Case-Based Reasoning

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Literature: Bergmann, Ralph: Experience Management. Springer-Verlag 2002

Case-Based Reasoning

Assumption: Similar problems have similar solutions

General approach:

■ experiences are stored as cases

■ To solve a new problem ...

…retrieve similar cases

…solution of the most similar case is reused in the new situation

Quelle: Bergmann

Humans use Cases for Problem Solving

■ Examples:

■ A medical doctor remembers the case history of another patient

■ A lawyer argues with similar original precedence

■ An architect studies the constrcution of existing building

■ A work scheduler remembers the construction steps of a similar workpiece (Variantenplanung)

■ A mathematician tries to transfers a known proof to a new problem

■ A service technician remembers a similar defect at another device

Case-Based Reasoning

- Case-based retrieval can be used for retrieval of
  - ♦ cases (problem-solution pairs) –
    - ● similarity of problem description with query (=problem)
  - ♦ documents or information –
    - ● similarity of metadata with query

- Case-based retrieval is useful if
  - ♦ relevant information cannot be specified exactly or
  - ♦ answers do not exactly fit to the query

- Applications examples
  - ♦ Information Retrieval: used cars
 - ♦ Lessons Learned databases: experience management

Example: Information Retrieval in Used Cars Database
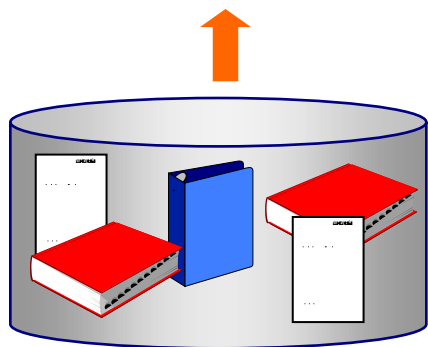
- Scenario: The database contains advertisements of used cars
- Assume you look for the following car:
  - ♦ Audi A4, limousine, 125 PS, colour silver, automatic transmission, 2 years, bis 30'000 Fr.
- Problem: The database does not contain a car with exact this equipment
- Objective: The system should suggest those cars that are most similar to the one I look for, e.g..
  - ♦ Audi A4, limousine, 150 PS, blue, 5 gears, 2 years, 28.000 Fr.
  - ♦ Audi A4, station waggon, 125 PS, silver, automatic transmission, 3 years, 26.000 Fr.
  - ♦ BMW 320, limousine, 138 PS, silver, automatic transmission, 2 years, 29.500 Fr.
  - ♦ Volvo S40, limousine, 125 PS, silver, automatic transmission, 18 months, 29.000 Fr.

What are the similarity criteria for these suggestions?

similarity

metadata

Lessons Learned Database

problem ↓ ↑ **solution**

similarity

lessons learned (problem + solution)
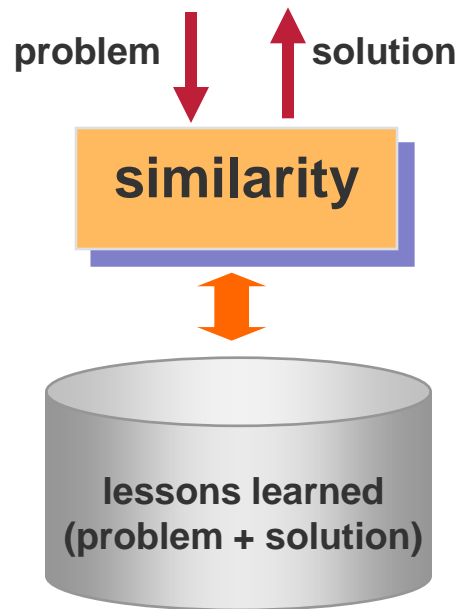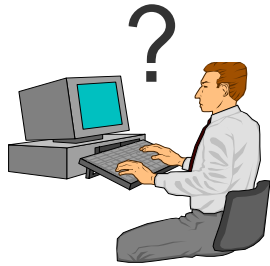
- Situation: Don't Worry Insurance DWI
  - ♦ Supplementary Health Insurance may require complex clarification
  - ♦ For specific situations there might be no general rules

- Objective: avoid extensive anaylsis if situation is already known
  - ➢ „Lessons Learned" database

- But: What if the siutation did not occurin exactly the same form?

- Example:
  - ♦ Lessons learned database contains a case of a 52 year old men, who had a cardiac infarction (heart attack) 4 years ago
  - ♦ Another case is of a 48 year old woman with cardiac insufficiency 2 years ago.
  - ♦ The new case is of a 49 year old man who had a cardiac insufficiency 3 years ago.

- How to find the old cases?

- Can a decision of the previous cases be transfered to the new case?

- How to transfer the solution?

A simple Example (Overview)

- **Technical Diagnosis of Car Faults**
  - ♦ Symptoms are observed (e.g. engine doesn't start) and values are measured (e.g. battery voltage = 6.3V)
  - ♦ Goal: Find the cause for the failure (e.g. battery empty) and a repair strategy (e.g. charge battery)

- **Case-Based Diagnosis:**
  - ♦ A case describes a diagnostic situation and contains:
    - • description of the symptoms
    - • description of the failure and the cause
    - • description of a repair strategy
  - ♦ Store a collection of cases in a case base
  - ♦ Find case similar to current problem and reuse repair strategy

A simple Example: What's in a Case

- A case describes one particular diagnostic situation

- A case records several features and their specific values occurred in that situation

- Each case describes one particular situation

- All cases are independent from each other

Feature   **Value**

Problem (Symptoms)		
	• *Problem*: Front light doesn't work	
	• *Car*: VW Golf II, 1.6 L	
C A S E	• *Year*: 1993	
	• *Battery voltage*: 13,6 V	
	• *State of lights*: OK	
	• *State of light switch*: OK	
Solution		
1	• *Diagnosis*: Front light fuse defect	
	• *Repair*: Replace front light fuse	

Source: Ralph Bergmann

Example: A Case Base with 2 Cases

Problem (Symptoms)

C A S E 1

- Problem: Front light doesn't work
- Car: VW Golf II, 1.6 L
- Year: 1993
- Battery voltage: 13,6 V
- State of lights: OK
- State of light switch: OK

Solution

- Diagnosis: Front light fuse defect
- Repair: Replace front light fuse

Problem (Symptoms)

C A S E 2

- Problem: Front light doesn't work
- Car: Audi A6
- Year: 1995
- Battery voltage : 12,9 V
- State of lights: surface damaged
- State of light switch: OK

Solution

- Diagnosis: Bulb defect
- Repair: Replace front light

Source: Ralph Bergmann
Adapted from Bergmann

Solving a new Diagnostic Problem

■ A new problem must be solved

■ We make several observations in the current situation

■ Observations define a new problem

■ Not all feature values must be known

■ Note: The new problem is a case without solution part
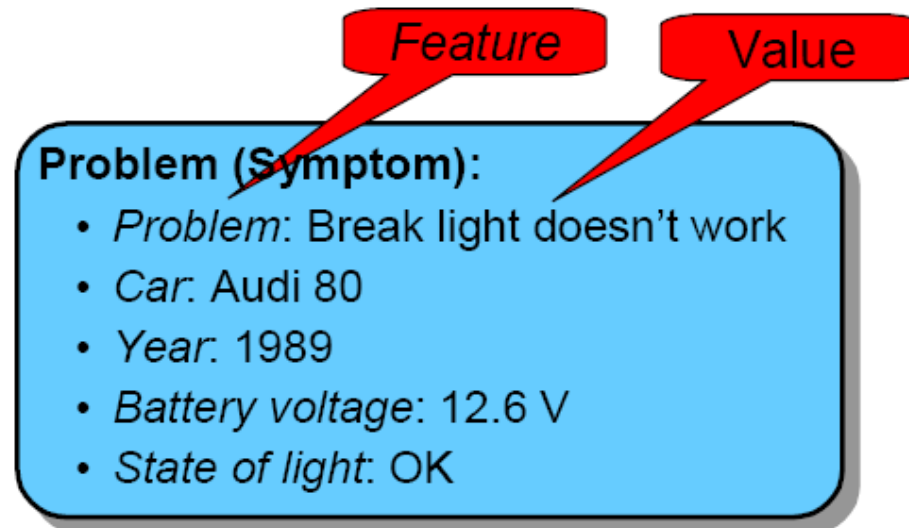
Source: Ralph Bergmann

Compare the New Problem with Each Case and Select the Most Similar Case

Similarity is the most important concept in CBR !!

