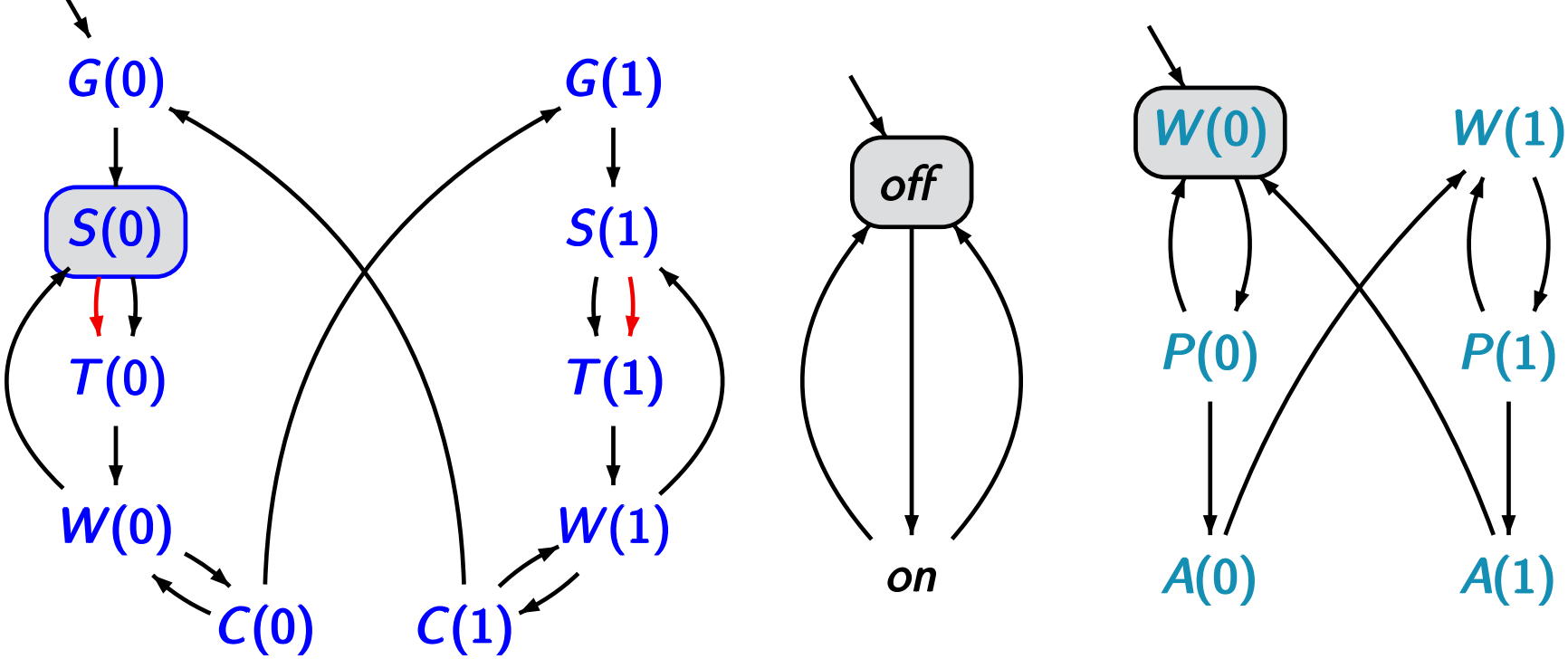


Generate(0) off Wait(0) $c{=}\varepsilon$ $d{=}\varepsilon$



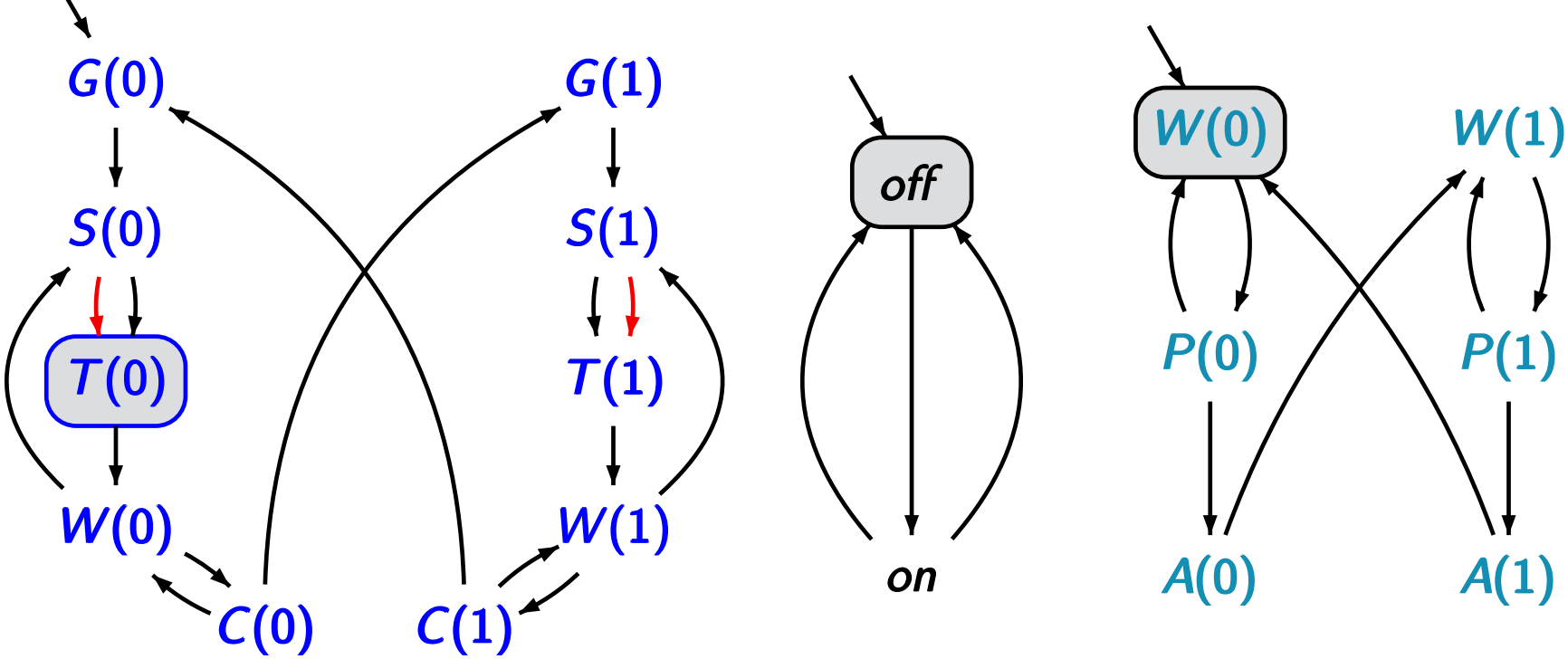
Generate(0) off Wait(0) $c{=}\varepsilon$ $d{=}\varepsilon$

