$E \to T \; \{ E'.m = T.t \} \; E' \; \{ E.t = E'.t \}$

$E' \to + E \; \{ E'.t = makeN('+', E'.m, E.t) \}$

$E' \to \varepsilon \; \{ E'.t = E'.m \}$

$T \to F \; \{ T'.m = F.t \} \; T' \; \{ T.t = T'.t \}$

$T' \to \cdot \; T \; \{ T'.t = makeN('\cdot', T'.m, T.t) \}$

$T' \to \varepsilon \; \{ T'.t = T'.m \}$

$F \to H \; \{ F'.m = H.t \} \; F' \; \{ F.t = F'.t \}$

$F' \to * \; \{ F'.t = makeUN('*', F'.m) \}$

$F' \to \varepsilon \; \{ F'.t = F'.m \}$

b - . . .

$H \to a \mid b \mid c \mid \_ \; \{ H.t = makeL('a') \}$

$H \to (E) \; \{ H.t = E.t \}$

PARTIAL SOLUTION!

AXIOM IS NOT TAKEN INTO ACCOUNT

$makeN : BOperator \times Node \times Node \to Node$

$makeUN : UOperator \times Node \to Node$

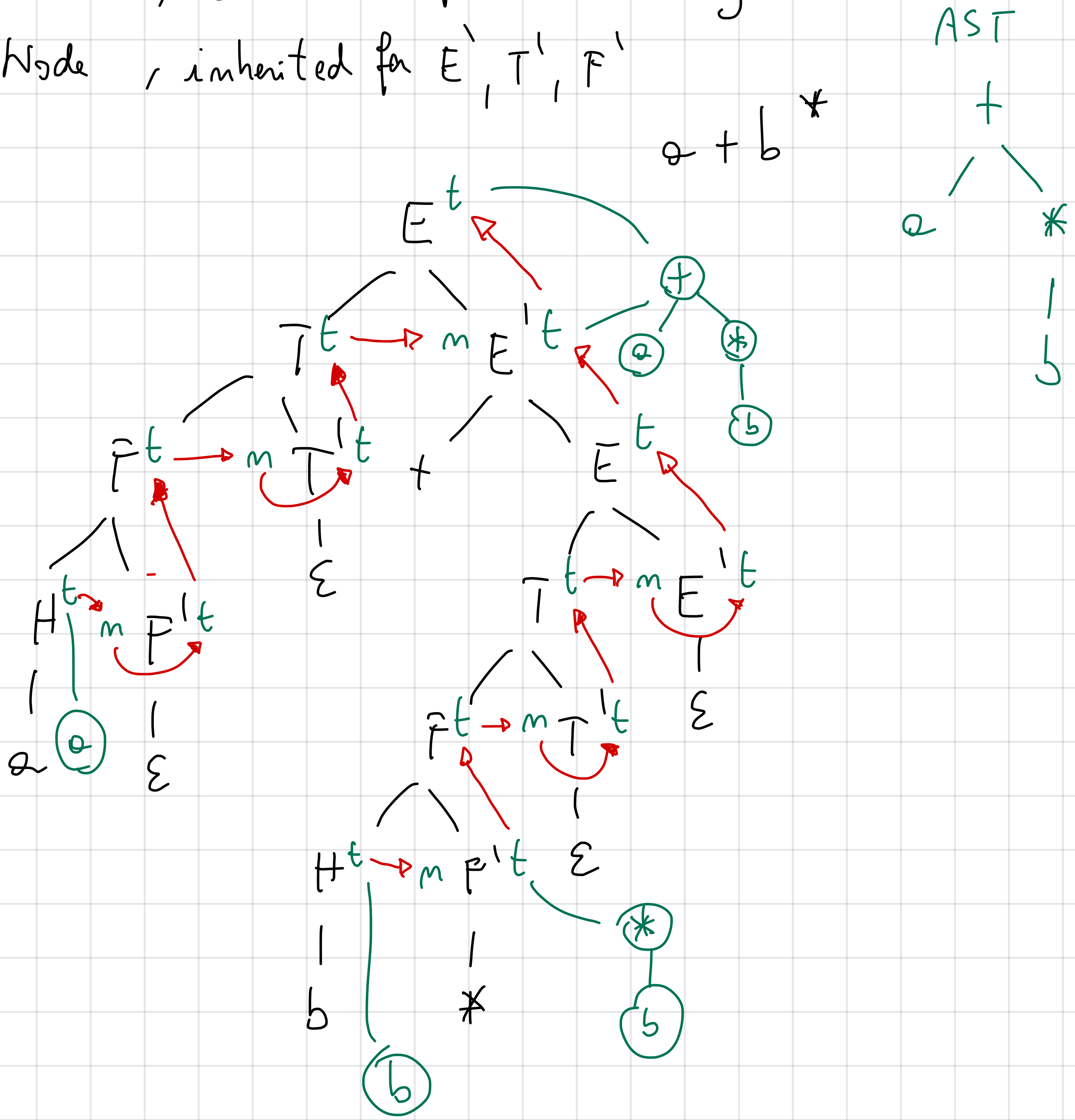
$makeL : \{ a, b, c, \_ \} \to Node$

Attributes

t : Node , synthesised for all the symbols

m : Node , inherited for E', T', F'

