Advanced Topics in Software Engineering: Operational Laws

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Advanced Topics in Software Engineering
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Operational Laws

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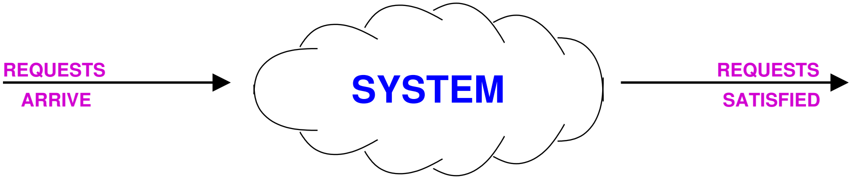
- The laws are very general and make almost no assumptions about the behaviour of the random variables characterising the system.
- Another advantage of the laws is their simplicity: this means that they can be applied quickly and easily by almost anyone.

Observable variables



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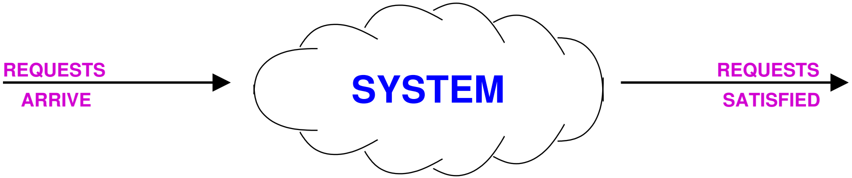

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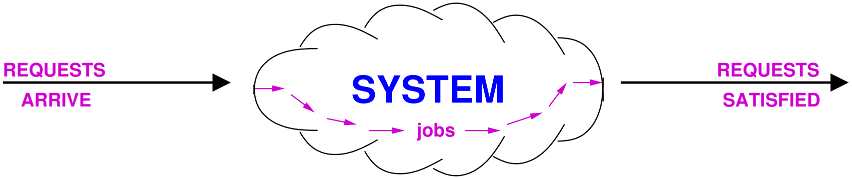

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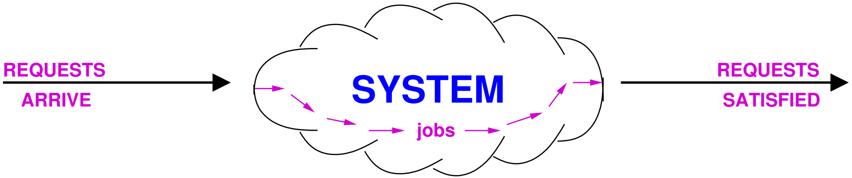

- Operational laws are based on observable variables — values which we could derive from watching a system over a finite period of time.
- We assume that the system receives requests from its environment.
- Each request generates a job or customer within the system.
- When the job has been processed the system responds to the environment with the completion of the corresponding request.

Observations and measurements

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- N, the average number of jobs in the system.

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$$S = B/C, \text{ the mean service time per completed job.}$$

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- We will assume that the system is job flow balanced. This means that the number of arrivals is equal to the number of completions during an observation period, i.e. $A = C$.

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- Note that if the system is job flow balanced the arrival rate will be the same as the completion rate, that is, $\lambda = X$.

Little's Law

$$N = XW$$

Little's Law

The average number of jobs N in a system is equal to the product of the throughput of the system X and the average time W spent in that system by a job.

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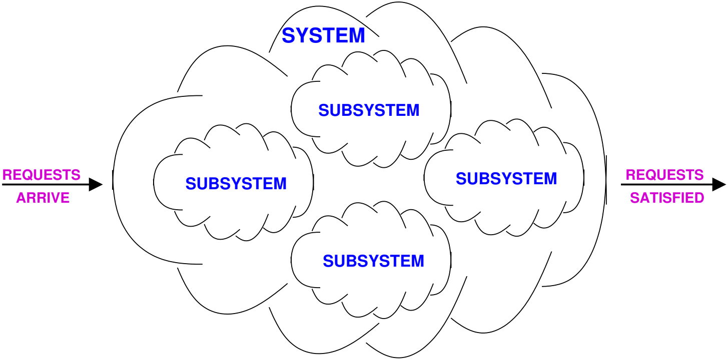
Little's law tells us that the average time spent at the disk by a request must be $4/40 = 0.1$ seconds.

If we know that each request requires 0.0225 seconds of disk service we can then deduce that the average queueing time is 0.0775 seconds.