Send(0) off Wait(0) $c{=}\varepsilon$ $d{=}\varepsilon$

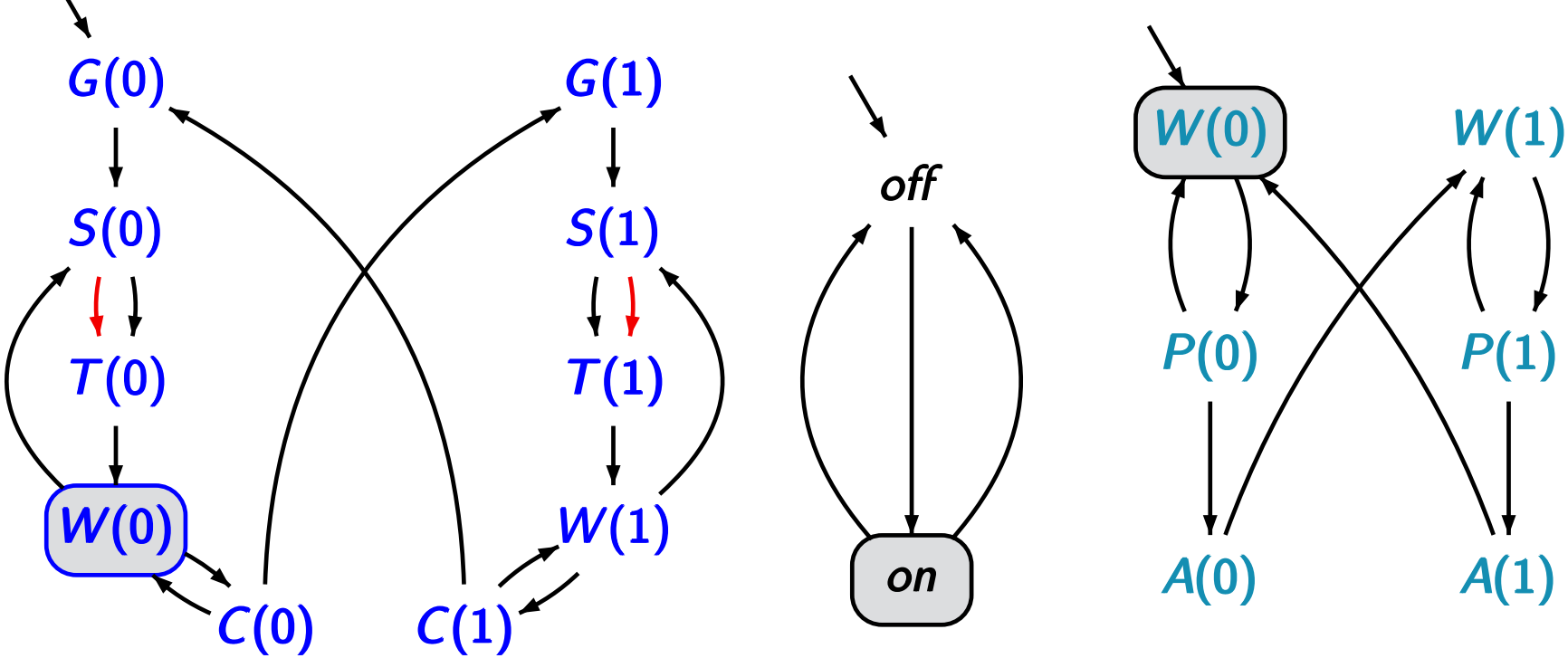


Generate(0) off Wait(0) $c{=}\varepsilon$ $d{=}\varepsilon$

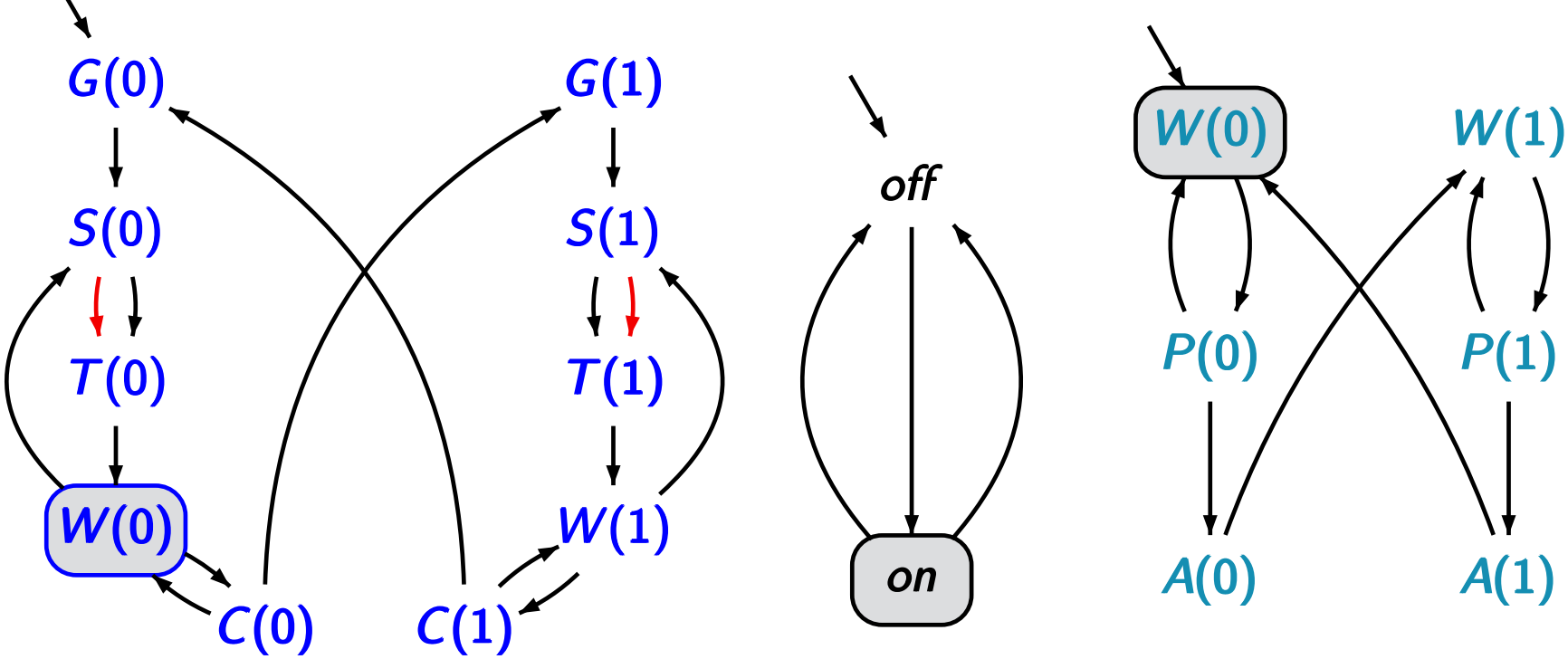
Send(0) off Wait(0) $c{=}\varepsilon$ $d{=}\varepsilon$ message lost



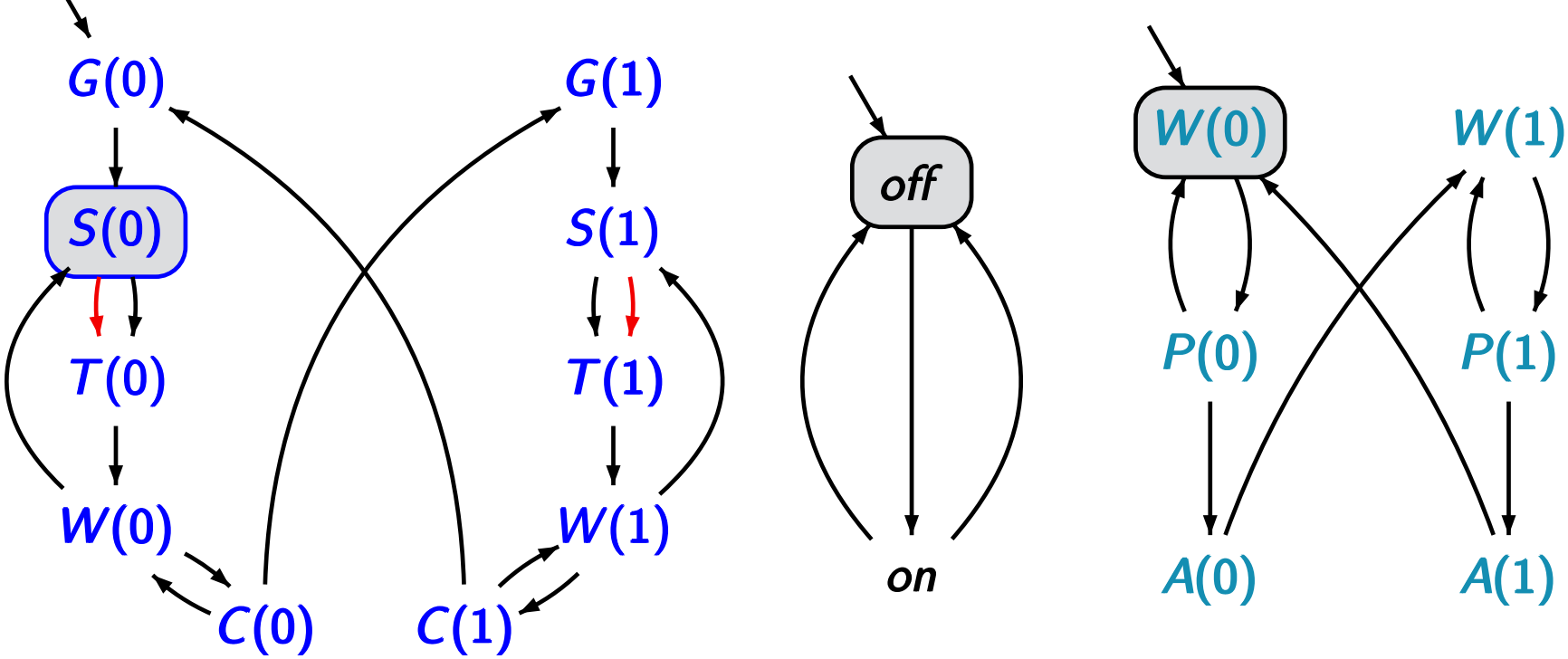
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message lost
Timer_on(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	



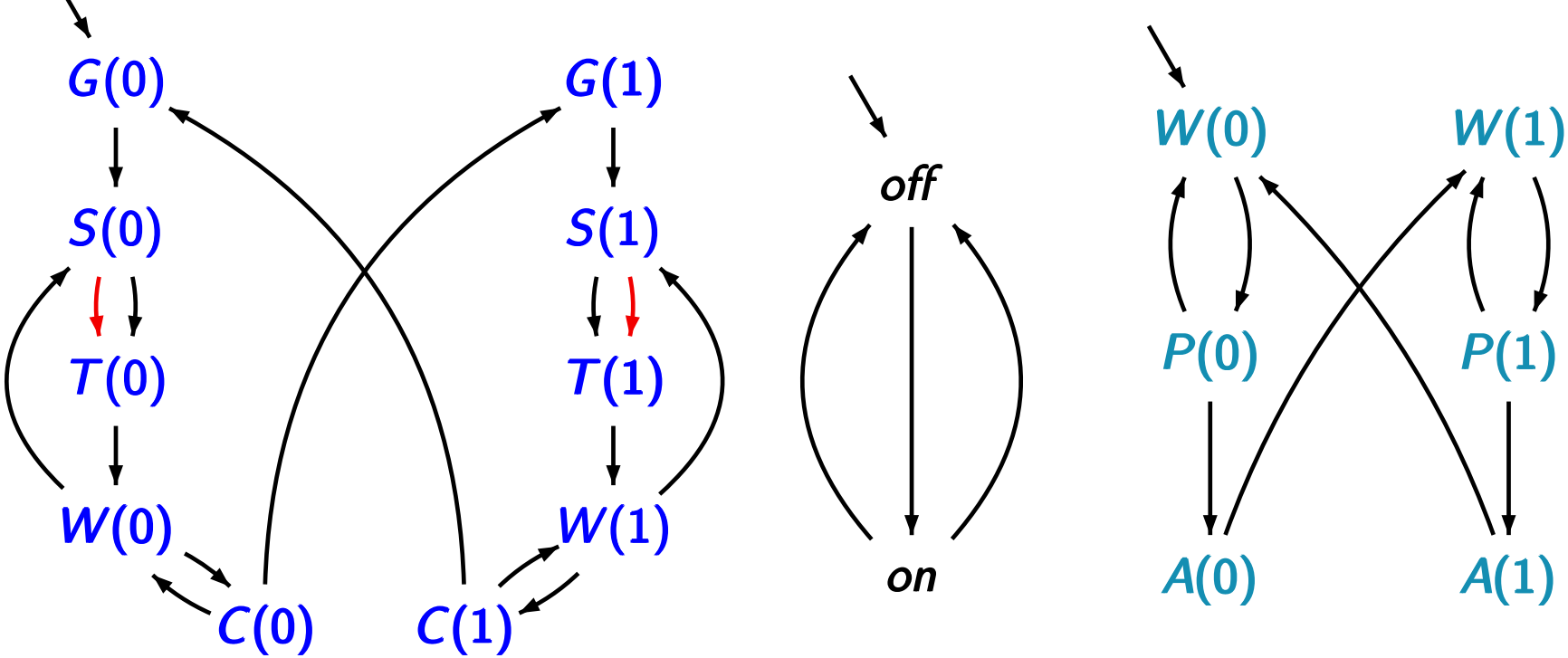
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message lost
Timer_on(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Wait(0)	on	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	

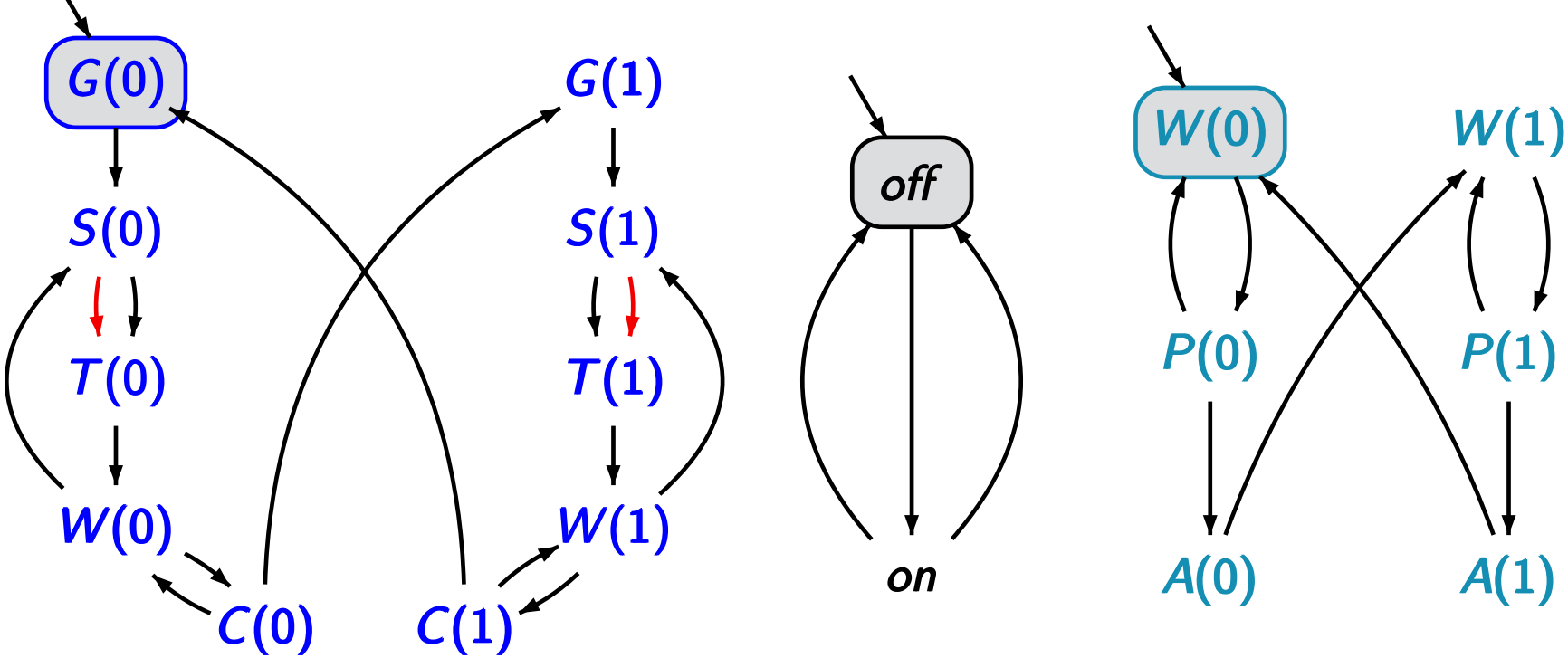


Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message lost
Timer_on(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Wait(0)	on	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	timeout