$a + b *$

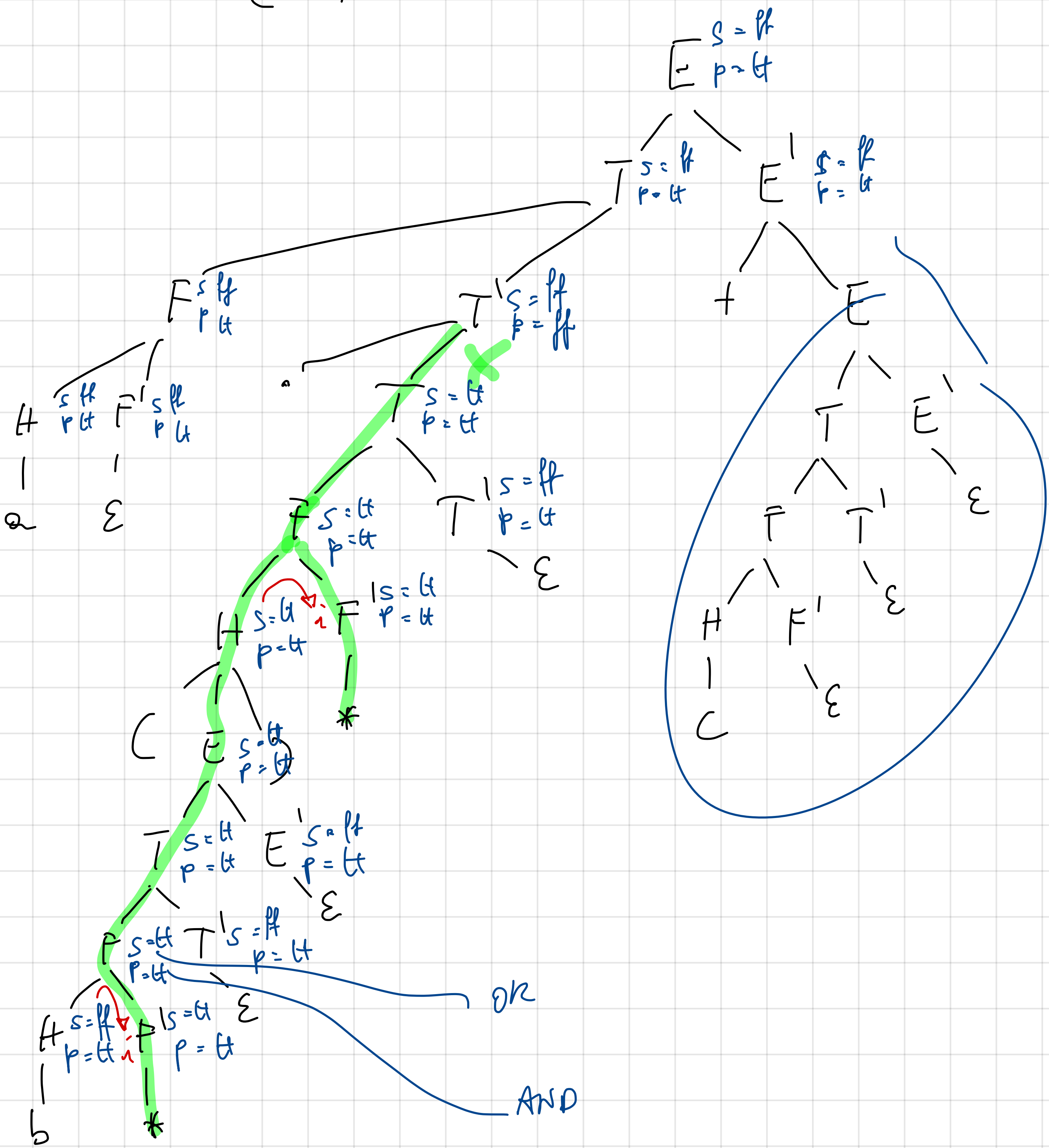


s : boolean "stored", synthesised, for all symbols