- When are two cases similar?

- How to rank the cases according to their similarity?

- We can assess similarity based on the similarity of each feature

- Similarity of each feature depends on the feature **value**.

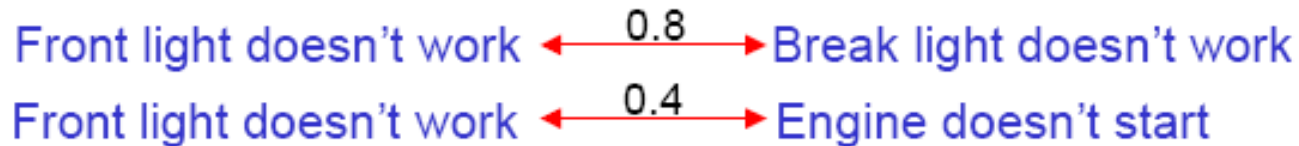- BUT: Importance of different features may be different

Source: Ralph Bergmann
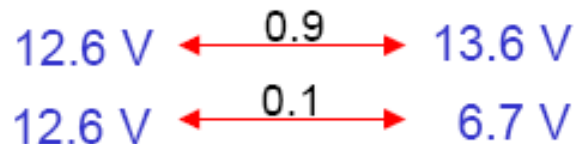
Similarity Computation

- Assignment of similarities for features values. **Not similar** **Very similar**
- Express degree of similarity by a real number between 0 and 1
  Examples:
  - Feature: *Problem*

    Front light doesn't work $\xleftrightarrow{\text{0.8}}$ Break light doesn't work

    Front light doesn't work $\xleftrightarrow{\text{0.4}}$ Engine doesn't start

  - Feature: *Battery voltage*   (similarity depends on the difference)

    12.6 V $\xleftrightarrow{\text{0.9}}$ 13.6 V

    12.6 V $\xleftrightarrow{\text{0.1}}$ 6.7 V

- **Different features have different importance (weights) !**
  - High importance: Problem, Battery voltage, State of light, ...
  - Low importance: Car, Year, ...

Source: Ralph Bergmann
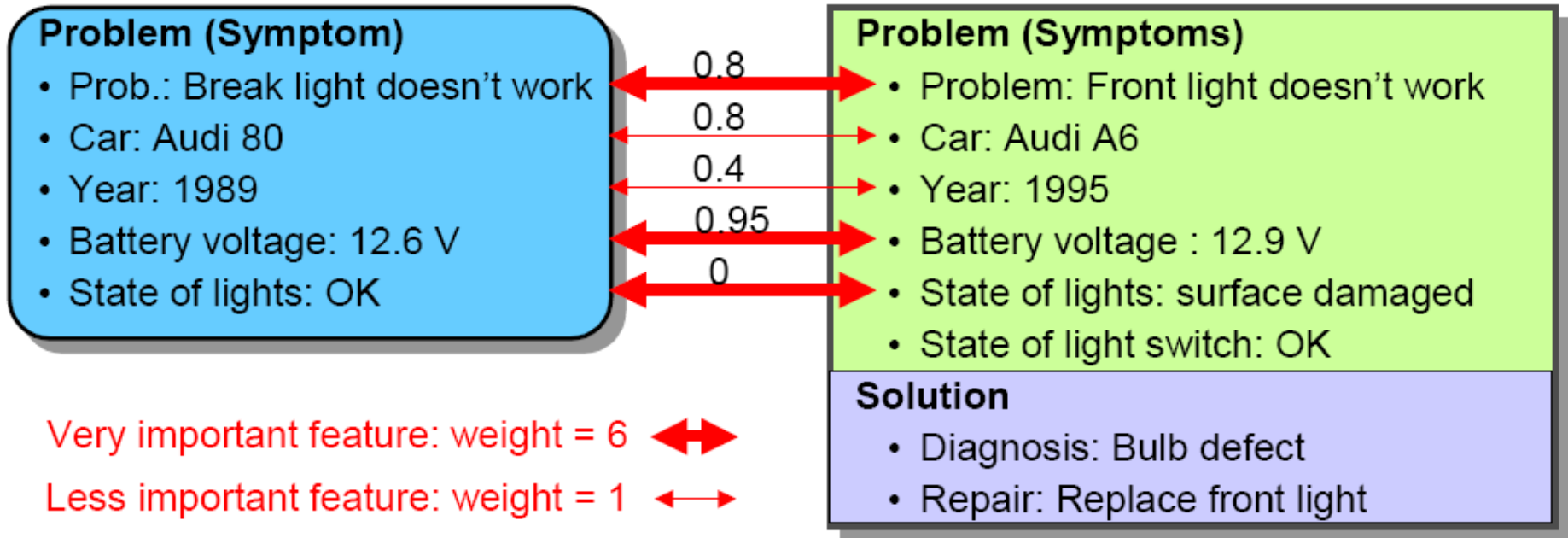
Compare New Problem and Case 1

Problem (Symptom)
- Prob.: Break light doesn't work
- Car: Audi 80
- Year: 1989
- Battery voltage: 12.6 V
- State of lights: OK

0.8
0.4
0.6
0.9
1.0

Problem (Symptoms)
- Problem: Front light doesn't work
- Car: VW Golf II, 1.6 L
- Year: 1993
- Battery voltage: 13.6 V
- State of lights: OK
- State of light switch: OK

Solution
- Diagnosis: Front light fuse defect
- Repair: Replace front light fuse

Very important feature: weight = 6 ◄►
Less important feature: weight = 1 ◄—►
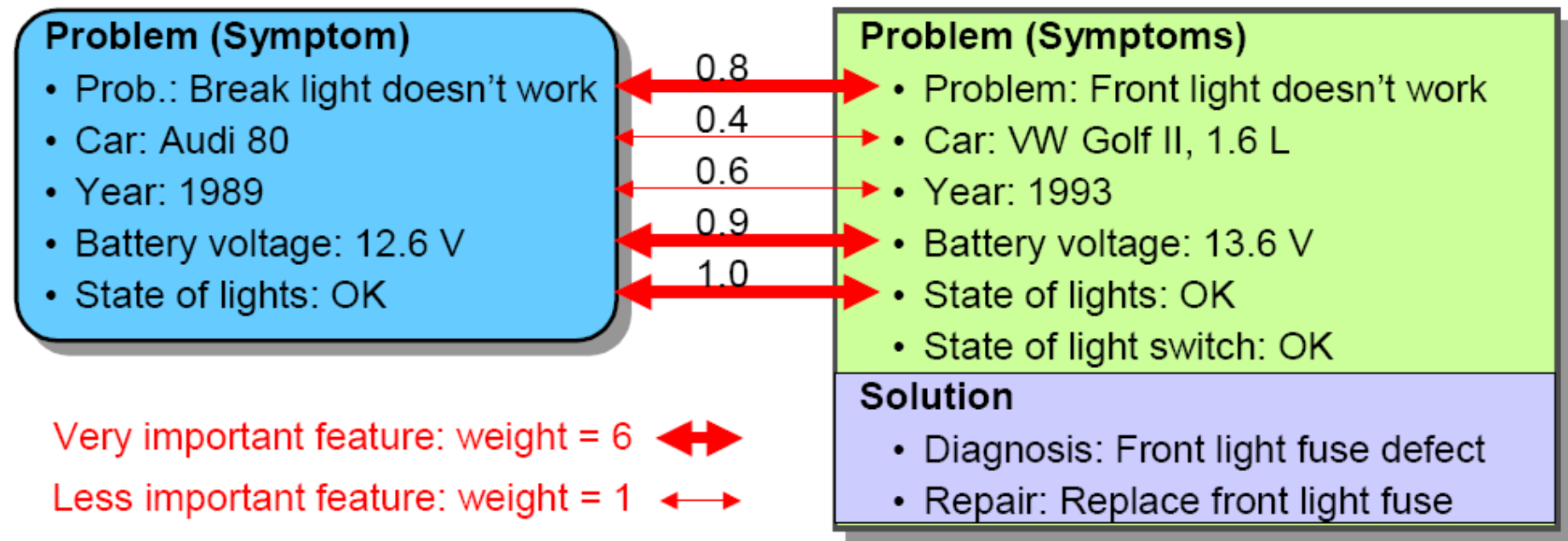
Similarity Computation by Weighted Average