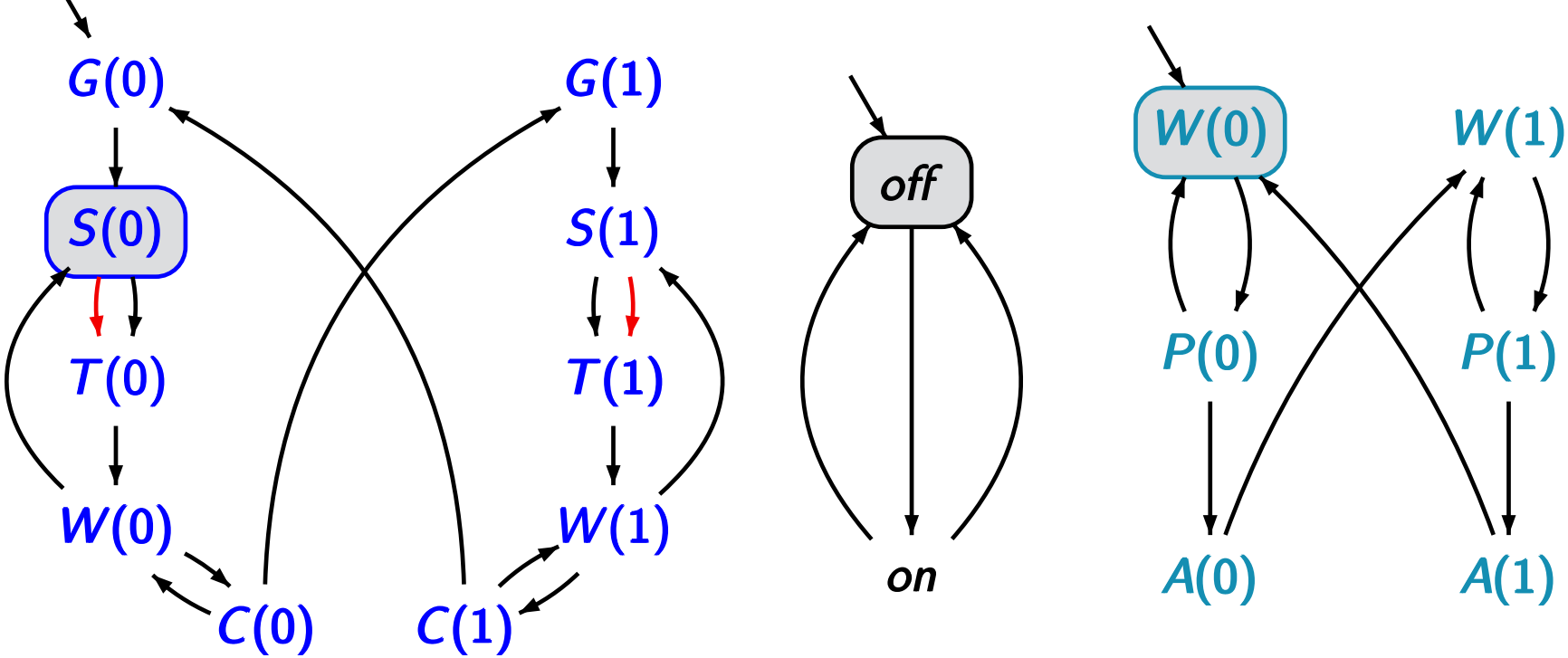


Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message lost
Timer_on(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Wait(0)	on	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	timeout
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
	⋮				try again

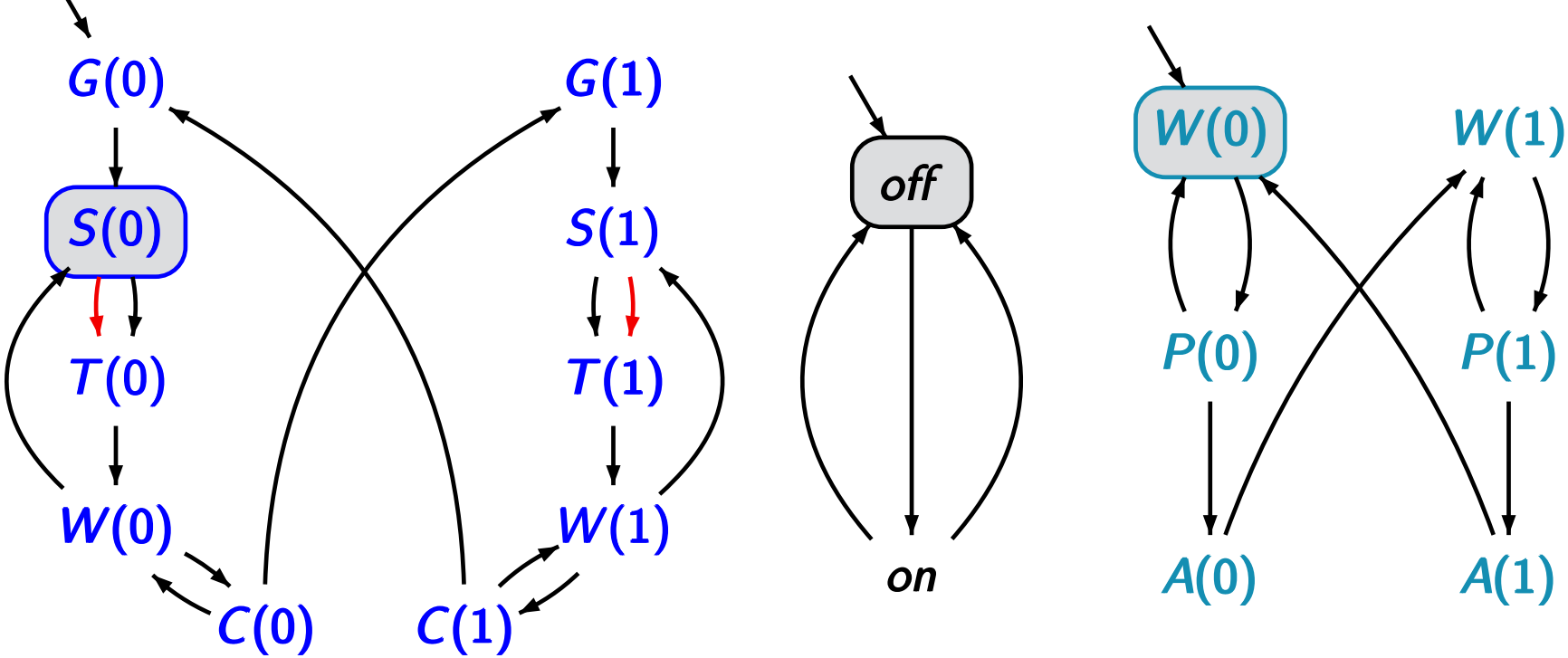

G(0)
S(0)
T(0)
W(0)
C(0)
G(1)
S(1)
T(1)
W(1)
C(1)
off
on
W(0)
W(1)
P(0)
P(1)
A(0)
A(1)



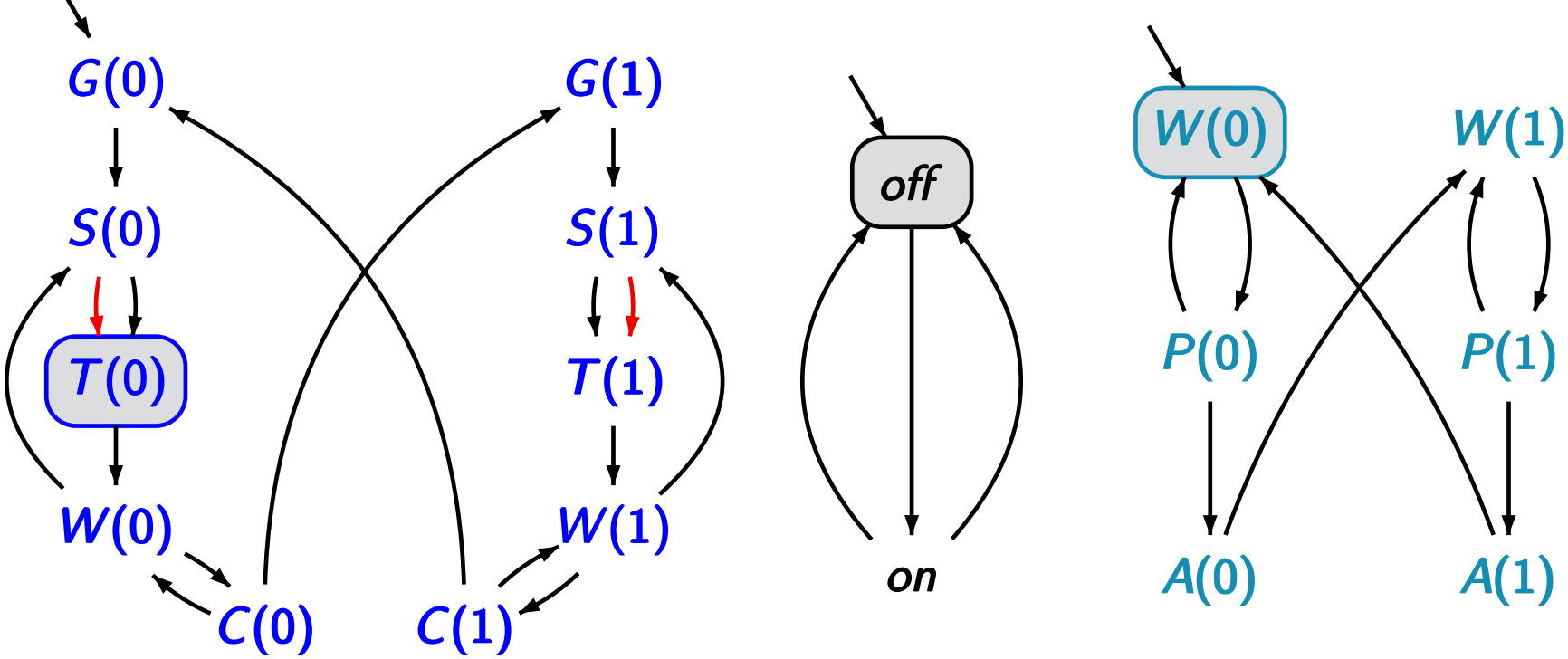
Generate(0) off Wait(0) $c{=}\varepsilon$ $d{=}\varepsilon$



