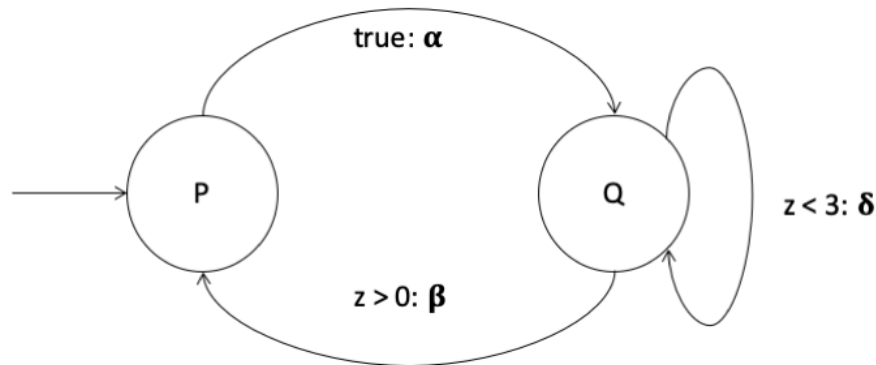


**EXERCISE 1 (8 points)**

Consider the following program graph



where  $z$  is a variable such that  $\text{dom}(z) = \{0, 1, 2, 3\}$ ,  $g_0 \equiv (z = 1)$ ,  $\text{Loc}_0 = \{P\}$ ,  $\text{Effect}(\alpha, \eta) = \eta[z := 0]$ ,  $\text{Effect}(\beta, \eta) = \eta[z := \eta(z) - 1]$  and  $\text{Effect}(\delta, \eta) = \eta[z := \eta(z) + 1]$ .

1. Translate the given program graph into an equivalent transition system.

**EXERCISE 2 (8 points)**

Consider the atomic propositions  $AP = \{A, B\}$  and the following linear time properties:

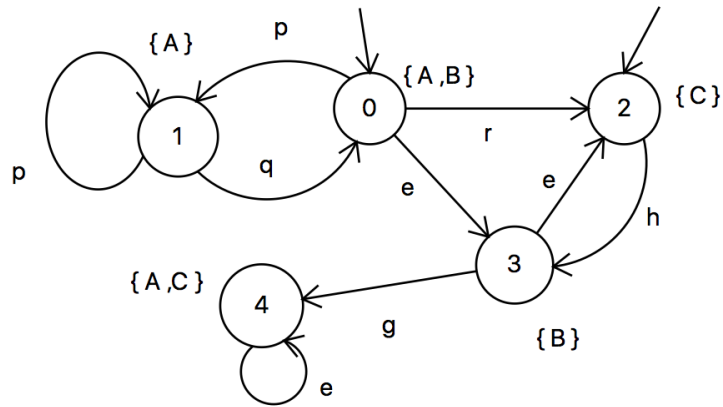
- (a) Whenever  $A$  holds then  $B$  holds after two steps
- (b)  $A$  and  $B$  hold together infinitely many times
- (c)  $A$  holds once and  $B$  never holds

For each property:

1. formalise it using set expressions and first order logic;
2. formalise it in LTL (you can use the operators next, until, box and diamond, together with all boolean connectives);
3. tell if it is a safety, liveness or mixed property. In case it is a pure safety property provide an NFA for the language of the **minimal bad prefixes**.

### EXERCISE 3 (8 points)

Consider the following transition system  $TS$  on  $AP = \{A, B, C\}$ .



Decide whether or not the following LTL formulas:

$$\begin{aligned} \varphi_0 &= \Box(C \rightarrow \Diamond A) & \varphi_1 &= \Diamond(B \wedge \bigcirc(B \vee C)) \\ \varphi_2 &= \Box\Diamond\neg B \end{aligned}$$

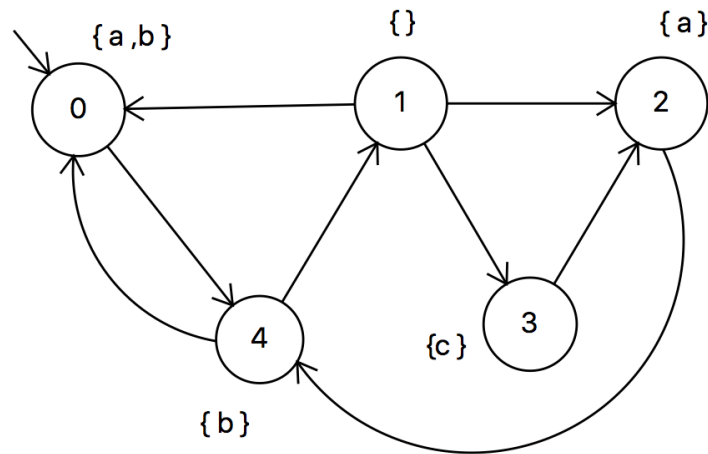
are satisfied by  $TS$  under the following fairness conditions (to be considered separately):

$$\begin{aligned} \psi_0^{\text{fair}} &= (\{\}, \{\}, \{\}) & \psi_1^{\text{fair}} &= (\{\}, \{\{g\}, \{e, r\}\}, \{\{q\}\}) \\ \psi_2^{\text{fair}} &= (\{\}, \{\}, \{\{g\}, \{e\}, \{q\}\}) \end{aligned}$$

Justify your answers! In case the answer is no, provide a counterexample.

### EXERCISE 4 (8 points)

Consider the following transition system  $TS$  on  $AP = \{a, b, c\}$ .



Decide whether or not the following CTL formulas:

$$\begin{aligned} \phi_0 &= \forall\Diamond c & \phi_1 &= \exists\Diamond(b \wedge \exists\bigcirc\neg b) \\ \phi_2 &= \forall\Box(a \rightarrow \forall\Diamond b) & \phi_3 &= \forall\Box(\neg b \rightarrow \forall\bigcirc(a \vee b)) \end{aligned}$$

are satisfied by  $TS$ . Justify your answers! When possible, provide a counterexample or a witness.