$$\pi_i \cdot \sum_{j \neq j} q_{ij} = \sum_{i \neq j} \pi_j q_{ji}$$

$$\left( \sum_{i \neq j} \pi_j \cdot q_{ji} \right) - \underbrace{\pi_i \cdot \sum_{i \neq j} q_{ij}} = 0 \quad \Rightarrow \quad \sum_{i \neq j} \pi_j \cdot q_{ji} + \pi_i \cdot q_{ii} = 0$$

$$\pi_i \cdot \left( - \sum_{i \neq j} q_{ij} \right) \qquad\qquad \equiv$$

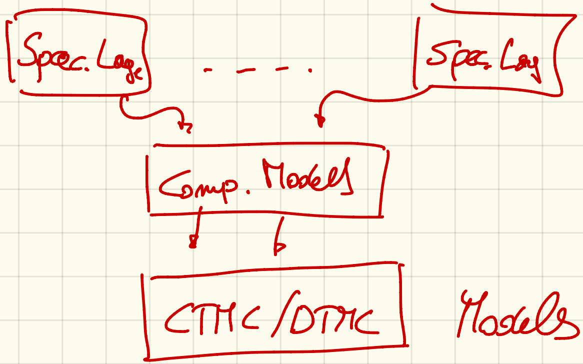
$$\parallel$$

$$\pi_i \cdot q_{ii} \qquad\qquad \sum_j \pi_j \cdot q_{ji} = 0$$

$$Q^T \pi = 0$$

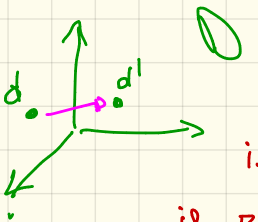
$$\overset{Q^T}{\begin{pmatrix} & & \\ & & \\ 1 & \cdots & 1 \end{pmatrix}} \overset{\pi}{\begin{pmatrix} \\ \\ \\ \end{pmatrix}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathbb{1} \cdot \pi = 1 \equiv \sum_i \pi_i = 1$$


Spec. Logic
- - - -
Spec. Log
Comp. Models
CTMC / DTMC
Models

# Population Models

- $S$ : a set of Species
- $\mathcal{D} \subseteq \mathbb{R}^{|S|}$
- $d_0$
- $\mathcal{R} = \{ \rho_0, \dots, \rho_m \}$ $\rho = (a, u, r)$




$\rho$ is "well-defined" iff

if $r(d) > 0$ then

$d + u \subseteq \mathcal{D}$

$d + u$

$r : \mathcal{D} \to \mathbb{R}_{\geq 0}$

Example: Bike Sharing

Users: $U, W, B, R$

Parking Stations: $P_{a,c}$ $(c = 10)$

$$S = \{ U, W, B, R \} \cup \bigcup_{i=0}^{c} \{ P_{i,c} \}$$

| $U$ | $W$ | $B$ | $R$ | $P_{0,c}$ | | $P_{c,c}$ |
|---|---|---|---|---|---|---|
| 100 | 10 | 50 | 5 | 0 | . . . . . | 25 |

$d$

$d(W) > 0$

$d(P_{k,c}) > 0$ $(k > 0)$

$d'$

$d'(W) = d(W) - 1$

$d'(B) = d(B) + 1$

$d'(P_{k,c}) = d(P_{k,c}) - 1$

$d'(P_{k-1,c}) = d(P_{k-1,c}) + 1$

Let $\mathcal{P} = (S, \mathcal{D}, d_0, \mathcal{R})$, $\mathcal{P}$ is "population preserving"

$$\forall \rho = (a, u, r) \in \mathcal{R} : \quad \sum_{s \in S} u(s) = 0$$

Notations:

- $\forall d \in \mathcal{D}, \rho = (a, u, r) \in \mathcal{R}$: $d \xrightarrow{\rho} d' \iff r(d) > 0 \quad d' = d + u$
- $d \xrightarrow{\mathcal{R}} d' \iff \exists \rho : d \xrightarrow{\rho} d' \quad (\rho \in \mathcal{R})$
- $d \in \mathcal{D}$ Reach$(d)$ is the smallest set $X$:

1. $d \in X$
2. $\forall d' : d' \in X . \ \forall d'' : d' \xrightarrow{\mathcal{R}} d'' . \ d'' \in X$

Let $\mathcal{P} = (S, \mathcal{D}, d_0, \mathcal{R})$

$$CTMC(\mathcal{P}) = (Reach(d_0),\ R_{\mathcal{P}})$$

$$R_{\mathcal{P}}[d_1, d_2] = \sum_{\rho = (a, v, r) \,:\, d_1 + v = d_2} r(d_1)$$

$$d_1 \xrightarrow{\rho_1} d_2$$
$$d_1 \xrightarrow{\rho_2} d_2 \qquad (\rho_1 \neq \rho_2)$$

SEIR

- S
- E
- I
- R

$$S \quad E \rightsquigarrow E \quad E$$

$$S \quad I \rightsquigarrow E \quad I$$

$$E \rightsquigarrow I$$

$$I \rightsquigarrow R$$

$$R \rightsquigarrow S$$

$1_x$ $\qquad$ $d_1 + d_2$ $\qquad$ $\kappa \cdot d$

$S \quad E \rightsquigarrow E \quad E \qquad \rho(SE, 1_E - 1_S, \lambda)$

$-1_S - \cancel{1_E} + \cancel{1_E} + 1_E$

$$r_{SE}(d) = \begin{cases} 0 & \text{if } d(S) = 0 \wedge d(E) = 0 \\ \lambda_m \cdot \frac{d(E)}{|d|} \cdot p_i \cdot d(S) & d(S) > 0 \wedge d(E) > 0 \end{cases}$$

---

$\rho(SI, 1_E - 1_S, r_{SI})$

$$r_{SI}(d) = \begin{cases} \lambda_m \cdot \frac{d(I)}{|d|} \cdot p_I \cdot d(S) & d(S) > 0 \wedge d(I) > 0 \\ 0 & \text{otherwise} \end{cases}$$

$E \longrightarrow I$

$$f_E = (E,\ 1_I - 1_E,\ r_E)$$

$$r_E(d) = \lambda_{EI} \cdot d(E)$$

$I \longrightarrow R$

$$f_I = (I,\ 1_R - 1_I,\ \lambda d.\ \lambda_{IR} \cdot d(I))$$

$R \longrightarrow S$

$$f_S = (R,\ 1_S - 1_R,\ \lambda d.\ \lambda_{RS} \cdot d(R))$$

$$(5,1,0,0)$$

$$(4,2,0,0) \qquad (5,0,1,0)$$

$$(3,3,0,0) \qquad (4,1,1,0) \qquad (5,0,0,1)$$

$$(6,0,0,0)$$

$$4 \cdot 4 \cdot \ldots \cdot 4$$

6

$$|S|^{|N|}$$

$N=6 \rightsquigarrow 4^6 \approx 2^{12}$

$N=10 \rightsquigarrow 2^{20}$

$N=100 \rightsquigarrow 2^{200}$