$$similarity(new, case\ 1) = 1/20 * [\ 6*0.8 + 1*0.4 + 1*0.6 + 6*0.9 + 6*1.0\ ] = \mathbf{0.86}$$

Source: Ralph Bergmann

Compare New Problem and Case 2

Problem (Symptom)
- Prob.: Break light doesn't work
- Car: Audi 80
- Year: 1989
- Battery voltage: 12.6 V
- State of lights: OK

0.8
0.8
0.4
0.95
0

Problem (Symptoms)
- Problem: Front light doesn't work
- Car: Audi A6
- Year: 1995
- Battery voltage : 12.9 V
- State of lights: surface damaged
- State of light switch: OK

Solution
- Diagnosis: Bulb defect
- Repair: Replace front light

Very important feature: weight = 6 ⬌

Less important feature: weight = 1 ↔

Similarity Computation by Weighted Average

$similarity(new, case\ 2) = 1/20 * [\ 6*0.8 + 1*0.8 + 1*0.4 + 6*0.95 + 6*0\ ] = \mathbf{0.585}$
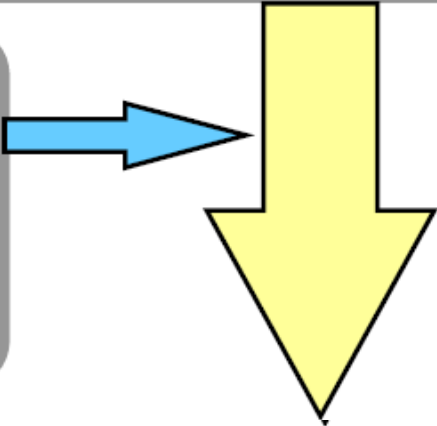
Case 1 is more similar: due to feature "State of lights"

Source: Ralph Bergmann

Reuse Solution of Case 1

C A S E	**Problem (Symptoms):** • **Front light** doesn't work • ... **Solution:** • Diagnosis: **Front light** fuse defect • Repair: Replace **front light** fuse

Problem (Symptom):1
- Prob.: Break light doesn't work
- Car: Audi 80
- Year: 1989
- Battery voltage: 12,6 V
- state of break light: OK

Adapt Solution:
How do differences in the problem affect the solution?

- **New Solution:**
  - Diagnosis: **Break light** fuse defect
  - Repair: Replace **break light** fuse

Source: Ralph Bergmann

Store the New Experience

■ If diagnosis is correct: store new case in the memory.

CASE 3

Problem (Symptoms):
- Problem: Break light doesn't work
- Car: Audi 80
- Year: 1989
- Battery voltage: 12.6 V
- State of break lights: OK
- light switch clicking: OK

Solution:
- Diagnosis: break light fuse defect
- Repair: replace break light fuse
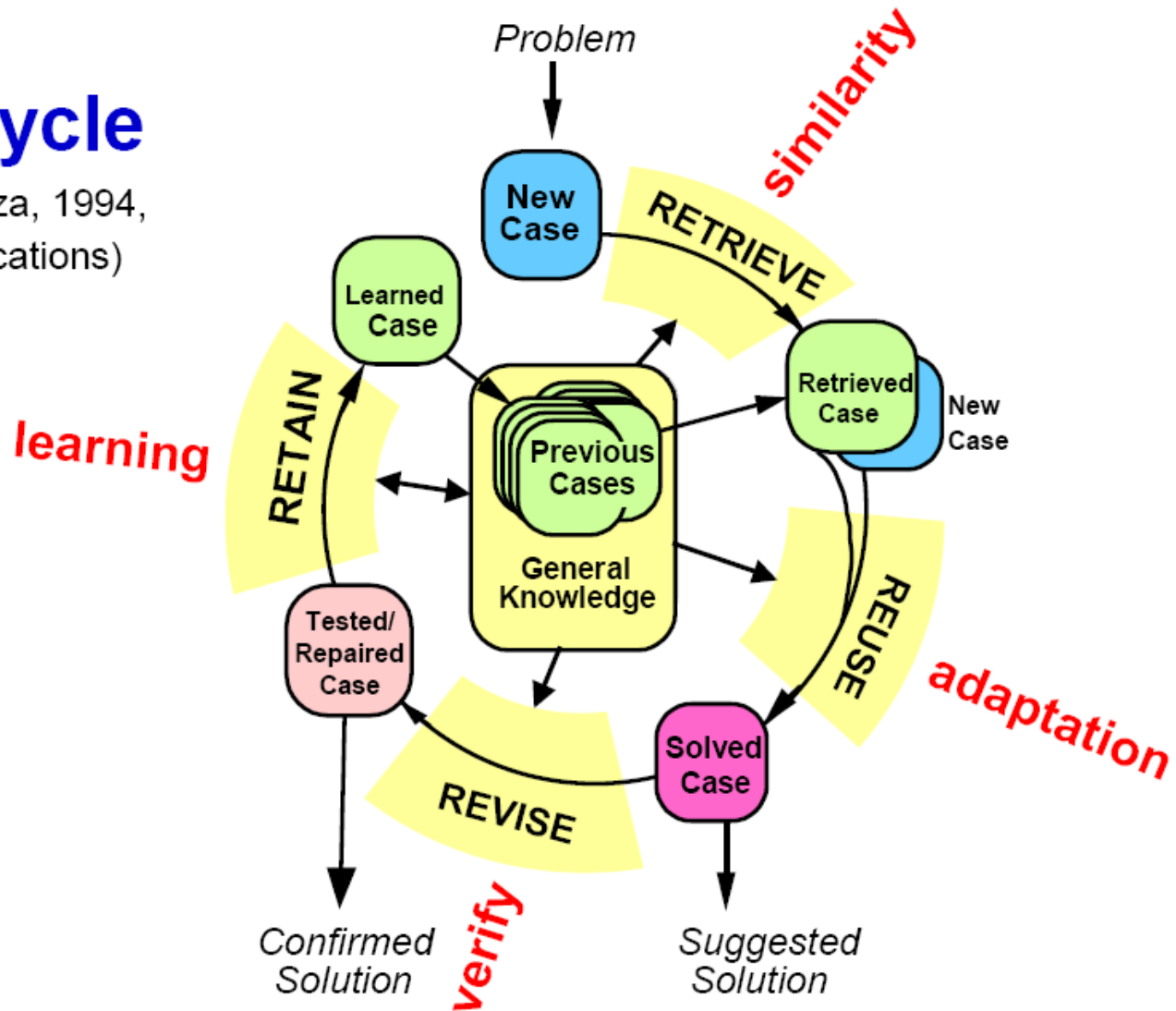
Source: Ralph Bergmann
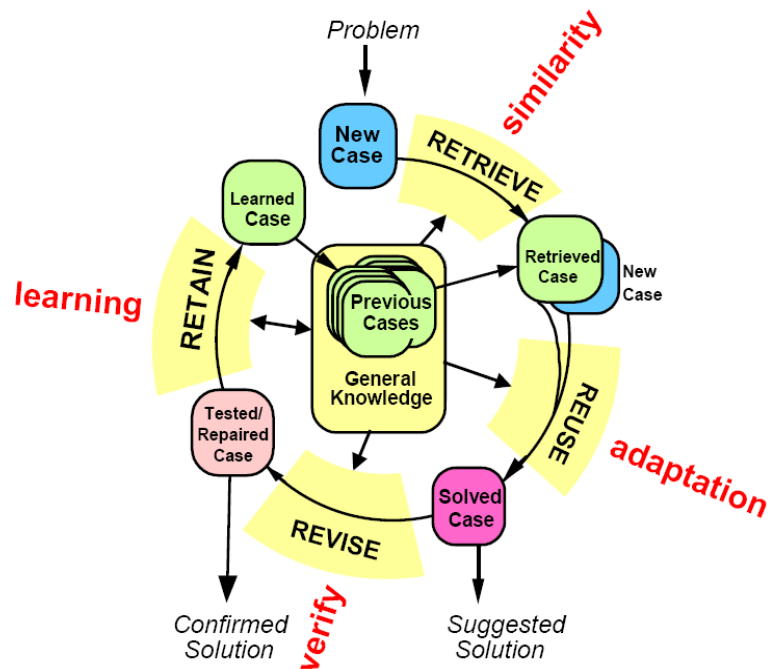
CBR Cycle

