

⊕ precedence over  $\otimes$

id

⊕ right ass.

$\otimes$  left ass.

$$E \rightarrow E + T \mid T$$

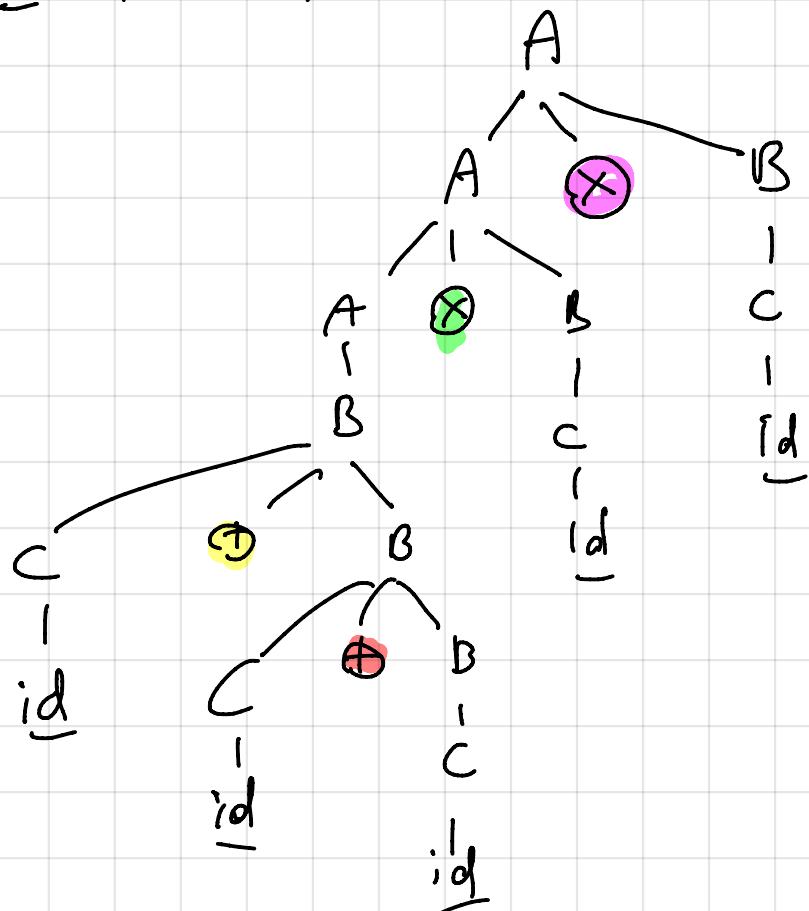
$$T \rightarrow T * F \mid F$$

$$F \leftarrow \text{---}$$

$\rightarrow A \rightarrow A \otimes B \quad | \quad B$

$B \rightarrow C \oplus B \quad | \quad C$

$C \rightarrow \underline{id} \quad | \quad (A)$