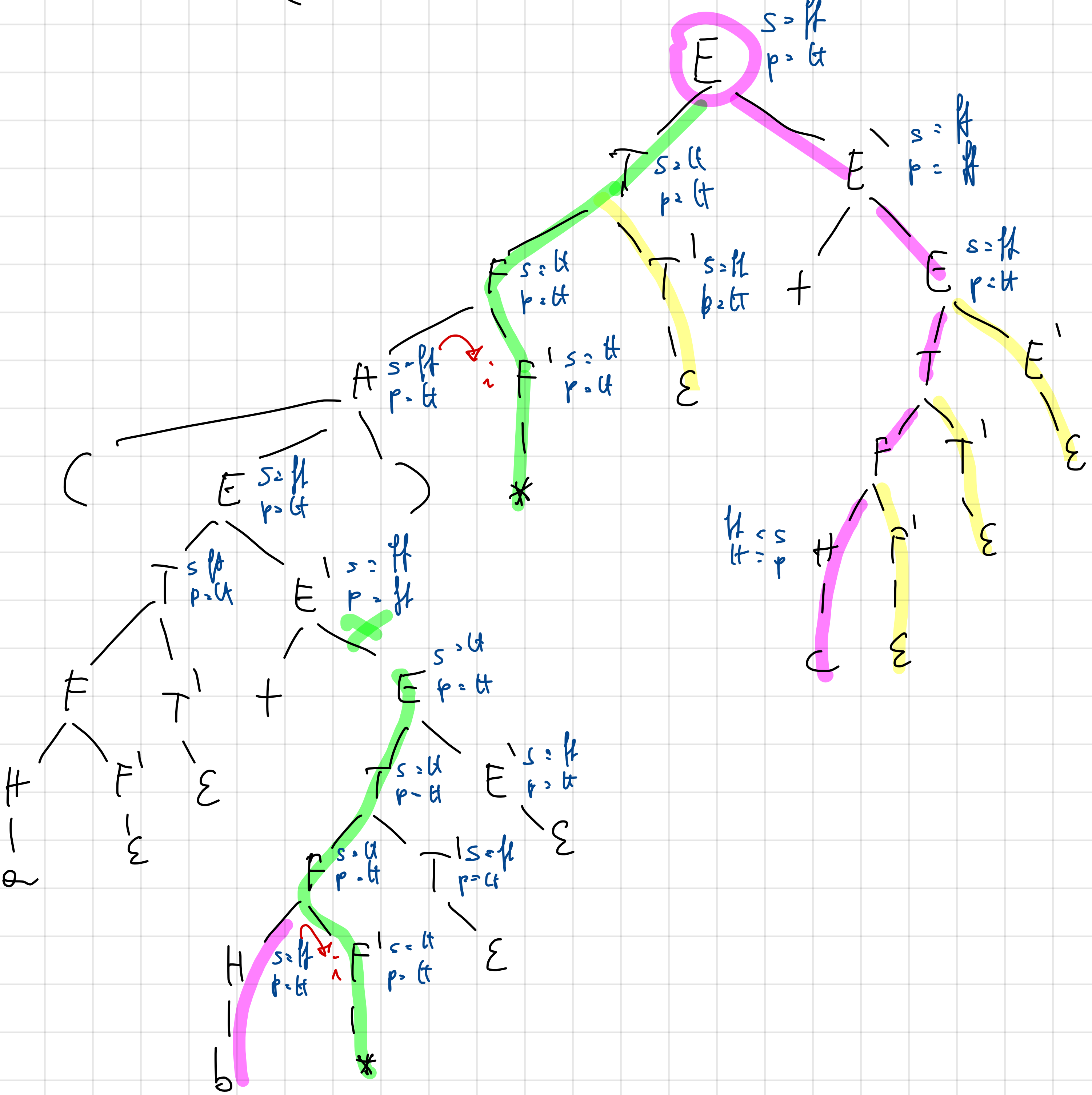
p : boolean "* can be propagated", for all symbols, synthesised