(Aamodt & Plaza, 1994,
AI Communications)

Source: Ralph Bergmann

CBR Cycle

■ Retrieve …

… most similar case or cases

■ Reuse …

… the information and knowledge in that case to solve the problem

■ Revise …

… the proposed solution if necessary

■ Retain …

… the parts of this experience likely to be useful for future problem solving

Source: K.-D. Althoff & A. Aamodt: Relating case-based problem solving and leanring methods to task and domain characteristics. AI Communications 1996

Case Representation

Textual Approach

Frequently Asked Question 241
Title: Order numbers of CPUs with which communications is possible.

Question: Which order numbers must the S7-CPUs have to be able to run basic communications with SFCs?

Answer: In order to participate in communications via SFCs without a configured connection table, the module concerned must have the correct order number. The following table illustrates which order number your CPU must have to be able to participate in these S7 homogeneous communications.

Conversational Approach

Case: 241
Title: Printer does not work in the new release.

Q1: *What kind of problem do you have?* Printer Problem
Q1: *Does the printer perform a self-test?* Yes
Q2: *Does the printer work with other software?* Yes
Q3: *Did you just install the software?* Yes
Q4: *Did you create a printer definition file?* Yes
Q5: *What release did you install?* 4.2

Problem: Installation procedure overrides printer definition
Action: Reinstall the printer from disk 2.3

Structural Approach

Reference : AD8009
Price : 2.25
Input offset voltage : 2 mV
Input bias current : 50 uA
Output voltage : 1.2 V
Output current drive : 175 mA
Single supply : No
PSPS : 70 dB
Number of devices per package : single
Available Package(s) : SOIC

(Bergmann 2002, p. 54ff)

Structural Cases: Attribute Value Representation

- A common approach for structural case representations are attribute value pairs

  Example: *Price:* 25.000 CHF

  attribute   value

- As in programming language, types define allowed values for attributes. Examples of types are
  - *numbers types* like integer, real or intervals
  - *symbols* defined by
    - enumaration of symbols {red, yellow, green}
    - elements of a knowledge structure, e.g. classification scheme
  - *text* such as strings or markups
  - *special types* like URLs

(Bergmann 2002, p. 62)

Example: Searching for Used Car

car 1:
- ♦ model: Audi A4
- ♦ year: 2004
- ♦ price: 15'000
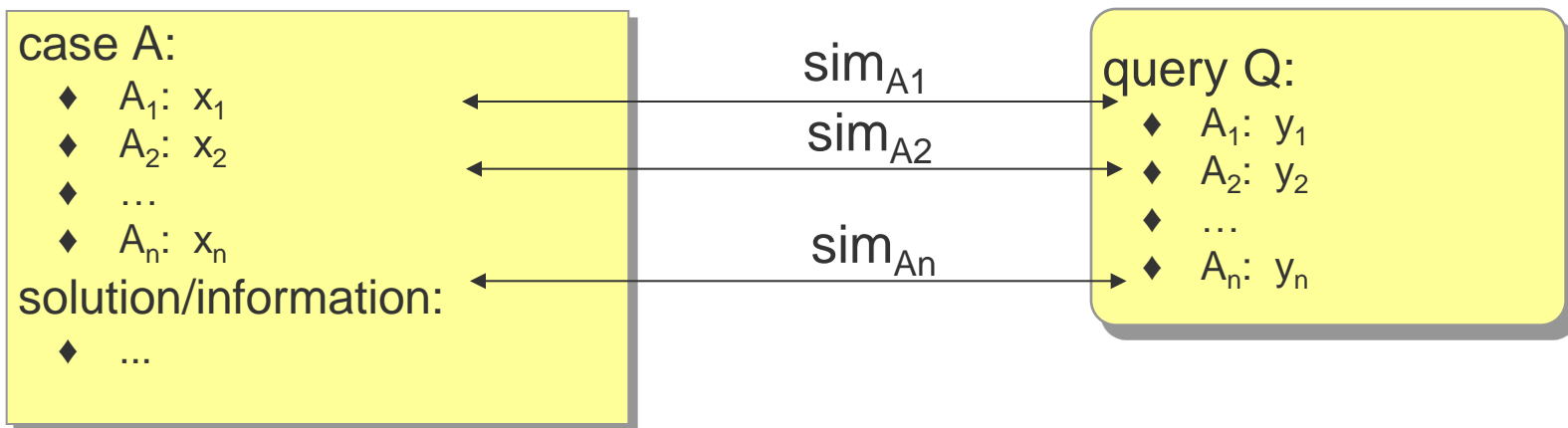- ♦ type: station waggon
- ♦ transmission: automatic
- ♦ extra: AC

query:
- ♦ model: VW Golf
- ♦ year: 2006
- ♦ price: 18'000
- ♦ type: station waggon
- ♦ transmission: 4 gear
- ♦ extra: AC, handsfree-kit

car 2:
- ♦ model: Audi A3
- ♦ year: 2005
- ♦ price: 20'000
- ♦ type: limousine
- ♦ transmission: 5 gear
- ♦ extra: CD, handsfree-kit

Similarity Calculation for Attribute-Value Pairs

Cases resp. meta-data are represented by n attributes $A_1, ..., A_n$

♦ each attribute A_i has type T_i

case A:
- A_1: x_1
- A_2: x_2
- …
- A_n: x_n

solution/information:
- …

sim_{A1}

sim_{A2}

sim_{An}

query Q:
- A_1: y_1
- A_2: y_2
- …
- A_n: y_n

Local similarity: for each attribute a similarity function is defined

♦ $sim_{Ai}(x_i, y_i): T_i \times T_i \rightarrow [0..1]$

♦ local similarity measures depend on the type of the attribute

Global similarity: combining values for local similarity

♦ $sim(A,Q) = F(sim_{A1}(x_1,y_1), sim_{A2}(x_1,y_1), …, sim_{An}(x_n,y_n))$

♦ $F:[0..1]n \rightarrow [0..1]$ is called an **aggregation function**

Similarity Measure

Definition: A *similarity measure* is a function *sim:* M x M $\rightarrow$ [0,1]

■ This definition restricts similarity to a number in the intervall [0,1].