A grammar for regular on  $\Sigma = \{a, b\}$

$$R \rightarrow R + A \mid A$$

$$A \rightarrow A \cdot B \mid B$$

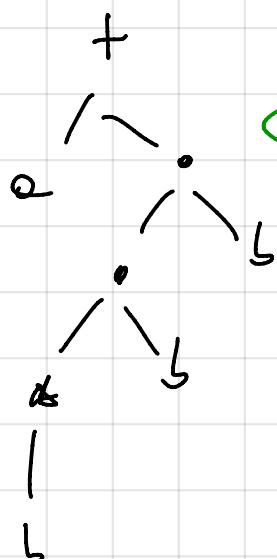
$$B \rightarrow B^* \mid C$$

$$C \rightarrow a \mid b \mid \text{eps} \mid (R)$$

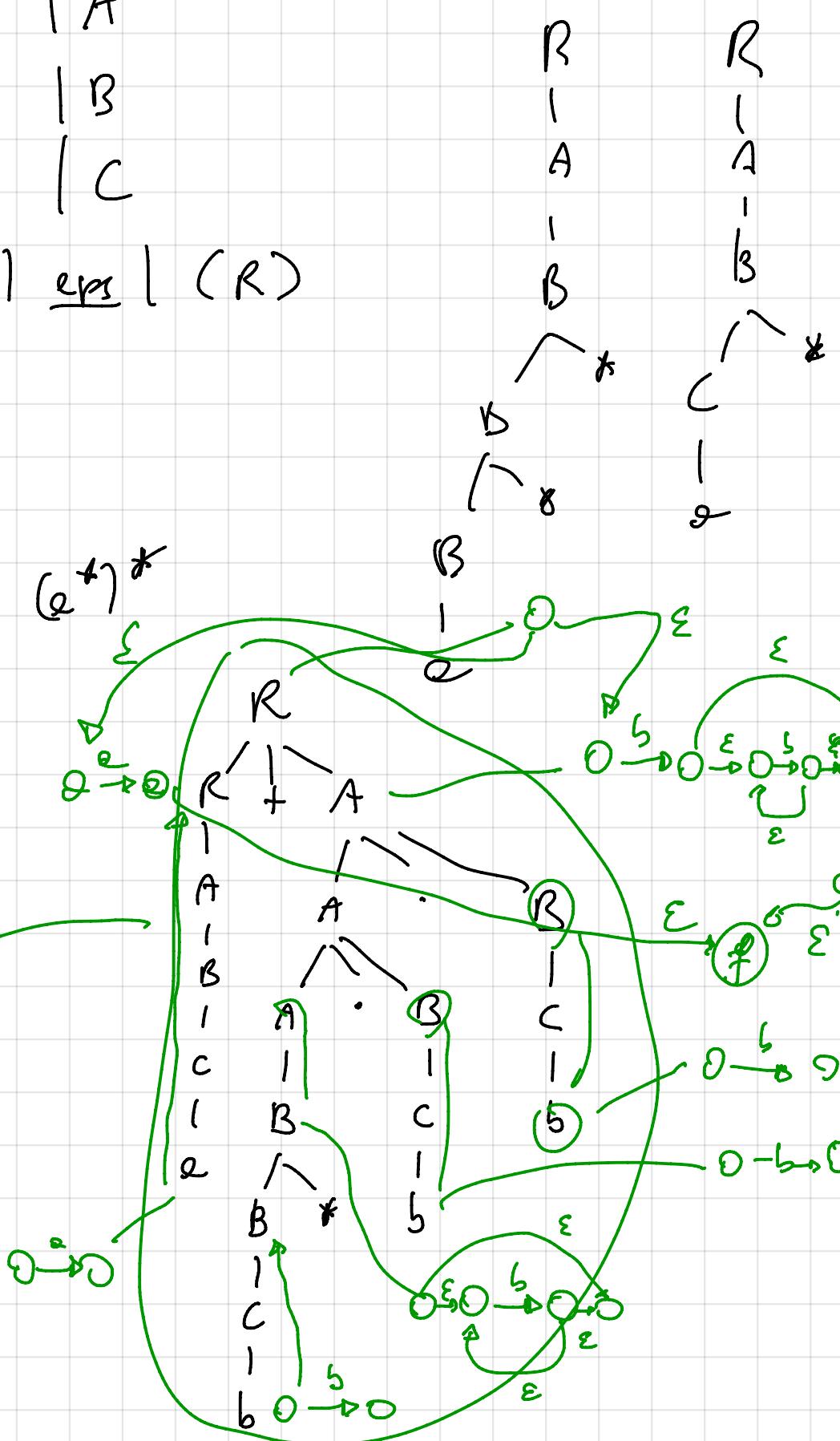
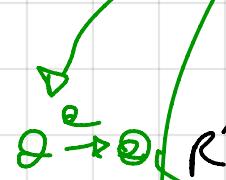
$C^*$