i : boolean, inherited for F'

$a(b^*)^* + c$



$(a + b^*)^* + c$


E
s = ff
p = tt
T
s = tt
p = tt
E'
s = ff
p = ff
F
s = tt
p = tt
T'
s = ff
p = tt
+
E
s = ff
p = tt
A
s = ff
p = tt
F'
s = tt
p = tt
ε
T
E'
(
E
s = ff
p = tt
)
*
F
T'
ε
T
s = ff
p = tt
E'
s = ff
p = ff
tt = s
tt = p
H
F'
ε
F
T'
+
E
s = tt
p = tt
c
ε
H
F'
ε
T
s = tt
p = tt
E'
s = ff
p = tt
a
ε
F
s = tt
p = tt
T'
s = ff
p = tt
ε
H
s = ff
p = tt
F'
s = tt
p = tt
ε
b
*

$E \rightarrow T \{ E'.m = T.t \} \; E' \{ E.t = E'.t, \; E.p = tt,$
$\text{if}(E'.p) \text{ then } E.s = T.s \text{ OR } E'.s \text{ else } E.s = ff \}$

$E' \rightarrow + E \{ E'.t = \text{makeN}('+', E'.m, E.t); \; E'.p = ff;$
$E'.s = ff \}$

$E' \rightarrow \varepsilon \; \{ E'.t = E'.m, \; E'.p = tt, \; E'.s = ff \}$

$T \rightarrow F \{ T'.m = F.t \} \; T' \{ T.t = T'.t, \; T.p = tt,$
$\text{if}(T'.p) \text{ then } T.s = F.s \text{ OR } T'.s \text{ else } T.s = ff \}$

$T' \rightarrow \cdot T \{ T'.t = \text{makeN}('\cdot', T'.m, T.t), \; T'.p = ff,$
$T'.s = ff \}$

$T' \rightarrow \varepsilon \; \{ T'.t = T'.m, \; T'.p = tt \;\; T'.s = ff \}$

$F \rightarrow H \; \{ F'.m = H.t, \; F'.i = H.s \} \; F' \{ F.t = F'.t,$
$F.p = tt; \; \text{if}(F'.p) \text{ then } F.s = H.s \text{ OR } F'.s \text{ else } F.s = ff \}$

$F' \rightarrow * \{ \text{if}(F'.i) \text{ then } F'.t = F'.m \text{ else } F'.t = \text{makeUN}('*', F'.m) \}$
$F'.s = tt; \; F'.p = tt$