♦ It allows to express the most similar (1) und the least similar (0) situation

♦ It also allows to express degrees of similarity: if *sim(x,y) > sim(x,z),* then x is more similar to y than to z

(Bergmann 2002, p. 96)

CBR vs. Database Query

- Remark: Query answering in SQL can be seen as a special case of a similarity measure where the value range is a set {0,1}
  - ◆ sim(x,y) = 1 (equal)   or
  - ◆ sim(x,y) = 0   (unequal)

Database System	CBR System
• simple search ⇒ "all or nothing"	• using same database but search for most similar cases
• often too many hits (underspecification) or no hits at all (overspecification)	• system can be told to show only, e.g., 10 cases by descending order
• no specific domain knowledge used for the search	• considers domain knowledge for search by using similarity measures, e.g., spatial or geographical relations
• pure database applications cannot be used for online consulting	• online consulting is the power of a CBR system

(Bergmann 2002, p. 96)

Usual Properties for Similarity Measures

■ A similarity measure is called *reflexive* if

$$sim(x,x) = 1$$

holds for all x [1]

■ If additionally *sim(x,y) = 1* implies that x = y, then the similarity measure is called *strong reflexive*

■ A similarity measure is called *symmetric* if for all x,y it holds that

$$sim(x,y) = sim(y,x)$$

Symmetric: it is not important for the similarity computation which attribute value belongs to the query and which one belongs to the case

Assymetric: it is important which value belongs to the query and which one belongs to the case; a major example is the attribute *price*.

(Bergmann 2002, p. 101f)

[1] This means that for every value x of M, x is maximally similar to itself

Meanings of Similarity. Similarity …

… always refers to a specific aspect
- ♦ Example: Two cars are similar if they
  - are of the same brand
  - have similar maximum speed

… is not necessarily transitive
- ♦ Example: For integers we can say that
  - 2 is similar to 4
  - 4 is similar to 6
  - ...
  - 99998 is similar to 100.000

But: Is 2 similar to 100.000?

… is not necessarily symmetric
- ♦ if I look for a limousine, I probable could accept a station wagon
- ♦ if I need a station wagon because of the space, a limousine might not be acceptable for me

Approximating Utility with Similarity

- Assumption
  - ♦ Similar problems have similar solutions
  - ♦ The solution of a problem is also useful for similar problems

- Utility of a case c for a problem p is approximated by the similarity between the problem p and problem d contained in case c
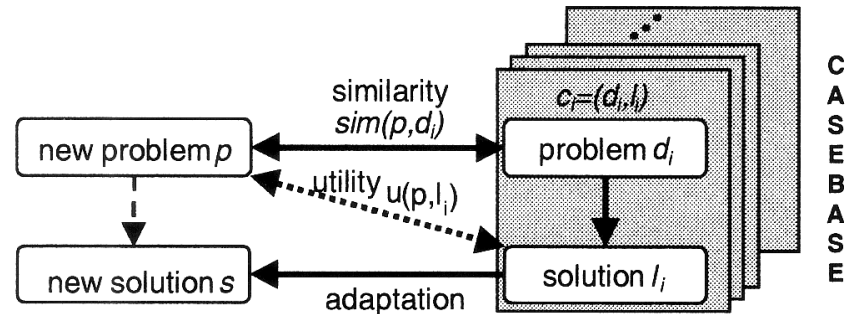
- Remark:
  - ♦ Utility is an *a posteriori* criteria: It can be assessed after after the problem was solved.
  - ♦ Similarity is an *a priori* criteria: It must be assessed before problem solving.
  - ♦ Utility corresponds to relevance in information retrieval

(Bergmann 2002, p. 94)

Local Similarity Measure for Numeric Attributes

■ For numeric attributes, similarity is computed as a function of distance d:

$$sim_A(x,y) = f(|d(x,y)|)$$

■ Typical distance functions are the following two:

♦ standard linear distance (= difference) $d(x,y) = |x - y|$

♦ logarithmic distance $d(x,y) = |\log(x) - \log(y)|$

(logarithmic distance is used for exponentially scaled value ranges. i.e. if the value range for the attribute spans several orders of magnitude)
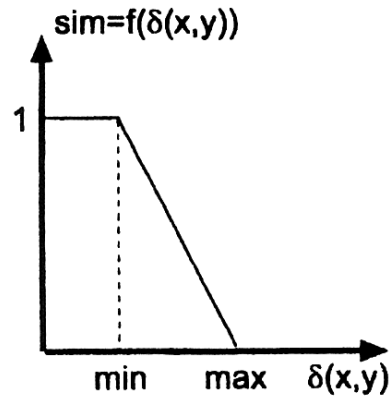
■ Examples of similarity measure for numeric attributes:

(Bergmann 2002, p. 107)

Further Functions for Numeric Similarity Measures

(Bergmann 2002, p. 183.)
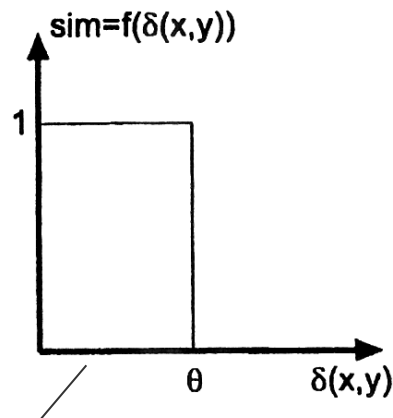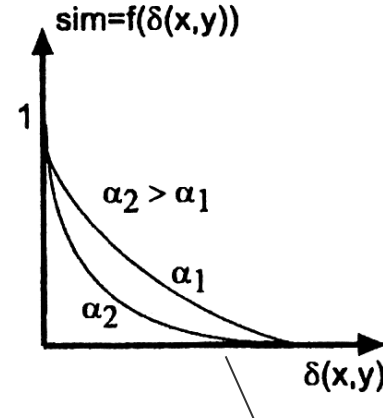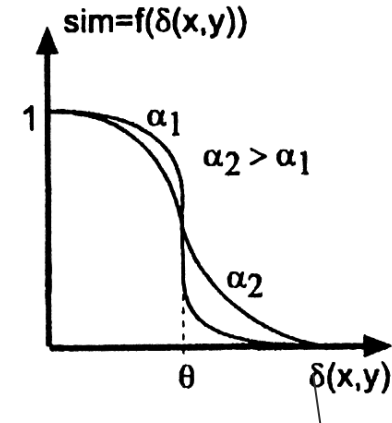
Symmetry for Numeric Values

■ Symmetric:

♦ it is not important for the similarity computation which attribute value belongs to the query and which one belongs to the case

♦ Example: for numeric values use the absolute distance *d = |query value - case value|*

● *sim(x.y) = f(d(x,y))*

■ Asymmetric:

♦ it is important which value belongs to the query and which one belongs to the case; a major example is the attribute *price*.

♦ similarity can be computed with two different similarity functions

$$sim(x,y) = \begin{cases} f_1(d(x,y)) & \text{if } x>y \\ 1 & \text{if } x=y \\ f_2(d(x,y)) & \text{if } x<y \end{cases}$$

Local Similarity for Ordered Symbols

- For symbolic attributes we can distinguish approaches depending on whether there is an order defined on the symbols or not.