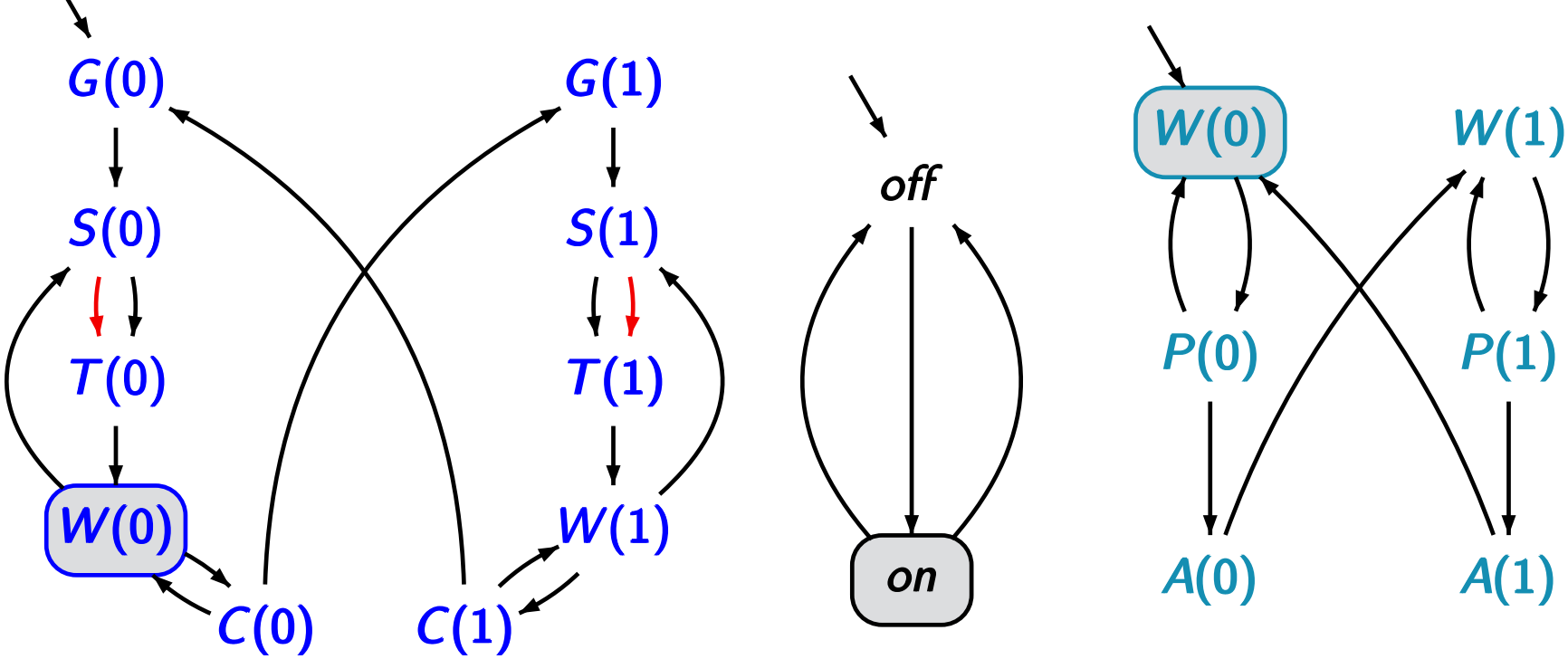
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$



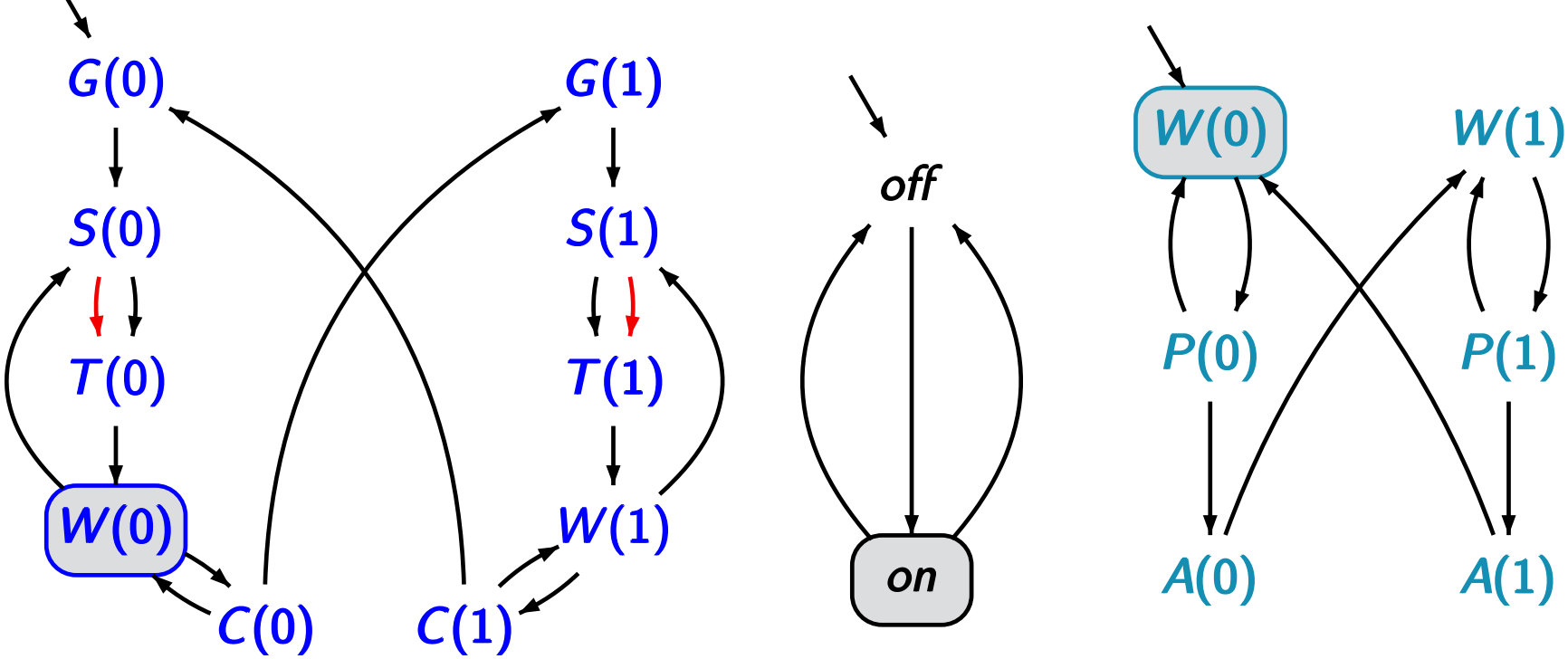
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message 0 sent



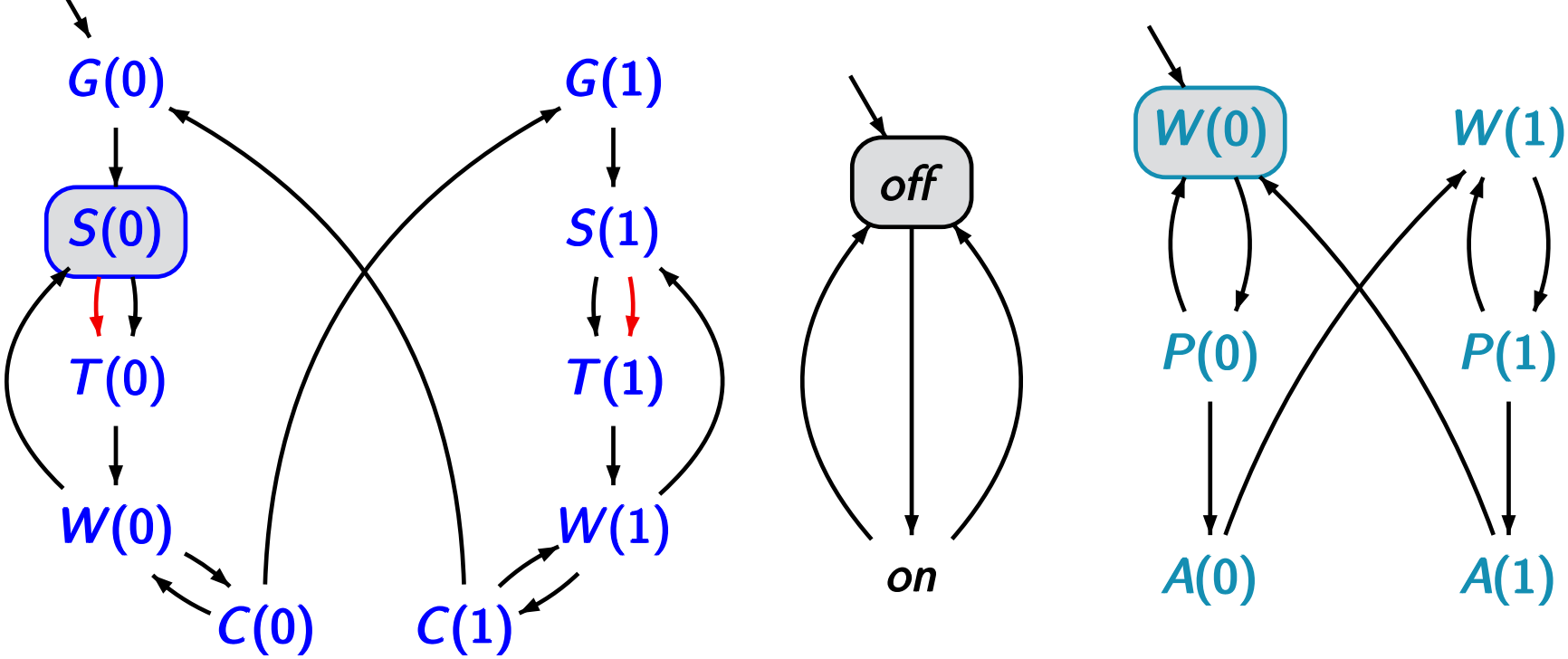
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	



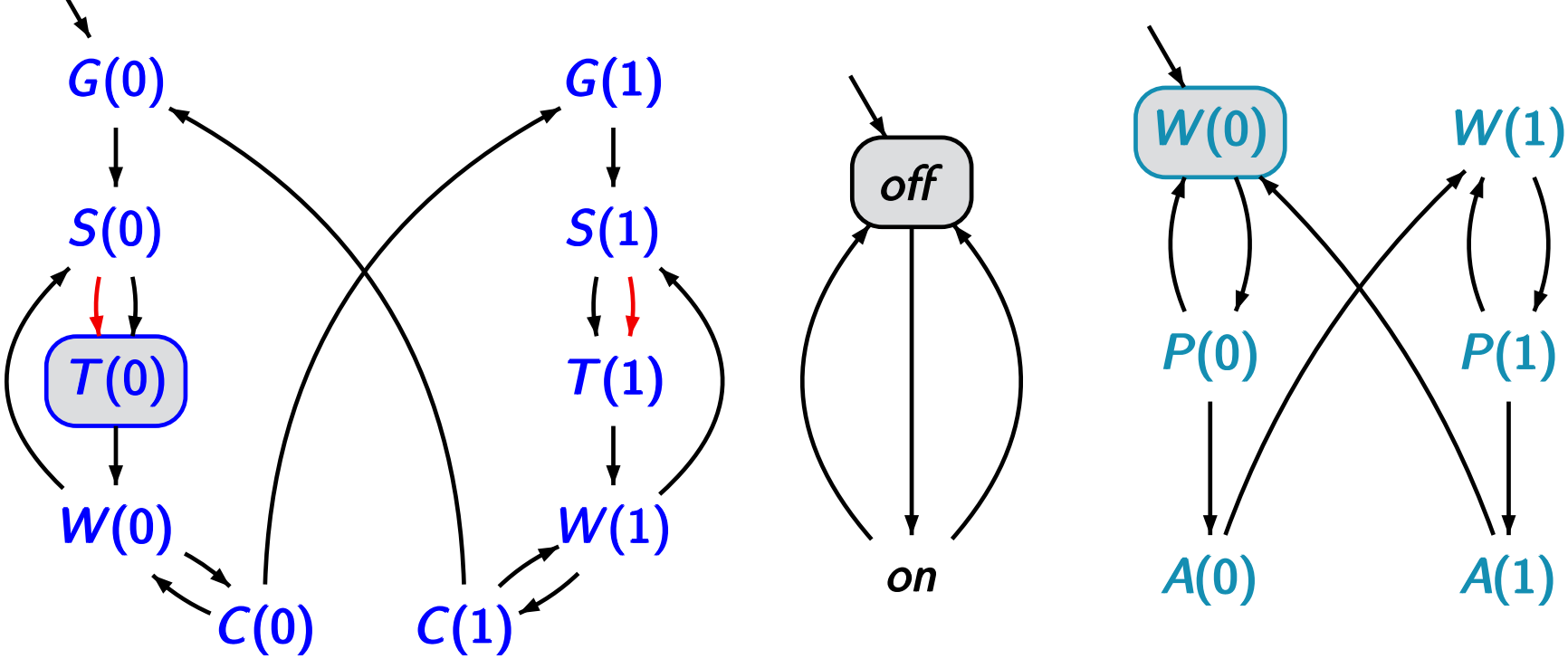
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	
Wait(0)	on	Wait(0)	$c{=}0$	$d{=}\varepsilon$	