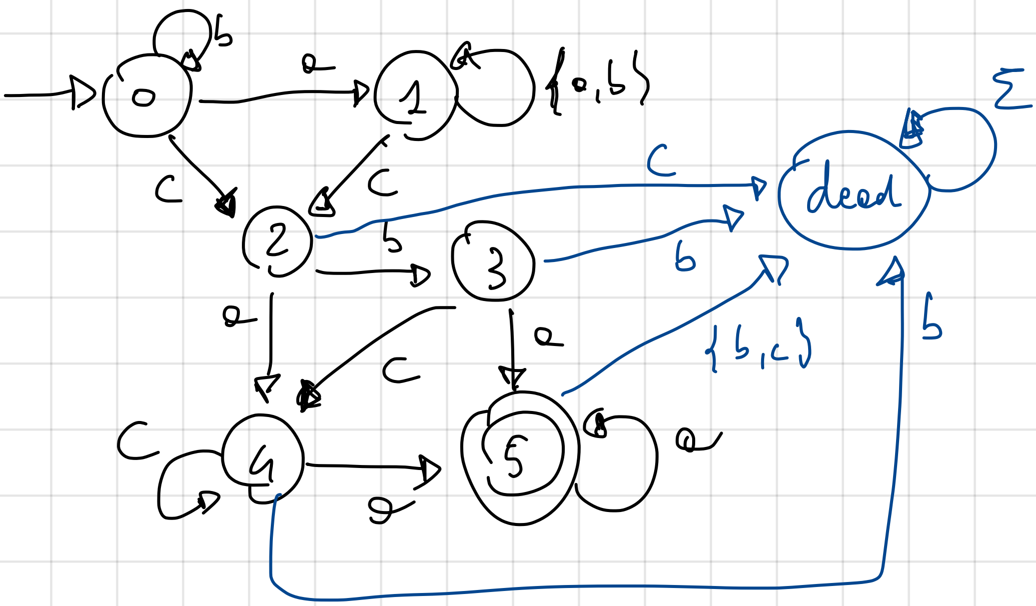
$F' \rightarrow \varepsilon \; \{ F'.t = F'.m, \; F'.p = tt, \; F'.s = ff \}$

$H \rightarrow a \mid b \mid c \mid \_ \; \{ H.t = \text{makeL}('a'), \; H.p = tt, \; H.s = ff \}$

$H \rightarrow (E) \; \{ H.t = E.t, \; H.p = E.p, \; H.s = E.s \}$

FINAL SOLUTION

23rd July 2019



$\Sigma = \{a, b, c\}$

1) Language as r.e.

$$b^*(\varepsilon \mid a(a|b)^*)\, c\, (a c^* \mid b c^*)\, a^+$$

$$\equiv \; b^*(\varepsilon \mid a(a|b)^*)\, c\, (a|b)\, c^* a^+$$

2) The automaton is deterministic

We apply the partition-refining algorithm to minimize the automaton augmented with the dead state

Π_1: (5) (0, 1, 2, 3, 4, dead)

$0 \xrightarrow{a} 1$

$2 \xrightarrow{a} 4$

$3 \xrightarrow{a} 5$

$1 \xrightarrow{a} 1$

dead $\xrightarrow{a}$ dead

$4 \xrightarrow{a} 5$

Π_2: (5) (3, 4) (0, 1, 2, dead)

$2 \xrightarrow{a} 4$

$0 \xrightarrow{a} 1$

$1 \xrightarrow{a} 1$

dead $\xrightarrow{a}$ dead

Π_3: (5) (3, 4) (2) (0, 1, dead)

$0 \xrightarrow{c} 2$

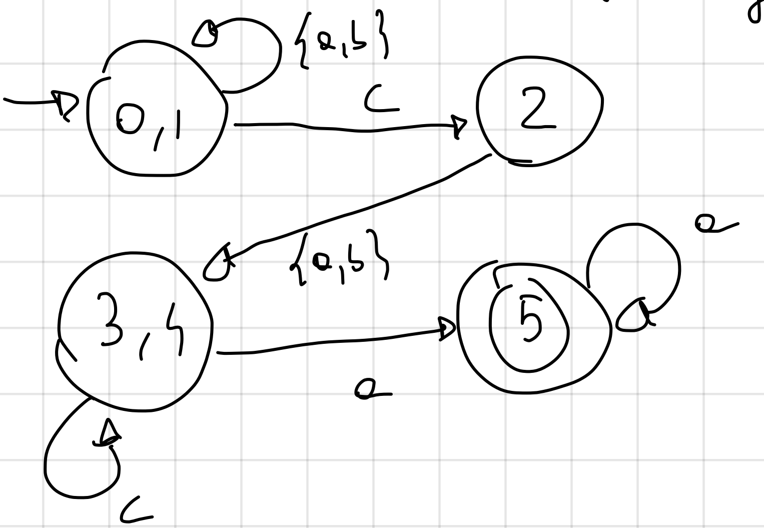
$1 \xrightarrow{c} 2$

dead $\xrightarrow{c}$ dead

Π_4: (5) (3, 4) (2) (dead) (0, 1)