- Example for ordered symbols: qualitative values, e.g. {small, medium, large}
  - ♦ small < medium < large

- With such an order defined, we can determine the similarity by using the ordinal number of the symbols, e.g.
  - ♦ small   -->   1
  - ♦ medium   -->   2
  - ♦ large   -->   3

  and applying similarity measure for numeric attributes

Local Similarity for unordered Symbols

■ If there is no obvious ordering on the set of attribute values and no odering can be defined, we can apply the tabular approach

♦ $sim_A(x,y) = s[x,y]$

cases y

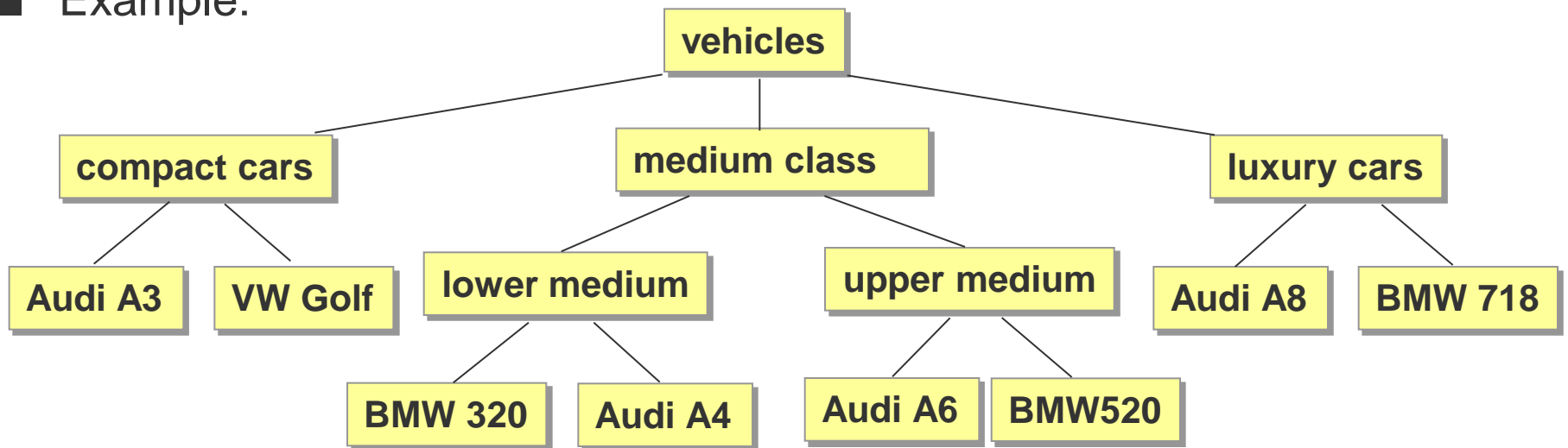| s[x,y]	limousine	convertible	station
limousine	1	0.3	0.7
convertible	0.4	1	0.2
station	0.5	0.2	1

query x

similarity of x and y

■ Reflexive similarity measure: diagonal values are 1

■ Symmetric similarity measure:
     upper triangular matrix = lower triangular matrix

cp. (Bergmann 2002, p. 110)

Taxonomically Ordered Symbolic Types

■ A special variant of symbolic types are taxonomies. A taxonomy is a tree in which the nodes represent symbolic values

■ A taxonomy represents an additional releationship between the symbols

  ♦ Leaf nodes represent concrete objects of the real world
  ♦ Inner nodes represent classes of real world objects.
  ♦ An inner node k stands for the set of real world objects represented by leaf nodes below it

■ Example:

```
                         vehicles
          ┌─────────────────┼─────────────────────┐
    compact cars      medium class            luxury cars
     ┌──────┴──────┐   ┌──────┴──────┐         ┌─────┴─────┐
  Audi A3      VW Golf lower medium  upper medium  Audi A8  BMW 718
                    ┌────┴────┐   ┌─────┴─────┐
                 BMW 320   Audi A4  Audi A6  BMW520
```
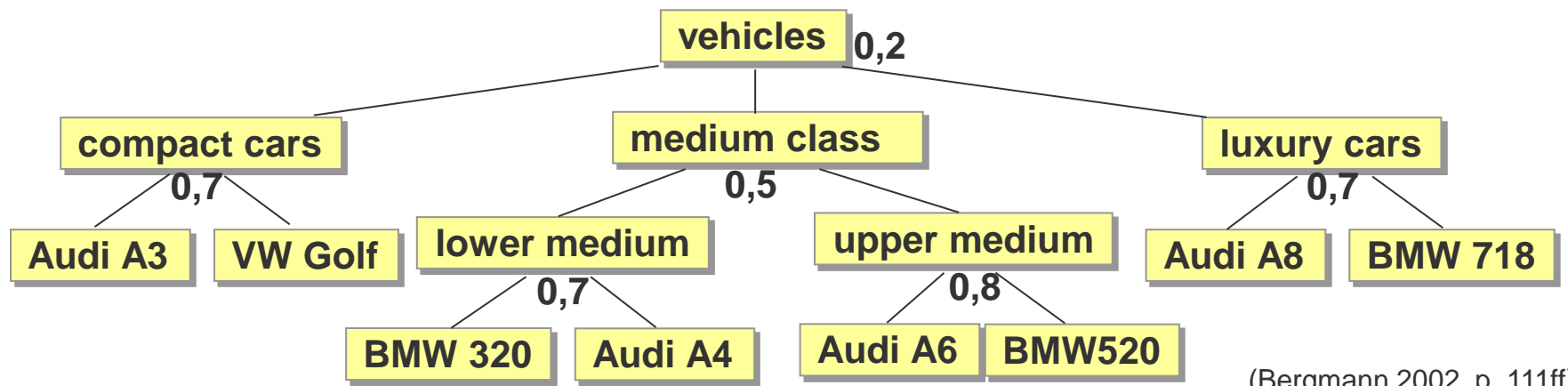
(Bergmann 2002, p. 111ff)

Similarity Measure for Taxonomies

- Inner nodes cluster real-world objects that have some properties in common. The deeper we decent in the taxonomy, the more features do the objects have in common

- Similarity measures in a taxonomy
  - Every inner node K_i is annotated with a similarity value S_i
  - The deeper the nodes in the hierarchy, the larger the similarity value can become
  - Meaning: The elements belonging to a class have the similarity measure assigned to the class

```
                        vehicles  0,2
              _____/   |   _____
             /                |                \
      compact cars        medium class        luxury cars
          0,7                 0,5                 0,7
        /    \             /        \            /     \
   Audi A3  VW Golf   lower medium  upper medium  Audi A8  BMW 718
                          0,7           0,8
                        /    \        /     \
                  BMW 320  Audi A4  Audi A6  BMW520
```
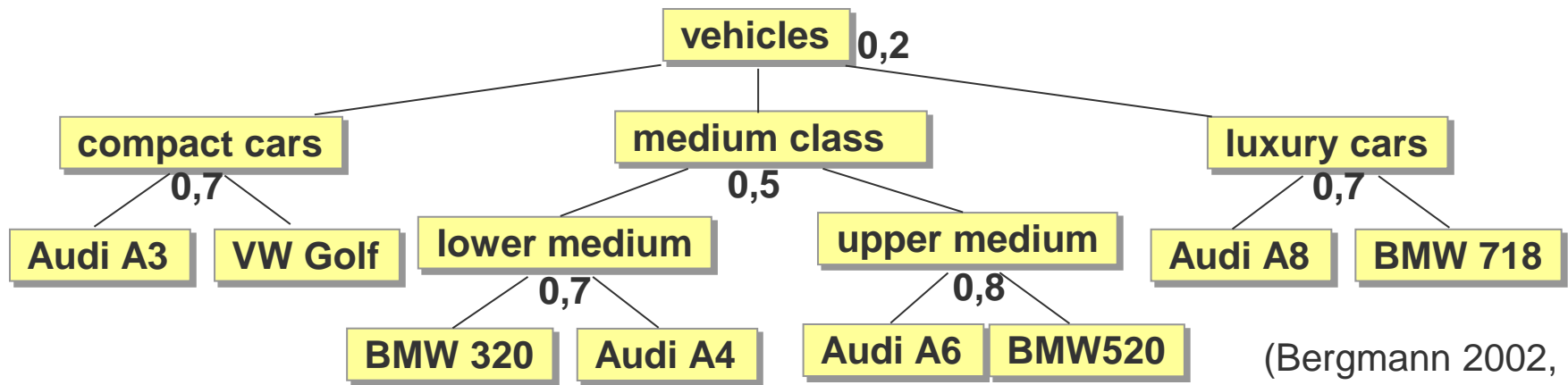
(Bergmann 2002, p. 111ff)