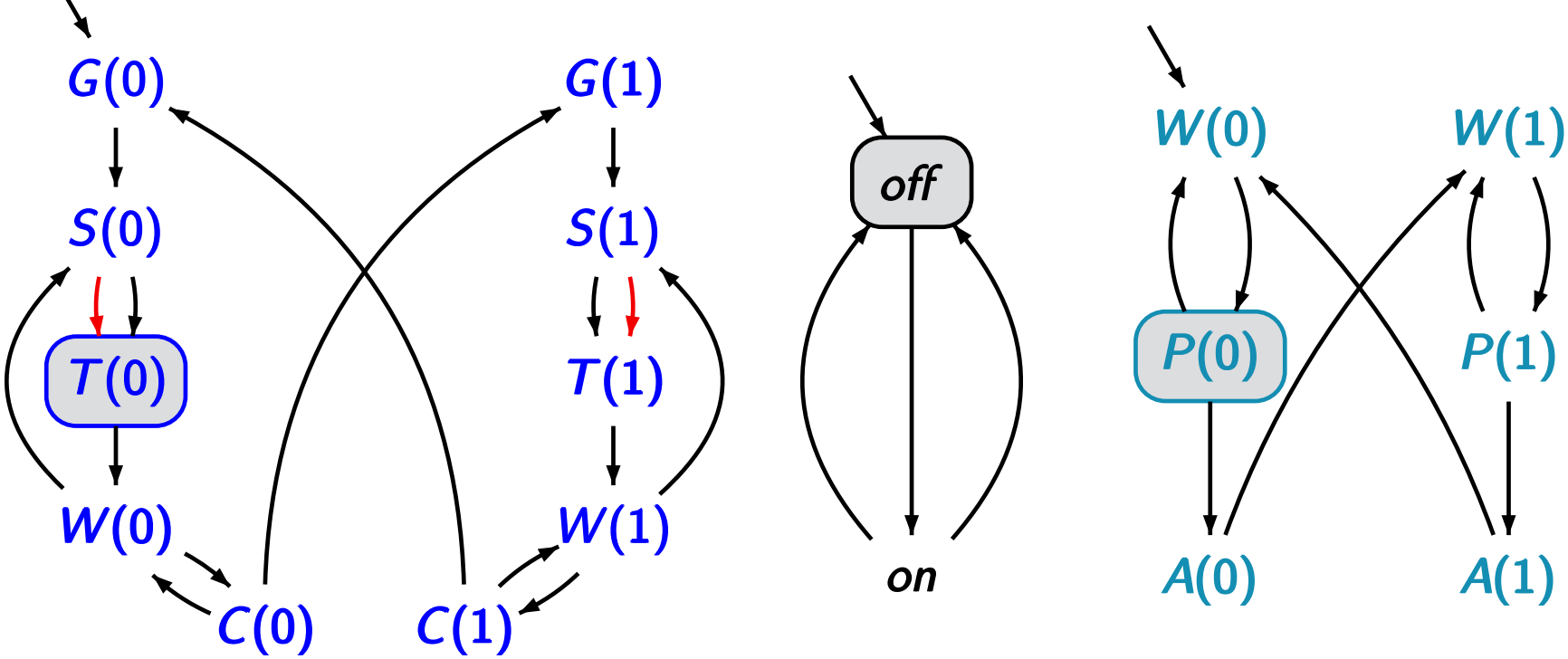
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	
Wait(0)	on	Wait(0)	$c{=}0$	$d{=}\varepsilon$	timeout



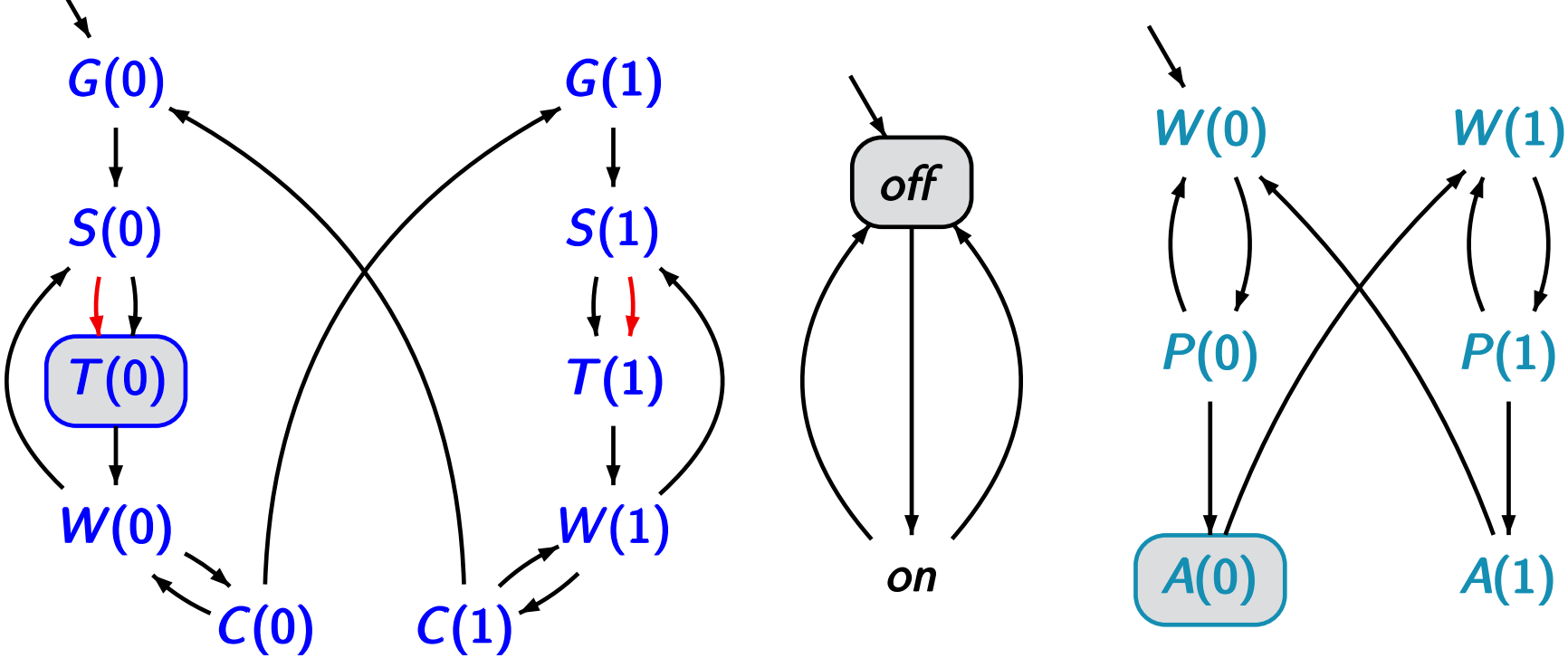
Generate(0)	off	Wait(0)	$c=\varepsilon$	$d=\varepsilon$	
Send(0)	off	Wait(0)	$c=\varepsilon$	$d=\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c=0$	$d=\varepsilon$	
Wait(0)	on	Wait(0)	$c=0$	$d=\varepsilon$	timeout
Send(0)	off	Wait(0)	$c=0$	$d=\varepsilon$	



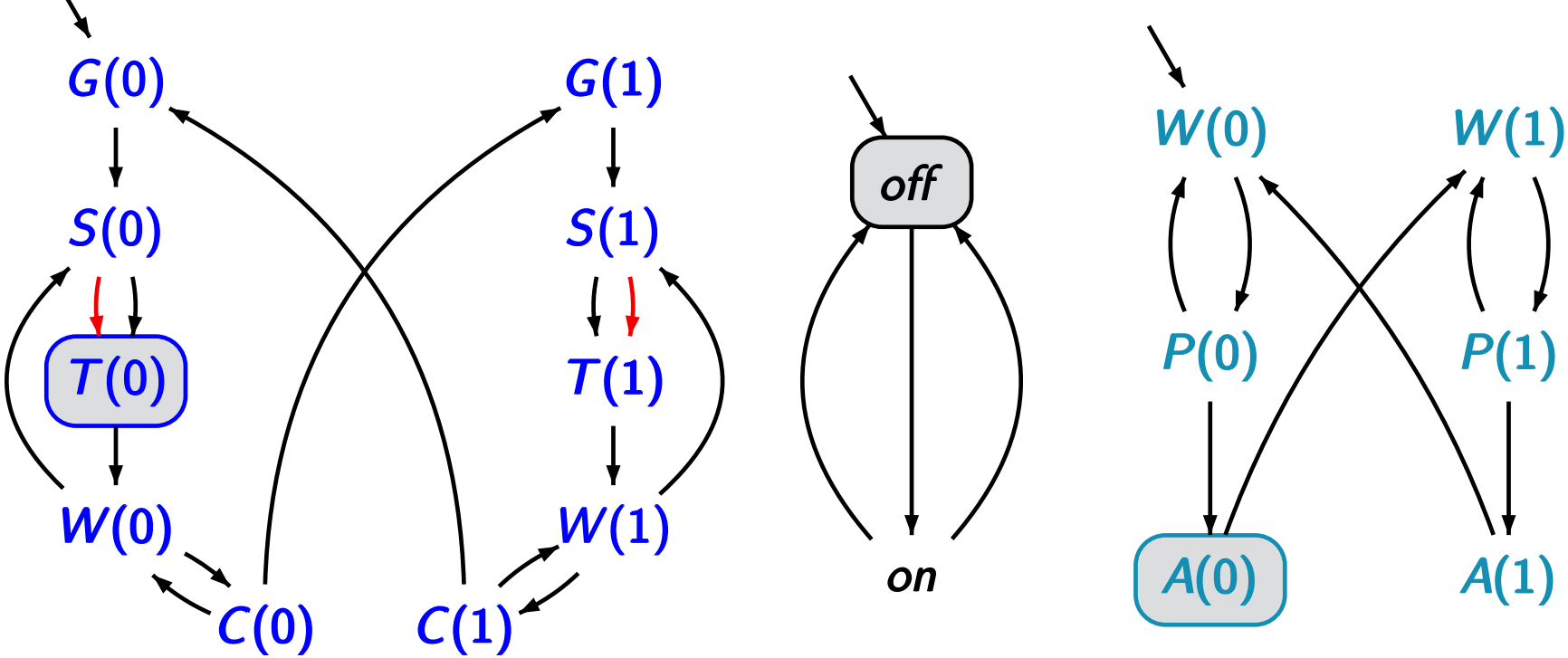
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	
Wait(0)	on	Wait(0)	$c{=}0$	$d{=}\varepsilon$	timeout
Send(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	0 sent again
Timer_on(0)	off	Wait(0)	$c{=}00$	$d{=}\varepsilon$	