no further split is possible

The automaton was <u>NOT</u> minimal. A minimal automaton is the following



$\rightarrow (a|b)^* c (a|b) c^* a^+$

$S' \rightarrow S$

$S \rightarrow aA \mid B \mid \varepsilon$

$A \rightarrow aA \mid C$

$C \rightarrow aCb \mid \varepsilon$

$B \rightarrow CD$

$D \rightarrow bD \mid b$

$L = \{\varepsilon\} \cup \{ a\, a^n b^m \mid n \geq 0, m \geq 0 \} \cup \{ a^n b^n b^m \mid n \geq 0, m > 0 \}$

$a\, a^n b^m \mid n \geq 0, m \geq 0$

$a^n b^n b^m \mid n \geq 0, m > 0$

$$\boxed{\begin{aligned}
I_0 = \; & S' \rightarrow \cdot S \\
& S \rightarrow \cdot aA \\
& S \rightarrow \cdot B \\
& S \rightarrow \cdot \\
& B \rightarrow \cdot CD \\
& C \rightarrow \cdot aCb \\
& C \rightarrow \cdot
\end{aligned}}$$

FIRST$(S) = \{\varepsilon, a, b\}$

FIRST$(A) = \{a, \varepsilon\}$

FIRST$(C) = \{a, \varepsilon\}$

FIRST$(B) = \{a, b\}$

FIRST$(D) = \{b\}$

FOLLOW$(S') = \{\$\}$

FOLLOW$(S) = \{\$\}$

FOLLOW$(A) = \{\$\}$

FOLLOW$(C) = \{b, \$\}$

FOLLOW$(B) = \{\$\}$

FOLLOW$(D) = \{\$\}$

The grammar is not SLR(1) because in state I_0 of the LR(0) item collection there is a reduce/reduce conflict on the symbol \$: it should trigger a reduce with production $S \to \varepsilon$ and with production $C \to \varepsilon$.