Computing Similarity Measure Leaf nodes of a Taxonomy

- Leaf nodes represent concrete element objects

- Similarity of two leaf nodes is the similarity value of the lowest common predecessor

$$sim(K_1, K_2) = \begin{cases} 1 & : & K_1 = K_2 \\ S_{\langle K_1, K_2 \rangle} & : & K_1 \neq K_2 \end{cases}$$

Example:
sim(BMW320, AudiA4) =
sim(BMW320, AudiA6) =
sim(BMW320, AudiA8) =

vehicles 0,2

compact cars 0,7

medium class 0,5

luxury cars 0,7

Audi A3   VW Golf

lower medium 0,7

upper medium 0,8

Audi A8   BMW 718

BMW 320   Audi A4   Audi A6   BMW520

(Bergmann 2002, p. 111ff)

Semantics and Similarity of Inner Nodes
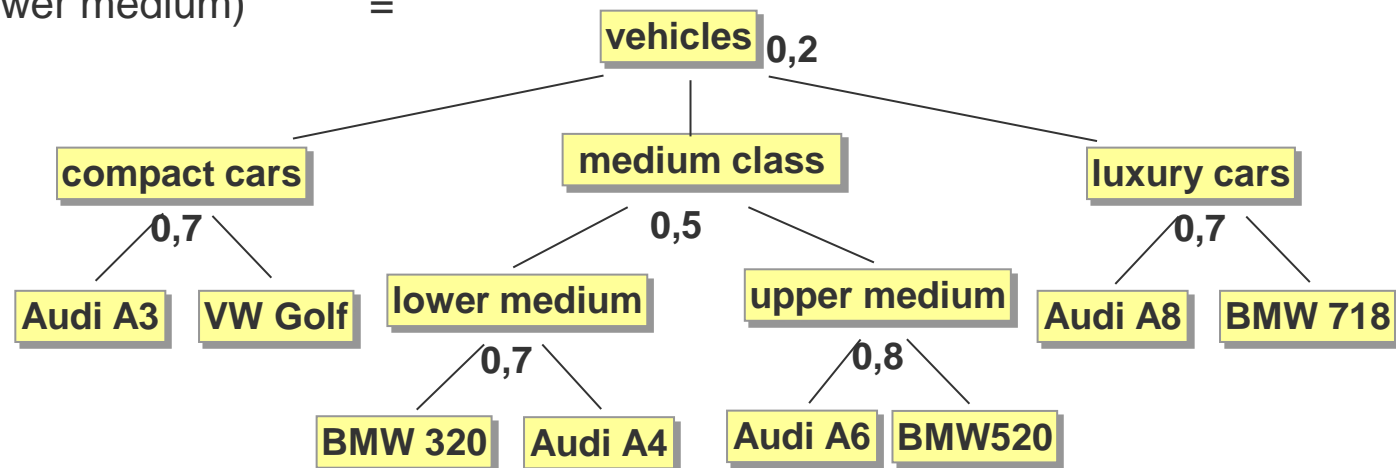
An inner node can have different meanings

- Any value in the query: The inner node in the query stands for any value below this node.

  - Example: A person is looking for a compact car but does not care whether it is an Audi A3 or a VW Golf

  - Sample query: „compact car"

- Any value in the Case: The inner value stands for any value below the node

  - Assume a car dealer specifies that he sells any compact cars

- Uncertainty: The use of an inner node K means that we do not know the exact value for this attribute, but we know that it must be a concrete value below this node

  - Assume a car dealer specifies that he sells a single compact car without saying whether it is a Audi A3 or a VW Golf

(Bergmann 2002, p. 111ff)

Similarity with inner Nodes of a Taxonomy: Any Value (1/2)

We are looking for the highest possible similarity sim(Q,C) [1]

- sim(AudiA3, compact car) =
- sim(compact car, AudiA3) =
- sim(AudiA4, medium class) =
- sim(medium class, AudiA4) =
- sim(AudiA3, medium class) =
- sim(medium class, AudiA3) =
- sim(medium class, compact car) =
- sim(lower medium, medium class) =
- sim(medium class, lower medium) =

```
                              vehicles 0,2
              ┌──────────────────┼──────────────────┐
      compact cars          medium class          luxury cars
         0,7                    0,5                   0,7
       ┌────┴────┐          ┌────┴────┐            ┌────┴────┐
   Audi A3   VW Golf   lower medium  upper medium  Audi A8  BMW 718
                          0,7           0,8
                        ┌──┴──┐       ┌──┴──┐
                   BMW 320  Audi A4  Audi A6  BMW520
```

[1] The first parameter of sim(Q.C) is the query, the second parameter is the case description

Similarity with inner Nodes of a Taxonomy: Any Values (2/2)

■ Inner node in the query or in the case
  ♦ All leaf nodes below the inner node have similarity 1
  ♦ For all other nodes: Take the similarity of the lowest common predecessor.

$$sim_{A_i}(Q,C) = \max\{sim(q,c)|q \in L_Q, c \in L_C\}$$

$$= \begin{cases} 1 & : \quad C < Q \text{ or } Q < C \\ S_{\langle Q,C \rangle} & : \quad \text{otherwise} \end{cases}$$

Notation:
Q   Query
C   Case
L_Q, L_C leaf nodes below Q or C
C < Q   C is below Q in the taxonomy
             (Q is predecessor of C)
$S_{<Q,C>}$ similarity of the lowest common predecessor of Q and C

(Bergmann 2002, p. 118)

Similarity with inner Nodes of a Taxonomy: Uncertainty in Query (1/2)

- Optimistic approach computes the upper bound for the similarity $sim(Q,c)$ [1]
  - ♦ sim(compact car, AudiA3)   =
  - ♦ sim(medium class, AudiA4)   =
  - ♦ sim(medium class, AudiA3)   =

- Pessimistic approach computes the lower bound for the similarity $sim(Q,c)$ [1]
  - ♦ sim(compact car, AudiA3)   =
  - ♦ sim(medium class, AudiA4)   =
  - ♦ sim(medium class, AudiA3)   =
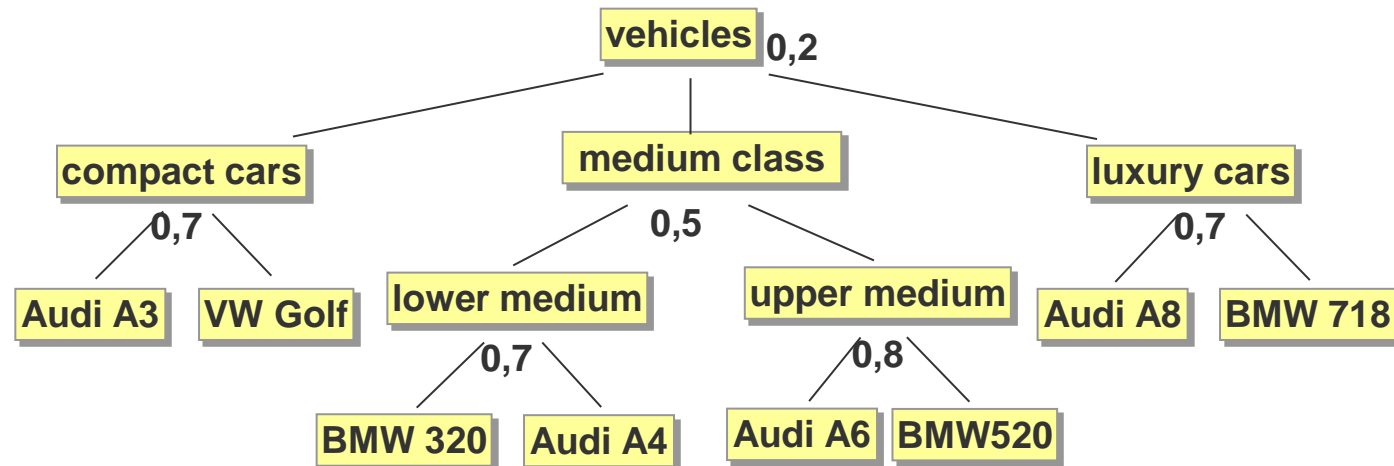
[1] The first parameter of sim(Q.C) is the query, the second parameter is the case description