$a + b^* bb$



$(a^*)^*$



$+ -$     $* /$

$E \rightarrow E + T \mid E - T \mid T$

$T \rightarrow T * F \mid F / T \mid F$

$F \rightarrow \underline{id} \mid (E)$

$+$  is left-  
 $-$  is left-  
 $*, /$  same precen-

$*$  is left-  
 $/$  is right-  
 $*, /$  have the same

precedence

$$L = \{ a^m b c \mid m \geq 0 \} \cup \{ b^m c b \mid m \geq 0 \} \cup \{ c a^m \mid m \geq 0 \}$$

$$S \rightarrow a A \mid b B \mid c C$$

$$A \rightarrow a A \mid b c$$

$$B \rightarrow b B \mid c b$$

$$C \rightarrow b \mid a D$$

$$D \rightarrow a D \mid \epsilon$$

$c b \$$   
 $c a e \$$

$$\text{FIRST}(S) = \{ a, b, c \} \quad \text{Follow}(S) = \{ \$ \}$$

$$\text{FIRST}(A) = \{ a, b \} \quad \text{Follow}(A) = \{ \$ \}$$

$$\text{FIRST}(B) = \{ b, c \} \quad \text{Follow}(B) = \{ \$ \}$$

$$\text{FIRST}(C) = \{ b, a \} \quad \text{Follow}(C) = \{ \$ \}$$

$$\text{FIRST}(D) = \{ \epsilon, a \} \quad \text{Follow}(D) = \{ \$ \}$$

	a	b	c	\$
S	$S \rightarrow a A$	$S \rightarrow b B$	$S \rightarrow c C$	
A	$A \rightarrow a A$	$A \rightarrow b c$		
B		$B \rightarrow b B$	$B \rightarrow c b$	
C	$C \rightarrow a D$	$C \rightarrow b$		
D	$D \rightarrow a D$			$D \rightarrow \epsilon$

MATCHED

STACK

INPUT

ACTION

$c b \$$   $S \rightarrow c C$

$c b \$$  match

$b \$$   $C \rightarrow b$

$b \$$  match

$\$$  ACCEPT

c  
c  
cb

c  
c  
ca  
ca  
ca  
ca

S \$

CC \$

C \$

a D \$

D \$

a D \$

D \$

\$

ca \$  $S \rightarrow c C$

ca \$ match

a \$  $C \rightarrow a D$

a \$ match

a \$  $D \rightarrow a D$

a \$ match

a \$  $D \rightarrow \epsilon$

a \$ ACCEPT

$$E \rightarrow E + T \mid T$$

id \* id

$$T \rightarrow T * F \mid F$$

$$F \rightarrow \underline{id} \mid (E)$$

$$E \Rightarrow \begin{array}{c} E \rightarrow T \\ z_m \end{array} \quad \begin{array}{c} T \rightarrow T * F \\ z_m \end{array} \quad \begin{array}{c} F \rightarrow id \\ z_m \end{array} \quad \begin{array}{c} T \rightarrow F \\ z_m \end{array} \quad \begin{array}{c} F \rightarrow id \\ z_m \end{array}$$

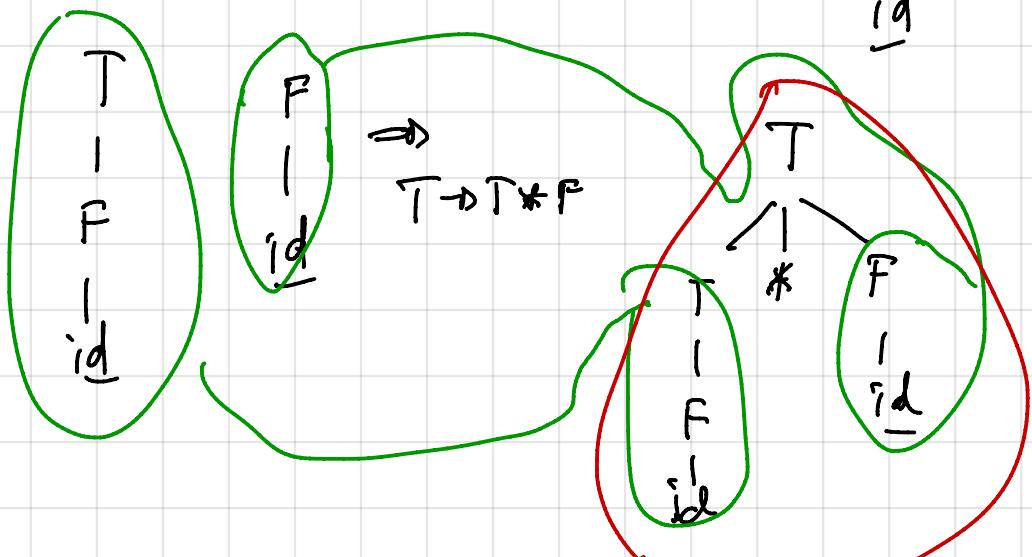
$$\underline{id} * \underline{id} \Rightarrow \underline{id} * \underline{id}$$