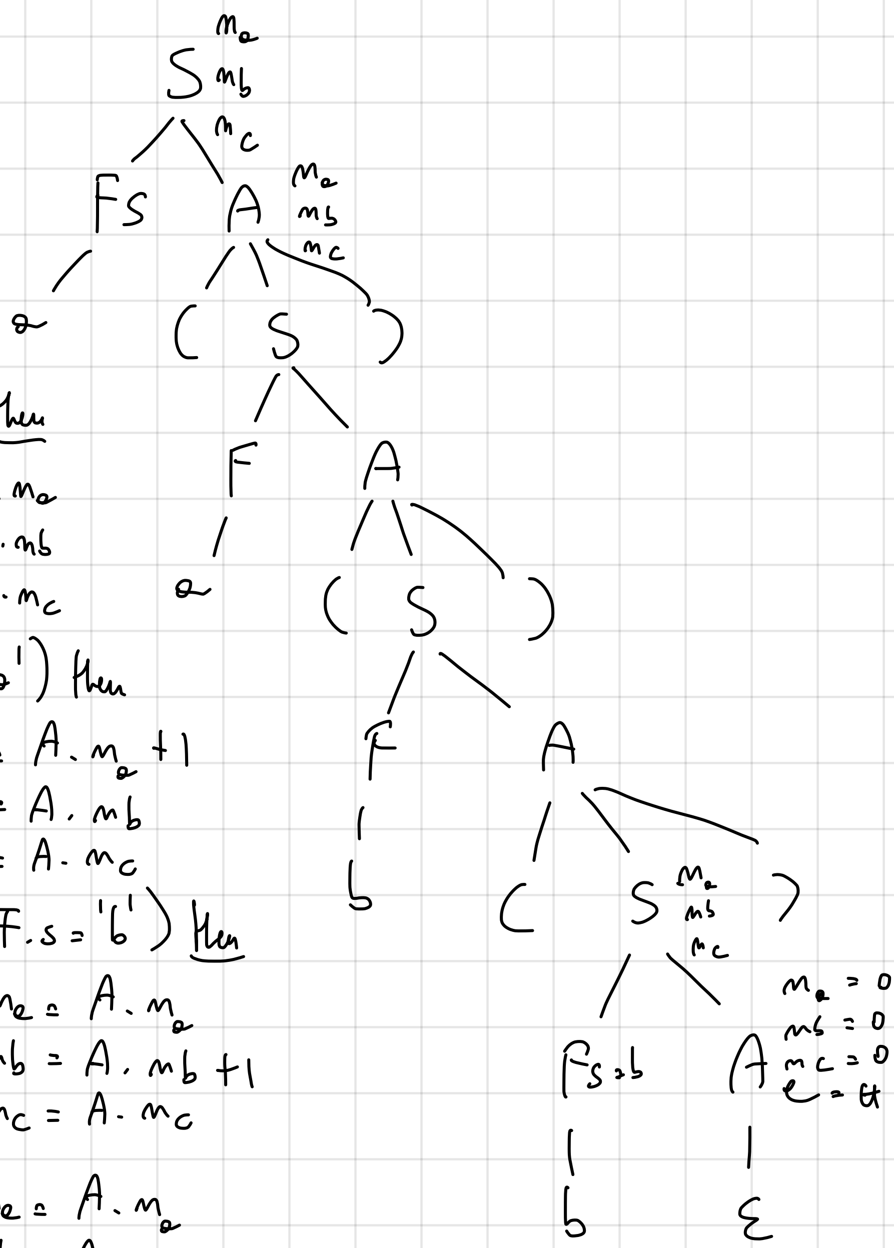
Grammar for expressions

$$S \to F(S) \mid F \qquad F \to a \mid b \mid c \quad \xrightarrow{\text{left-factorise}} \quad \begin{cases} S \to FA \\ A \to (S) \mid \varepsilon \\ F \to a \mid b \mid c \end{cases}$$

$FIRST(S) = \{a, b, c\}$ $FOLLOW(S) = \{\$,)\}$
$FIRST(A) = \{(, \varepsilon\}$ $FOLLOW(A) = \{\$,)\}$
$FIRST(F) = \{a, b, c\}$ $FOLLOW(F) = \{(,) \ \$\}$

	a	b	c	(	)	\$
S	$S \to FA$	$S \to FA$	$S \to FA$			
A				$A \to (S)$	$A \to \varepsilon$	$A \to \$$
F	$F \to a$	$F \to b$	$F \to c$			

$a(a(b(b)))$ $\qquad n_a = 2 \qquad n_b = 1 \qquad n_c = 0$



$$S \to F\,A \quad \{\ \text{if } (A.e) \ \underline{\text{then}}$$
$$S.n_a = A.n_a \qquad S.n_b = A.n_b \qquad S.n_c = A.n_c$$

$\underline{\text{else}}$ if $(F.s = \text{'}a\text{'})$ then
$$S.n_a = A.n_a + 1 \qquad S.n_b = A.n_b \qquad S.n_c = A.n_c$$

$\underline{\text{else}}$ $\underline{\text{if}}$ $(F.s = \text{'}b\text{'})$ $\underline{\text{then}}$
$$S.n_a = A.n_a \qquad S.n_b = A.n_b + 1 \qquad S.n_c = A.n_c$$

$\underline{\text{else}}$
$$S.n_a = A.n_a \qquad S.n_b = A.n_b \qquad S.n_c = A.n_c + 1\ \}$$

$$A \to \varepsilon \quad \{\ A.n_a = 0 \quad A.n_b = 0 \quad A.n_c = 0 \quad A.e = tt\ \}$$

$$A \to (\,S\,) \quad \{\ A.n_a = S.n_a \quad A.n_b = S.n_b \quad A.n_c = S.n_c \quad A.e = ff\ \}$$

$$F \to a \quad \{F.s = \text{'}a\text{'}\}$$

$$F \to b \quad \{F.s = \text{'}b\text{'}\}$$

$$F \to c \quad \{F.s = \text{'}c\text{'}\}$$