Similarity with inner Nodes of a Taxonomy: Uncertainty in Query(2/2)

- Optimistic approach: Upper bound

$$sim_{A_i}(Q, c) = \max\{sim(q, c) | q \in L_Q\} \quad = \begin{cases} 1 & : \quad c < Q \\ S_{\langle Q, c \rangle} & : \quad \text{otherwise} \end{cases}$$
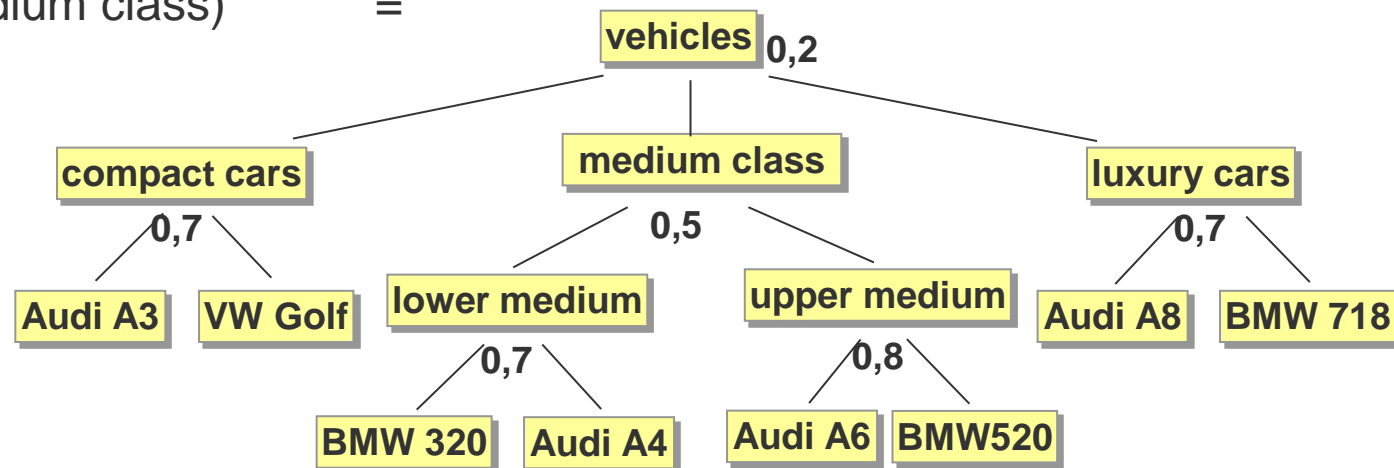
- Pessimistic approach: Lower bound

$$sim_{A_i}(Q, c) = \min\{sim(q, c) | q \in L_Q\} = S_{\langle Q, c \rangle}$$

(Bergmann 2002, p. 119)

Similarity with inner Nodes of a Taxonomy: Uncertainty in Case (1/2)

- Optimistic approach computes the upper bound for the similarity sim(q,C) [1]
  - ♦ sim(AudiA3, compact car)   =
  - ♦ sim(AudiA4, medium class)   =
  - ♦ sim(AudiA3, medium class)   =

- Pessimistic approach computes the lower bound for the similarity sim(q,C) [1]
  - ♦ sim(AudiA3, compact car)   =
  - ♦ sim(AudiA4, medium class)   =
  - ♦ sim(AudiA3, medium class)   =

[1] The first parameter of sim(Q.C) is the query, the second parameter is the case description

Similarity with inner Nodes of a Taxonomy: Uncertainty in Case (2/2)

- Optimistic approach: Upper bound

$$sim_{A_i}(q, C) = \max\{sim(q, c) | c \in L_C\} = \begin{cases} 1 & : & q < C \\ S_{\langle q, C \rangle} & : & \text{otherwise} \end{cases}$$

- Pessimistic approach: Lower bound

$$sim_{A_i}(q, C) = \min\{sim(q, c) | c \in L_C\} = S_{\langle q, C \rangle}$$

(Bergmann 2002, p. 118)

Multiple Attribute Values

- Attributes can contain multiple values
  - $A_{query} = \{a_1, ..., a_n\}$
  - $A_{case} = \{b_1, ..., b_m\}$

- Similarity measure for sets:
  - Compute all pairs of similarity measrues $sim_A(a_i, b_j)$
  - Aggregate the local similarity

  $sim_A(A_{query}, A_{case})$
  $$= MF(sim_A(a_1, b_1), ..., sim_A(a_1, b_m), ..., sim_A(a_n, b_1), ..., sim_A(a_n, b_m))$$

- There are various possible approaches for the aggregate function MF, e.g.
  - minimum
  - maximum
  - average

Derived Attributes

- Occasionally, the attributes themselves are not significant for similarity measurement

- A derived attribute is an attribute, whose value is computed from other attributes

- Example: credits

derived Attribut

Attribute	case 1	case 2	case 3	query
income:	1000	**2000**	5000	2000
expenses:	**1500**	5000	4500	1500
creditworthy:	no	no	yes	?
balance	-500	-3000	**500**	**500**

The derived attribute „balance" is used for similarity measurement, i.e. first the balance has to be computed for each case and the query and then the similarity for the balances is computed.

Unknown Attribut Values

■ It often occurs that attribute values are not known (NULL):

■ Strategies to deal with unknown values

- ♦ **optimistic strategy**: Assume that unknown values are most similar: sim(NULL,x) = 1

- ♦ **pessimistic strategy**: Assume that unknown values are least similar: sim(NULL,x) = 0.

- ♦ **strategy of expected value**: Use an expected value, e.g. based on probability or average

- ♦ ignore the attributes

Global Similarity

■ Global similary measures are defined by applying an aggregation function $F : [0..1]^n \rightarrow [0..1]$ to the local similarity values.

♦ Input: Local similarity measures $sim_{Ai}(x_i, y_i)$ for each attribute A_i

♦ Global similarity:

$$sim(x,y) \quad = F\,(sim_{A1}(x_1, y_1), ..., sim_{An}(x_n, y_n))$$

■ Possible properties for F

● F is monotone in each argument

● $F(0,...,0) = 0$

● $F(1,...,1) = 1$

Basic Aggregation Functions

- Weighted Average: $F(s_1,...,s_n) = \sum_{i=1}^{n} w_i \cdot s_i$ with $\sum_{i=1}^{n} w_i = 1$

- Generalized
  weighted average: $F(s_1,...,s_n) = \sqrt[\alpha]{\sum_{i=1}^{n} w_i \cdot s_i^{\alpha}}$ with $\alpha \in IR^+$ und $\sum_{i=1}^{n} w_i = 1$

- Maximum: $F(s_1,...,s_n) = \max_{i=1}^{n}(w_i \cdot s_i)$

- Minimum: $F(s_1,...,s_n) = \min_{i=1}^{n}(w_i \cdot s_i)$

(Bergmann 2002, p. 120f)