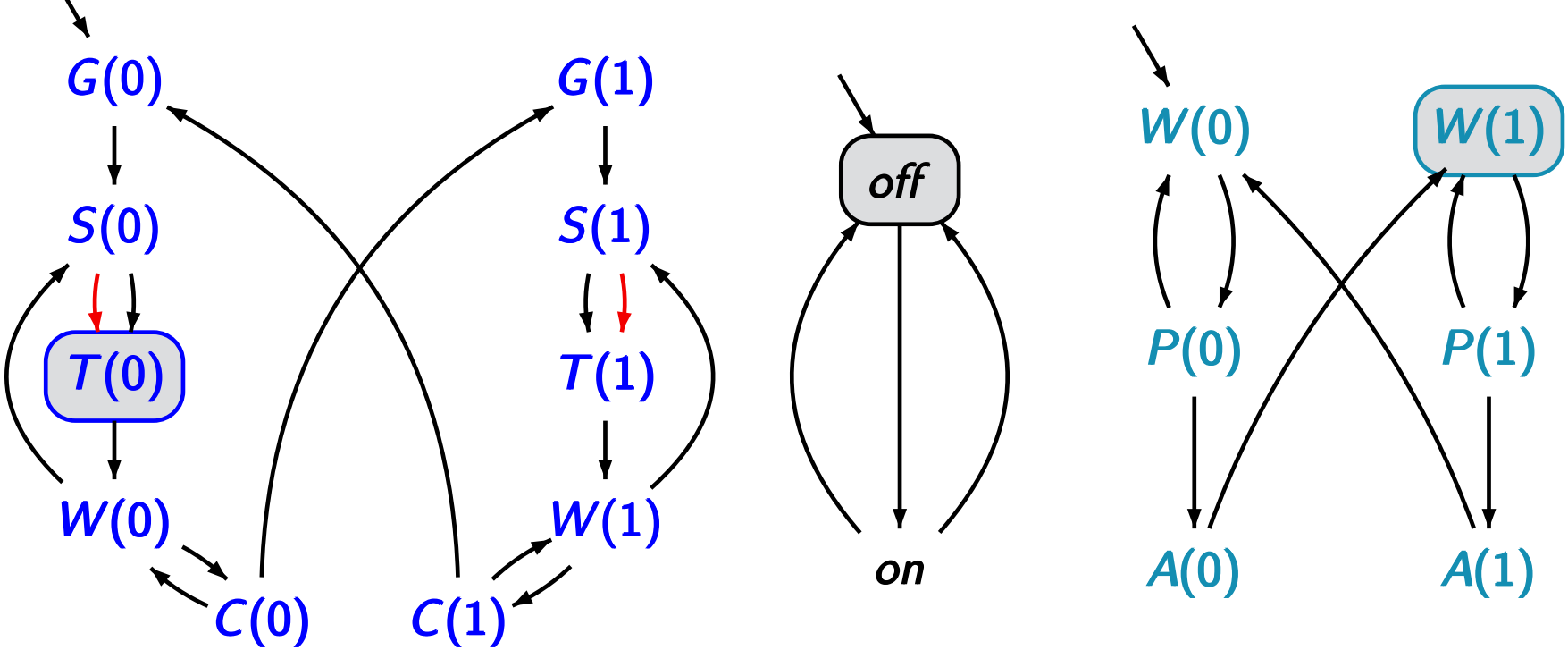
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	
Wait(0)	on	Wait(0)	$c{=}0$	$d{=}\varepsilon$	timeout
Send(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	0 sent again
Timer_on(0)	off	Wait(0)	$c{=}00$	$d{=}\varepsilon$	
Timer_on(0)	off	Proc(0)	$c{=}0$	$d{=}\varepsilon$	message received



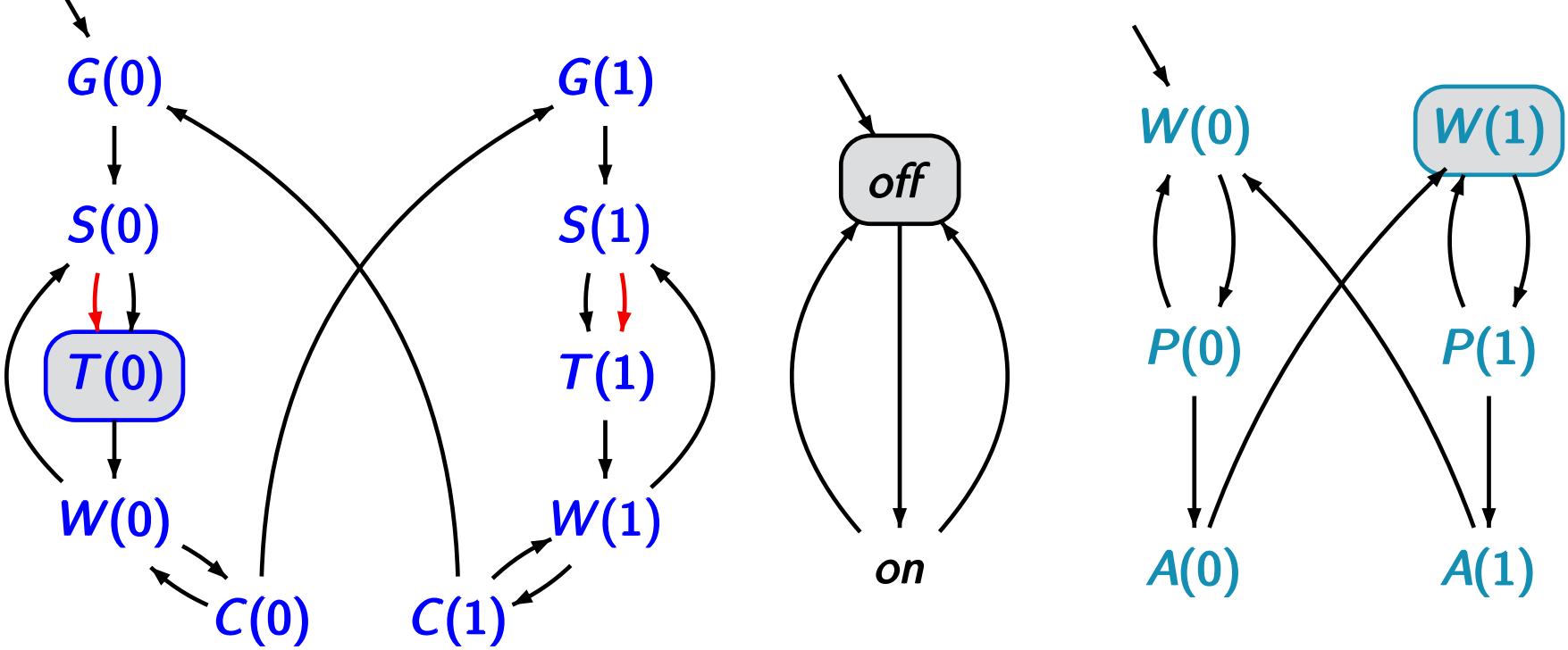
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	
Wait(0)	on	Wait(0)	$c{=}0$	$d{=}\varepsilon$	timeout
Send(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	0 sent again
Timer_on(0)	off	Wait(0)	$c{=}00$	$d{=}\varepsilon$	
Timer_on(0)	off	Proc(0)	$c{=}0$	$d{=}\varepsilon$	message received
Timer_on(0)	off	Ack(0)	$c{=}0$	$d{=}\varepsilon$	



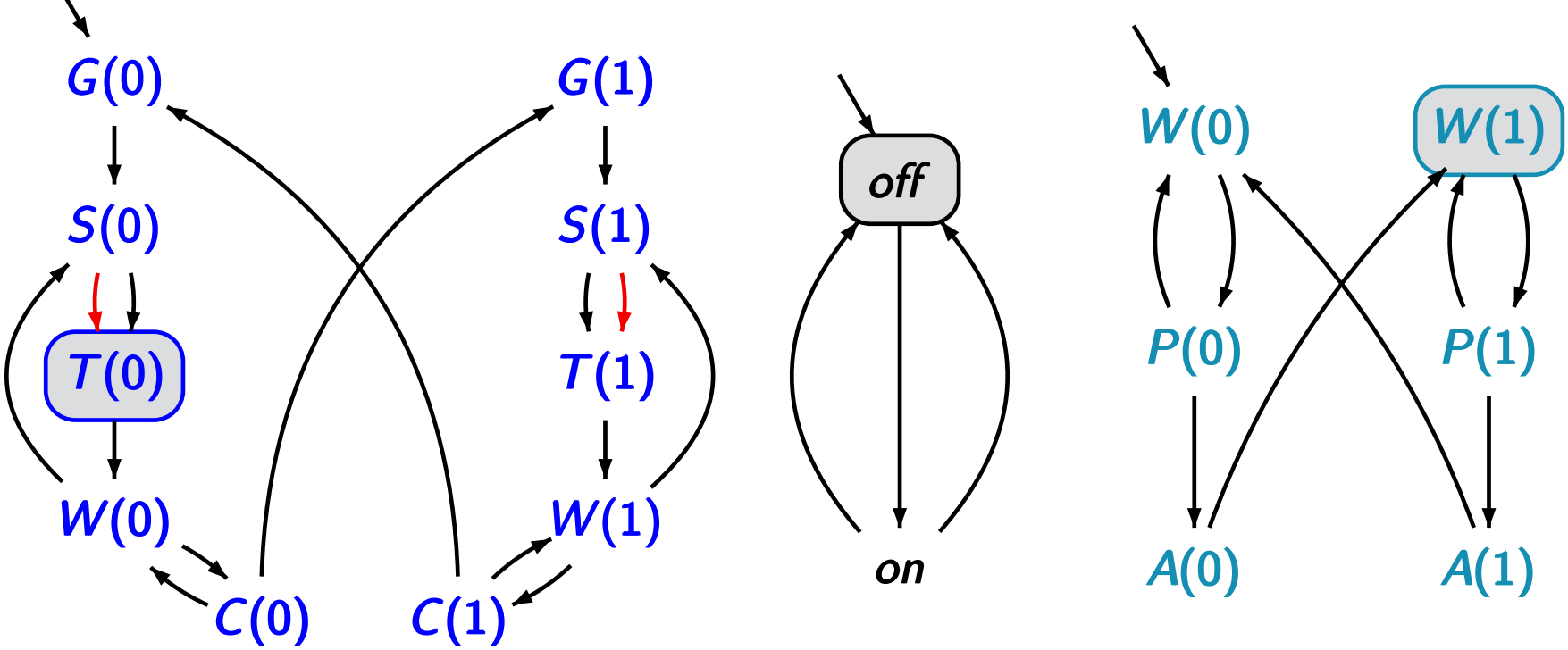
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	
Wait(0)	on	Wait(0)	$c{=}0$	$d{=}\varepsilon$	timeout
Send(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	0 sent again
Timer_on(0)	off	Wait(0)	$c{=}00$	$d{=}\varepsilon$	
Timer_on(0)	off	Proc(0)	$c{=}0$	$d{=}\varepsilon$	message received
Timer_on(0)	off	Ack(0)	$c{=}0$	$d{=}\varepsilon$	send ack via d