Subsystems within Systems



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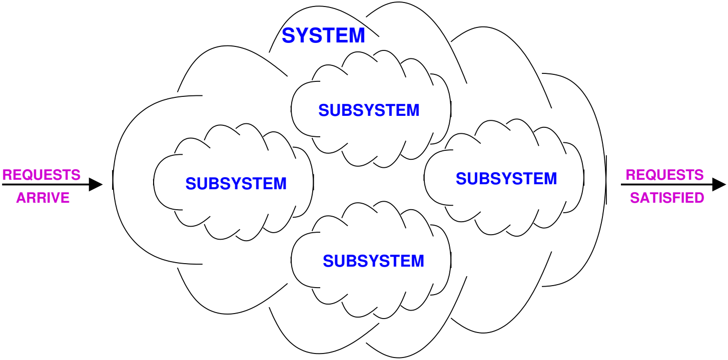

- A system may be regarded as being made up of a number of devices or resources.
- Each of these may be treated as a system in its own right from the perspective of operational laws.

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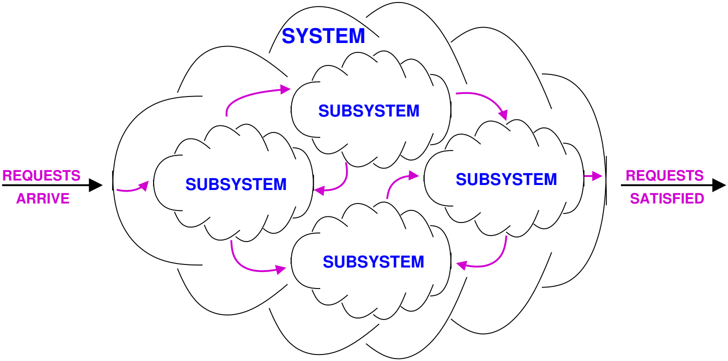

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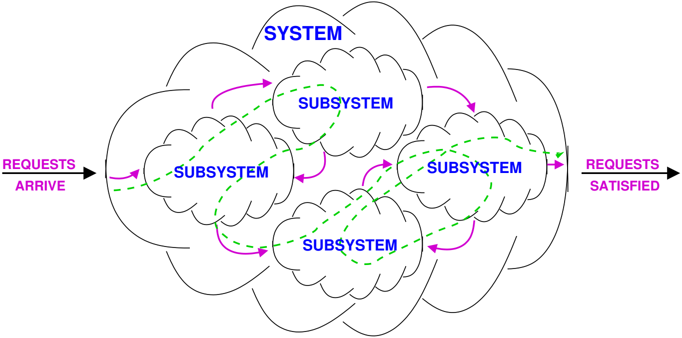

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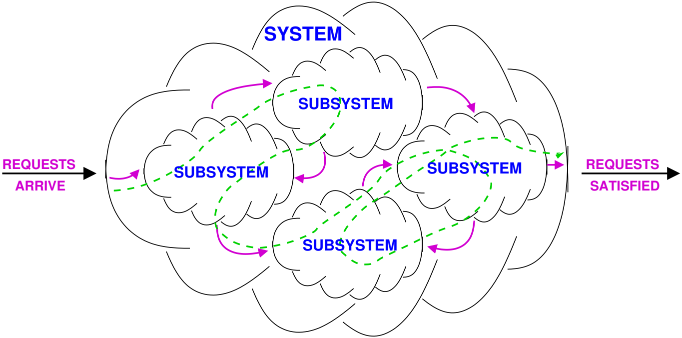
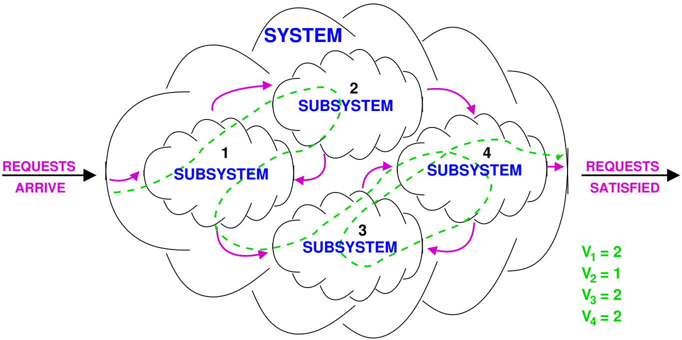
Visit count



In an observation interval we can count not only completions external to the system, but also the number of completions at each resource within the system.

Visit count



We define the visit count, V_i, of the ith resource to be the ratio of the number of completions at that resource to the number of system completions $V_i \equiv C_i/C$.

Visit count: example

For example, if, during an observation interval, we measure...

... 10 system completions

... and 150 completions at a specific disk

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Forced Flow Law

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Forced Flow Law

The throughput at the ith resource is equal to the product of the throughput of the system and the visit count at that resource.

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- Thus the press throughput is 4 widgets per minute.

