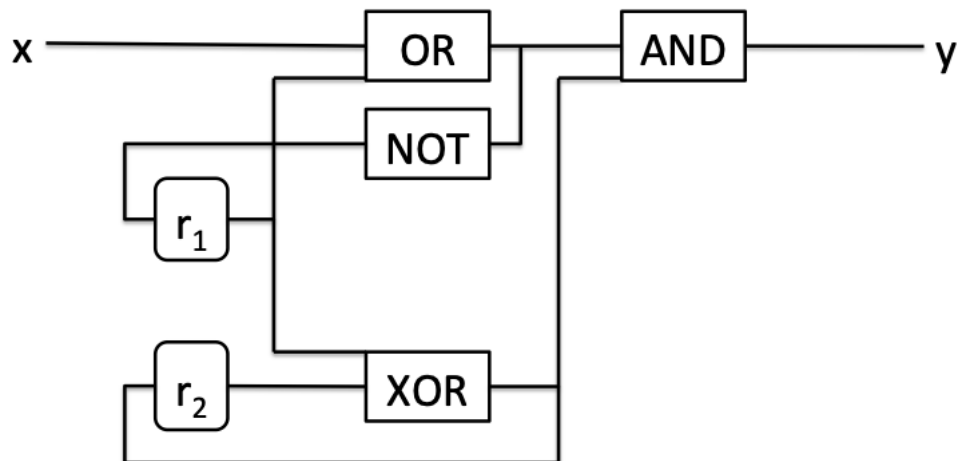


**EXERCISE 1 (8 points)**

Consider the following circuit.



Draw the transition system describing the behaviour of the circuit. Use  $AP = \{y\}$  as set of atomic propositions to label states. Registers are initialised to 0.

**EXERCISE 2 (8 points)**

Consider the atomic propositions  $AP = \{P, Q, R\}$  and the following linear time properties:

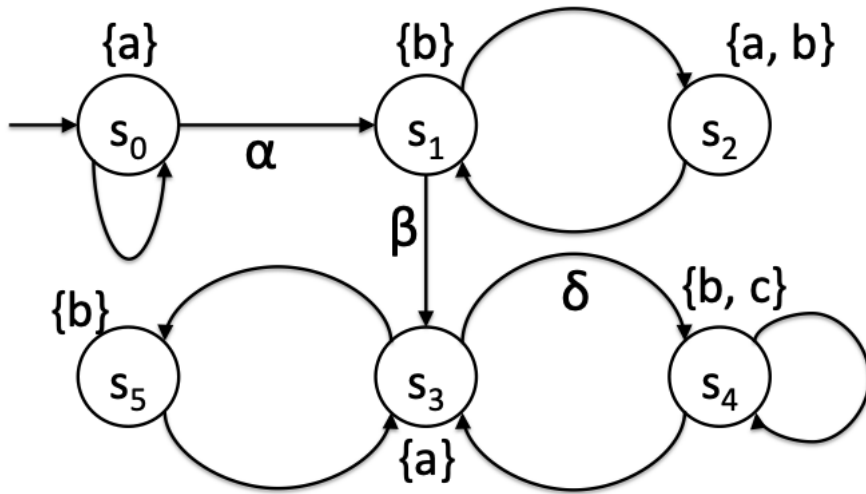
- (a)  $P$  always holds the step before  $Q$  holds unless  $R$  holds together with  $Q$ , in which case  $P$  may hold or not.
- (b)  $Q$  holds only finitely many times and whenever  $Q$  holds also  $R$  holds.
- (c) Whenever  $P$  holds then  $Q$  and  $R$  will hold together afterwards (or immediately).

For each property:

1. formalise it using set expressions and first order logic;
2. formalise it in LTL (you can use the operators next, until, box and diamond, together with all boolean connectives);
3. tell if it is a safety, liveness or mixed property. In case it is a pure safety property provide an NFA for the language of the **minimal bad prefixes**.

### EXERCISE 3 (8 points)

Consider the following transition system  $TS$  on  $AP = \{a, b, c\}$ .



Decide whether or not the following LTL formulas:

$$\begin{aligned} \varphi_0 &= \Box \Diamond b & \varphi_1 &= \Box (a \rightarrow \bigcirc (a \vee b)) \\ \varphi_2 &= \Box \Diamond c \end{aligned}$$

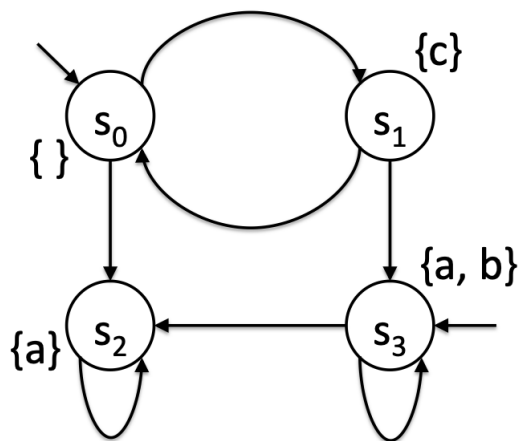
are satisfied by  $TS$  under the following fairness conditions (to be considered separately):

$$\begin{aligned} \psi_0^{\text{fair}} &= (\{\}, \{\}, \{\}) & \psi_1^{\text{fair}} &= (\{\}, \{\{\alpha, \beta, \delta\}\}, \{\}) \\ \psi_2^{\text{fair}} &= (\{\}, \{\}, \{\{\alpha, \beta, \delta\}\}) \end{aligned}$$

Justify your answers! In case the answer is no, provide a counterexample.

### EXERCISE 4 (8 points)

Consider the following transition system  $TS$  on  $AP = \{a, b, c\}$ .



Decide whether or not the following CTL formulas:

$$\begin{aligned} \phi_0 &= \forall \Diamond b & \phi_1 &= \exists \Diamond c \\ \phi_2 &= \forall \Box (c \rightarrow \exists \Diamond a) & \phi_3 &= \forall \Box \forall \Diamond (a \vee c) \end{aligned}$$

are satisfied by  $TS$ . Justify your answers! When possible, provide a counterexample or a witness.