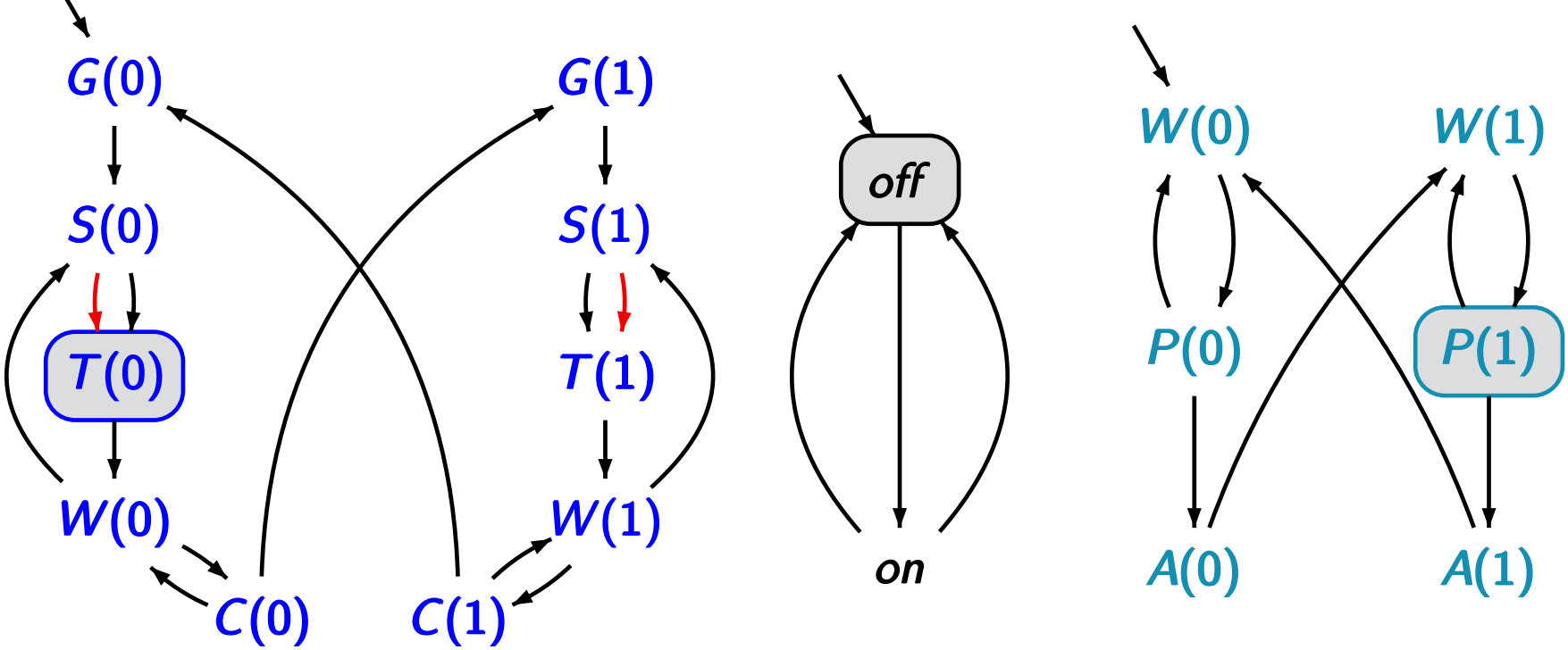
Wait(0)	on	Wait(0)	$c{=}0$	$d{=}\varepsilon$	timeout
Send(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	0 sent again
Timer_on(0)	off	Wait(0)	$c{=}00$	$d{=}\varepsilon$	
Timer_on(0)	off	Proc(0)	$c{=}0$	$d{=}\varepsilon$	message received
Timer_on(0)	off	Ack(0)	$c{=}0$	$d{=}\varepsilon$	send ack via d
Timer_on(0)	off	Wait(1)	$c{=}0$	$d{=}0$	receiver changes its mode



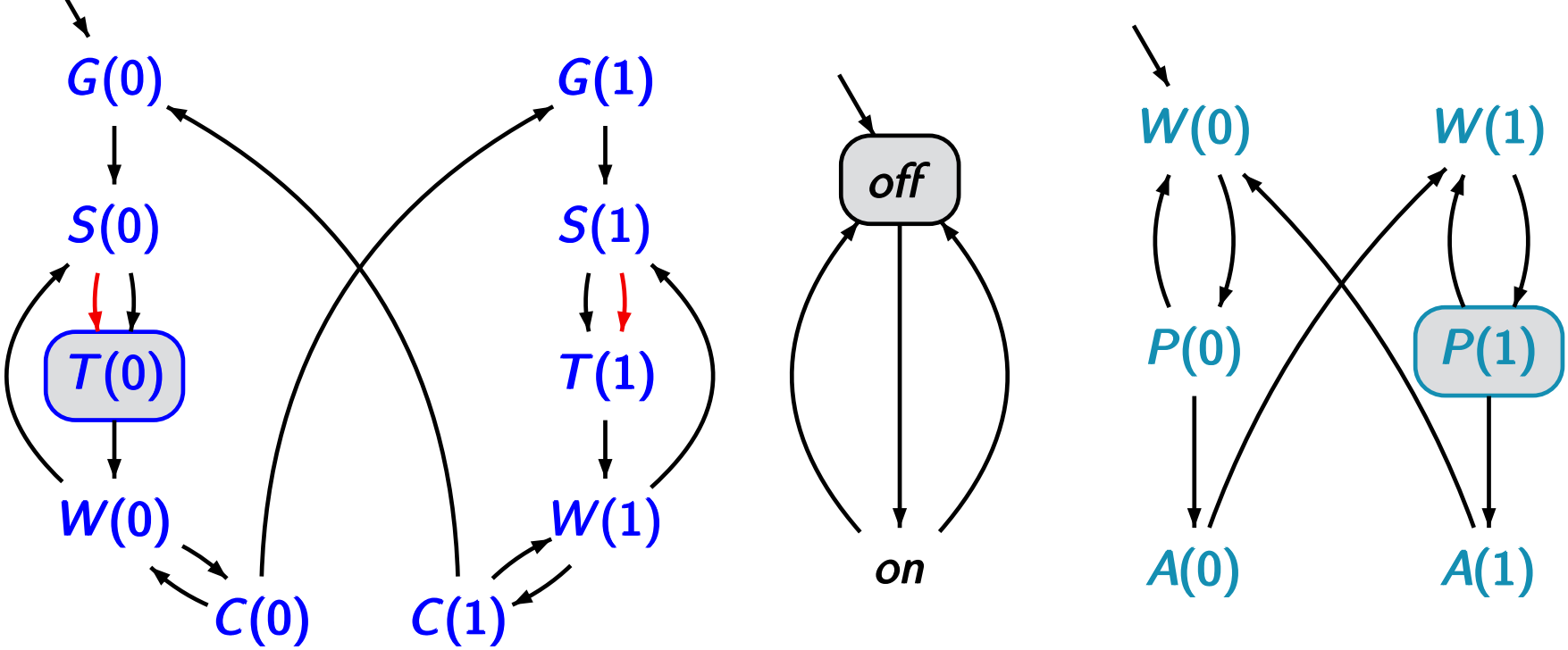
Timer_on(0) off Wait(1) $c=0$ $d=0$



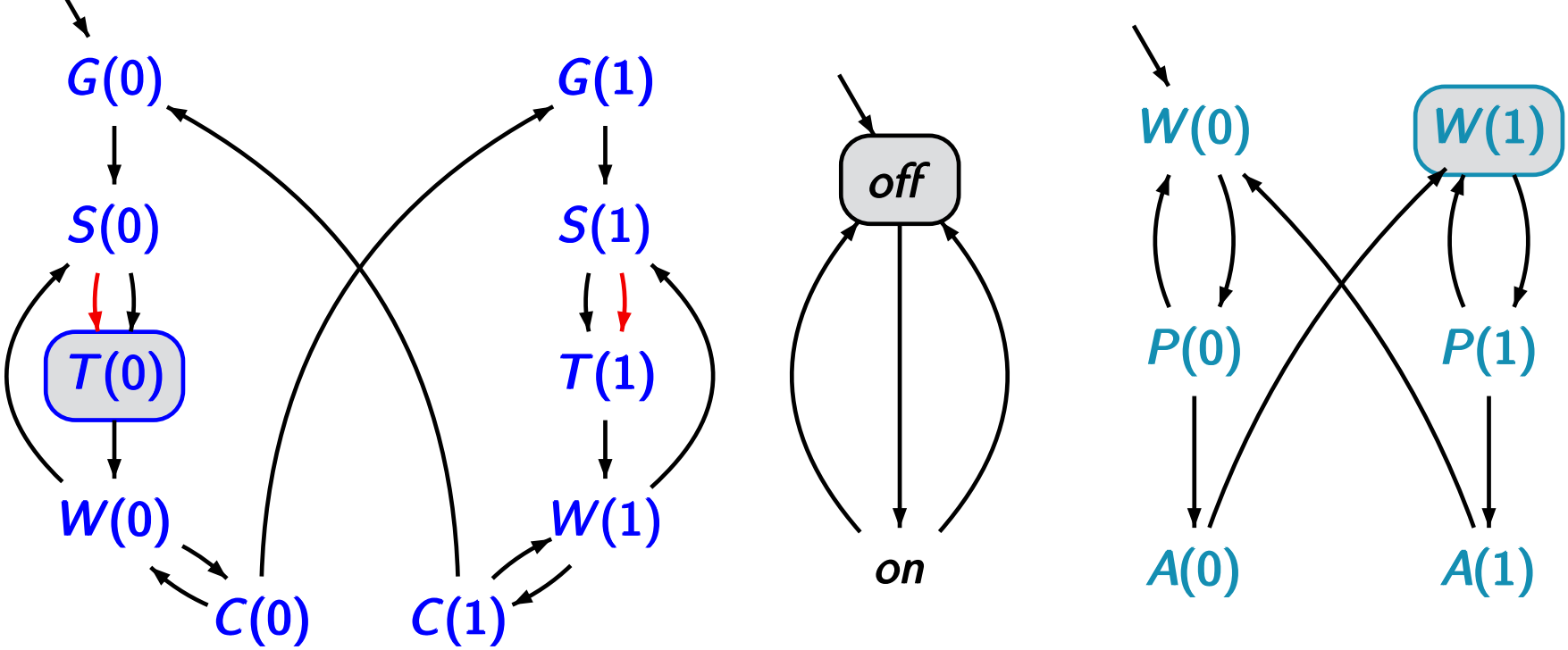
Timer_on(0) off Wait(1) $c=0$ $d=0$ receiver reads the same message again



Timer_on(0)	off	Wait(1)	$c{=}0$	$d{=}0$	receiver reads the same message again
Timer_on(0)	off	Proc(1)	$c{=}\varepsilon$	$d{=}0$	



Timer_on(0)	off	Wait(1)	$c{=}0$	$d{=}0$	receiver reads the same message again
Timer_on(0)	off	Proc(1)	$c{=}\varepsilon$	$d{=}0$	receiver discards the message