$$\underline{id} * \underline{id} \Rightarrow \underline{T} \rightarrow \underline{id}$$

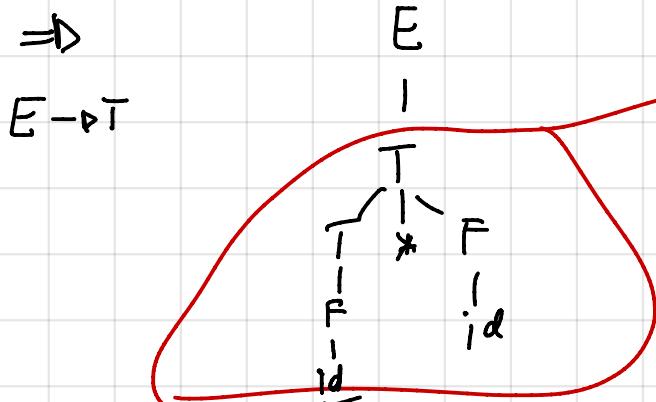
$$\begin{array}{c} F \rightarrow \\ | \\ \underline{id} \end{array} \Rightarrow \begin{array}{c} T \rightarrow F \\ | \end{array}$$

$$\begin{array}{c} T \\ | \\ F \\ | \\ \underline{id} \end{array}$$

$$\Rightarrow \begin{array}{c} F \rightarrow id \end{array}$$



$$\Rightarrow \begin{array}{c} E \\ | \\ T \end{array}$$



$S \rightarrow OS_2 | OI$

$OOS111$

$S \rightarrow OI$

|

$S \rightarrow OS_1 \xrightarrow{zm} OOS_11 \xrightarrow{zm} OOS111 \xrightarrow{zm} OOS111$

↑  
handle

$OOS11$

$S \rightarrow OS_1$

$S \rightarrow OS_1 \xrightarrow{zm} OOS_11 \xrightarrow{zm} OOS111$

↑  
handle ..

↑  
handle

$$S \rightarrow SS+ \mid SS* \mid \epsilon \quad \text{start} \leftarrow \$$$

SHIFT - REDUCE

PARSER

STACK

INPUT

ACTION

handle \$  
 \$  
 \* \$  
 S \$  
 S S \$  
 S S S \$  
 S S S S \$  
 \* S S S S \$  
 + S S S S \$  
 S S S S \$  
 + S S S S \$  
 S S S S \$  
 + S S S S \$  
 S S S S \$

QQQ \* Q ++ \$  
 Q Q \* Q ++ \$  
 Q Q \* Q ++ \$  
 Q \* Q ++ \$  
 Q \* Q ++ \$  
 \* Q ++ \$  
 \* Q ++ \$  
 Q ++ \$  
 Q ++ \$  
 ++ \$  
 ++ \$  
 ++ \$  
 ++ \$  
 ++ \$  
 ++ \$  
 ++ \$

SHIFT  
 Reduce  $S \rightarrow \epsilon$   
 ACCEPT

$\underset{2m}{\Rightarrow} S \underset{2m}{\Rightarrow} SS+ \underset{2m}{\Rightarrow} SSS+ \underset{2m}{\Rightarrow} SSS* \underset{2m}{\Rightarrow} SSS*\epsilon ++$

$\underset{2m}{\Rightarrow} SS* \underset{2m}{\Rightarrow} SSS* \underset{2m}{\Rightarrow} SSS* \epsilon ++ \underset{2m}{\Rightarrow} SSS* \epsilon ++$