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- (Note that service time is not necessarily the same as the residence time of the job at that resource: in general a job might have to wait for some time before processing begins.)
- The total amount of service that a system job generates at the ith resource is called the service demand, D_i:

$$D_i = S_i V_i$$

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- The utilisation law tells us that the utilisation of the disk must be $40 \times 0.0225 = 90\%$.

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- Applying Little's Law to the ith resource we see that $N_i = X_i W_i$, where N_i is the mean number of jobs at the resource and W_i is the average response time of the resource.
- From the Forced Flow Law we know that $X_i = XV_i$. Thus we can deduce that

$$N_i/X = V_i W_i.$$

General Residence Time Law

The total number of jobs in the system is clearly the sum of the number of jobs at each resource, i.e. $N = N_1 + \cdots + N_M$ if there are M resources. From Little's Law that $W = N/X$ and so:

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General Residence Time Law

The average residence time of a job in the system will be the sum of the product of its average residence time at each resource and the number of visits it makes to that resource.

General Residence Time Law: Example

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- On average each CPU burst requires 30 milliseconds (waiting + processing time).
- Monitoring has shown that the throughput of disk A is 15 requests per second and the average number in the buffer is 4 whilst at disk B the throughput is 10 requests per second and the average number in the buffer is 3.

General Residence Time Law: Example (continued)

Using Little's Law we calculate the residence time at each of the disks (remembering that the number in the system is the number in the buffer $+1$):

$$W_{diskA} = \frac{N_{diskA}}{X_{diskA}}$$

General Residence Time Law: Example (continued)

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$$W_{diskB} = \frac{N_{diskB}}{X_{diskB}} = \frac{4}{10/1000} = \frac{4000}{10}$$

General Residence Time Law: Example (concluded)

Then

$$W = W_{CPU} V_{CPU} + W_{diskA} V_{diskA} + W_{diskB} V_{diskB}$$

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$$\begin{aligned} W &= W_{CPU}V_{CPU} + W_{diskA}V_{diskA} + W_{diskB}V_{diskB} \\ &= 30 \times 126 + \frac{5000}{15} \times 75 + \frac{4000}{10} \times 50 \end{aligned}$$

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Interactive Response Time Law

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- More generally in interactive systems, jobs spend time in the system not engaged in processing, or waiting for processing: this may be because of interaction with a human user, or may be for some other reason.
- The key feature of such a system is that the residence time can no longer be taken as a true reflection of the response time of the system.

Example

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- At the end of this non-processing period the job generates a fresh request.

Think time, residence time, response time

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- The think time represents the time between processing being completed and the job becoming available as a request again.
- Thus the residence time of the job, as calculated by Little's Law as the time from arrival to completion, is greater than the system's response time.

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Note that if the think time is zero, $Z = 0$ and $R = W$, then the interactive response time law simply becomes Little's Law.

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- Then the interactive response time law tells us that the response time must be $64/2 - 30 = 2$ seconds.

Bottleneck analysis

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- The bottleneck resource is important because it limits the possible performance of the system.
- This will be the resource which has the highest utilisation in the system.

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- In general, there will be some contention in the system meaning that jobs have to wait for processing so the residence time will be larger than this, i.e. $W \geq D$

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- It follows that if we wish to improve throughput we should first concentrate on this resource—improving throughput at other resources in the system might have little effect on the overall performance.

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- Applying the Interactive Response Time Law to the throughput bound, $X \leq 1/D_{max}$ we obtain:

$$R = N/X - Z \geq ND_{max} - Z$$

- Applying Little's Law we obtain $W \geq ND_{max}$.

Asymptotic bound

Thus the asymptotic bound for residence time or response time is:

$$\underbrace{W \geq \max\{D, ND_{max}\}}_{\textbf{Residence Time Bound}}$$

$$\underbrace{R \geq \max\{D, ND_{max} - Z\}}_{\textbf{Response Time Bound}}$$

Bound on a lightly loaded system

- The bound on the throughput of an interactive system may be made tighter when the system is lightly loaded (i.e. the mean number of jobs, N, is small).

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$$X \leq \min\{1/D_{max}, N/(D + Z)\}$$

Throughput Bound (lightly loaded system)

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- If we change the number of visits that each job makes to a resource we might move the bottleneck.

Assumptions

- As mentioned in the introduction, the operational laws do not rely on many assumptions.
- The only explicit assumption we have made is that the system is job flow balanced—the same number of requests are completed by the system as arrive at the system.
- We are also implicitly assuming that this holds at each of the resources or devices within a system. A consequence of this is that jobs are not created or destroyed anywhere in the system. This is sometimes called conservation of work.
- We have also assumed that the system is homogeneous, that is, that the behaviour of jobs or resources within a system does not depend on the global state of the system.

To be continued. . .