Timer_on(0)	off	Wait(1)	$c{=}0$	$d{=}0$	receiver reads the same message again
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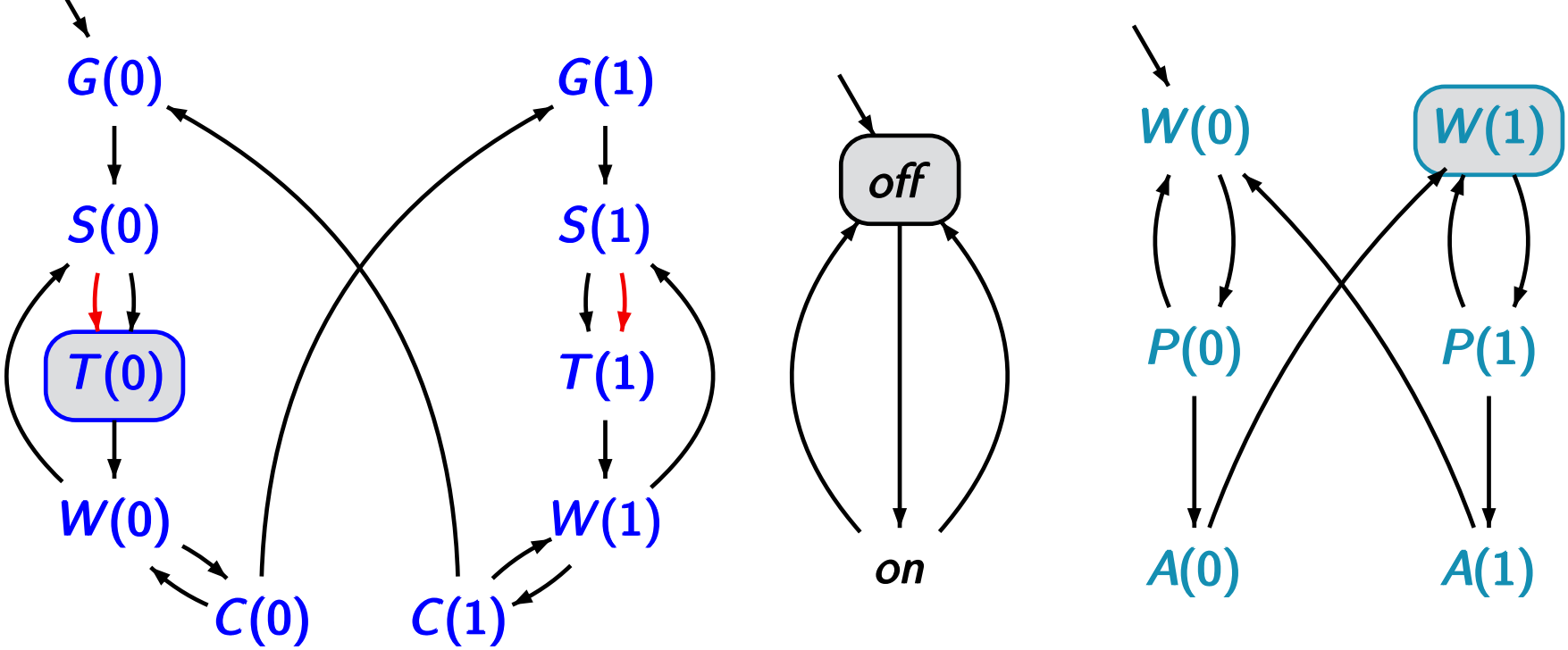
Timer_on(0)	off	Wait(1)	$c{=}\varepsilon$	$d{=}0$	



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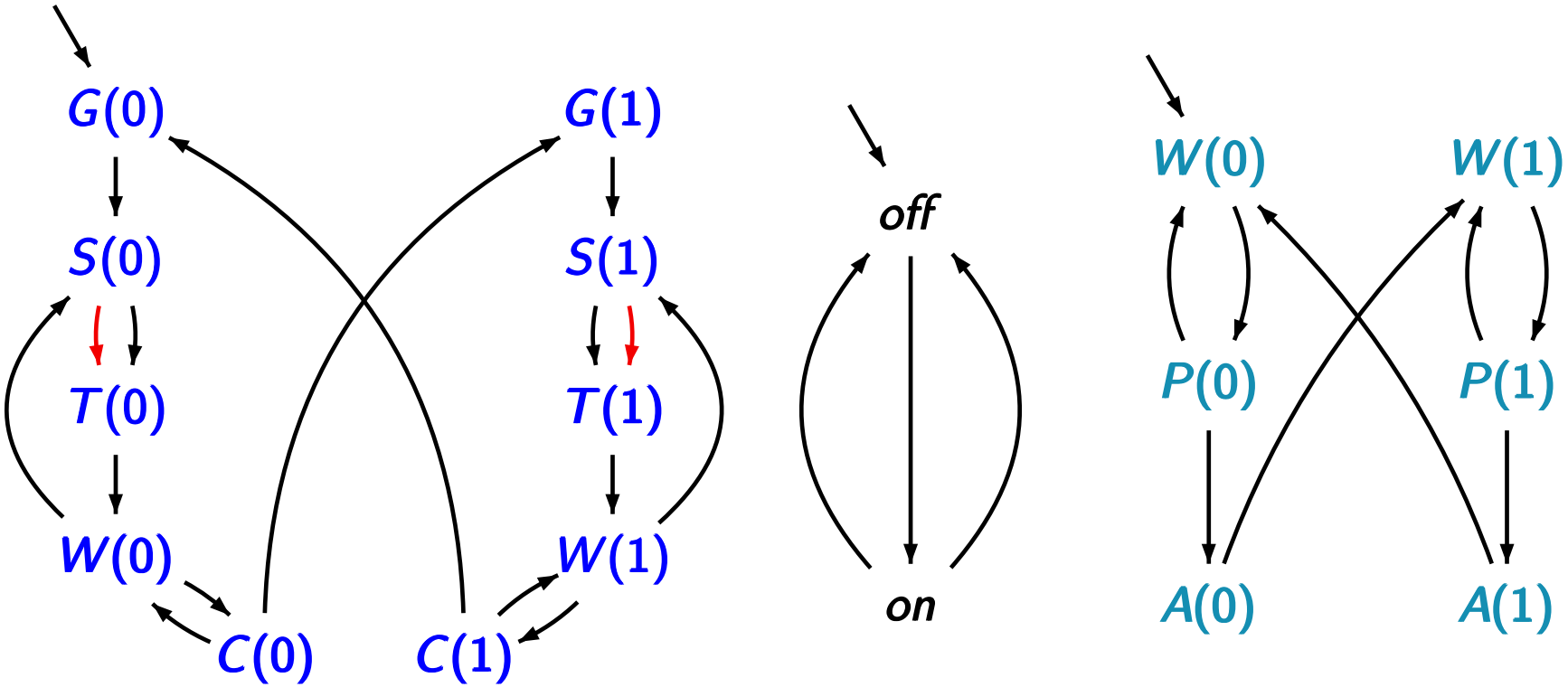
Alternating bit protocol (ABP)



number of states in the TS:

$$\mathbf{10} \cdot 2 \cdot \mathbf{6} \cdot \#\text{channel evaluations}$$

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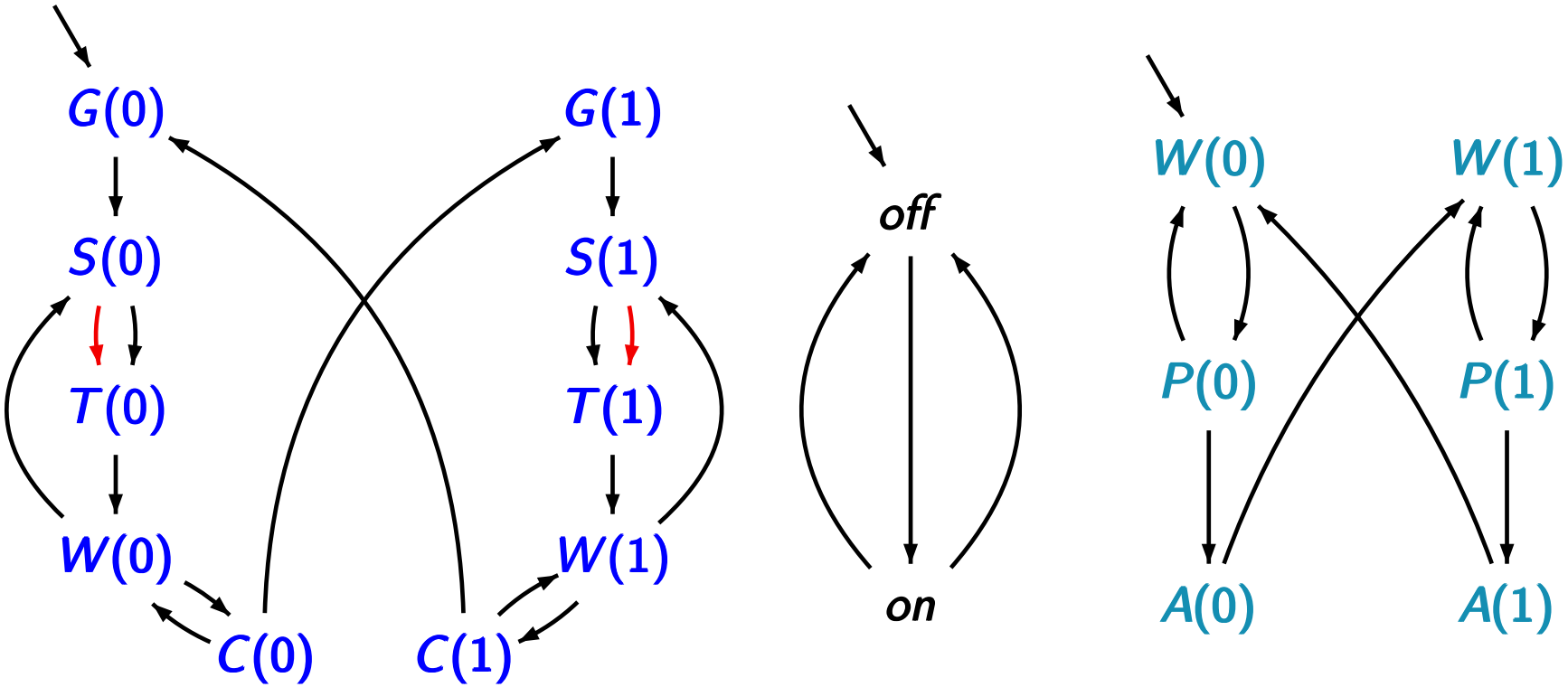

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$> \mathbf{10^8}$ for FIFOs with capacity **10**

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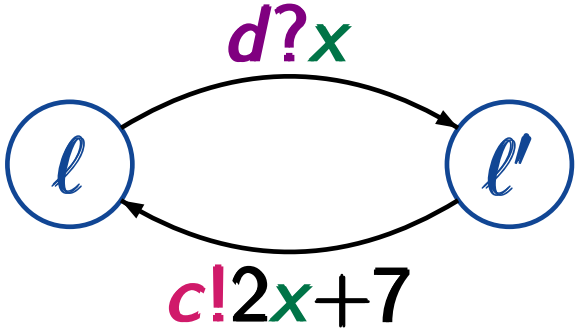

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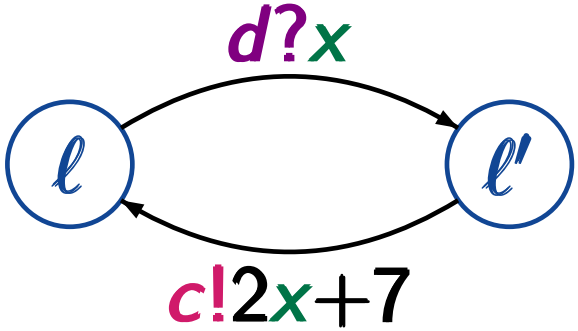

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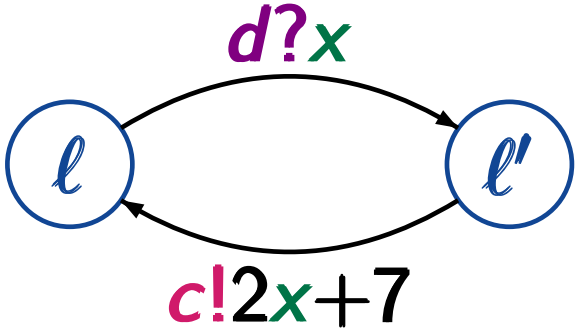

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$\rightsquigarrow$ more compact TS-representations

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