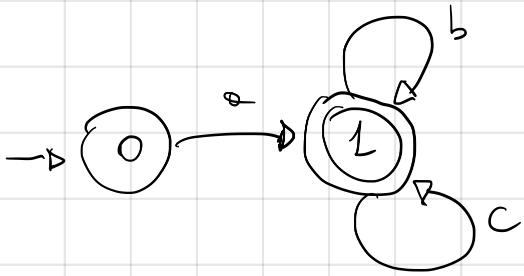
$a(b|c)^*$ find a minimal DFA

CANONICAL STRATEGY

1) $a(b|c)^* \longrightarrow$ NFA Thompson's algorithm
2) NFA $\rightarrow$ DFA Subset construction algorithm
3) DFA $\rightarrow$ minimal DFA Partition Refinement Alg.

To solve this particular problem we can skip 1) and 2) easily and give directly a DFA

$$\mathcal{L}(a(b|c)^*) = \{ax \mid x \in \{b,c\}^*\}$$



$\Pi^{(1)} = \{\{0\},\{1\}\}$

This partition cannot be refined so this automaton is also minimal for the language.

Ex: Given r_1, r_2 regexps, are they equivalent?

$r_1 \equiv r_2$ iff $\mathcal{L}(r_1) = \mathcal{L}(r_2)$

Strategy:

1) $r_1 \longrightarrow NFA_1$
$r_2 \longrightarrow NFA_2$

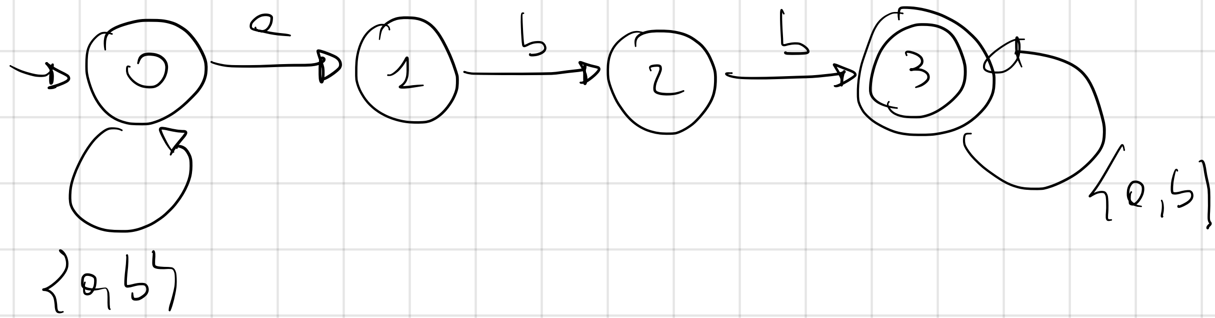
2) $NFA_1 \longrightarrow DFA_1$
$NFA_2 \longrightarrow DFA_2$

3) $DFA_1 \longrightarrow DFA_{1\_min}$
$DFA_2 \longrightarrow DFA_{2\_min}$

4) if ($DFA_{1\_min} \approx DFA_{2\_min}$)
then return YES
else return NO

isomorphic

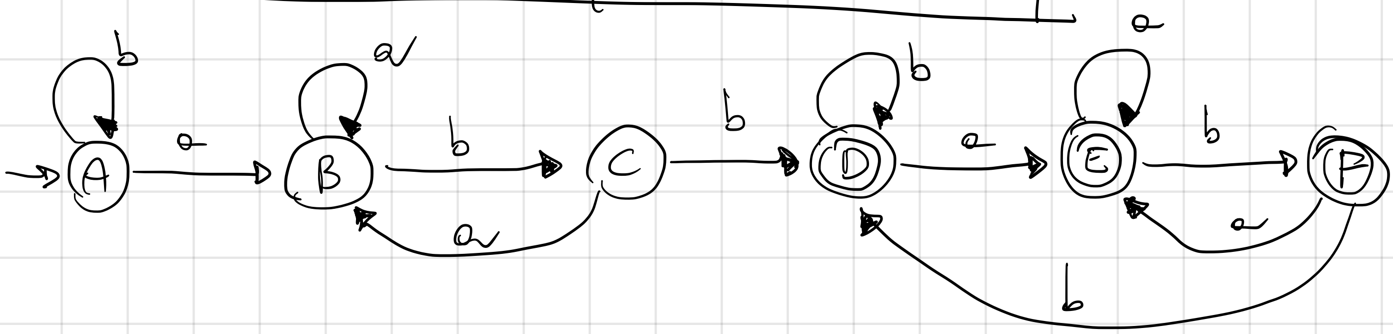
Transform

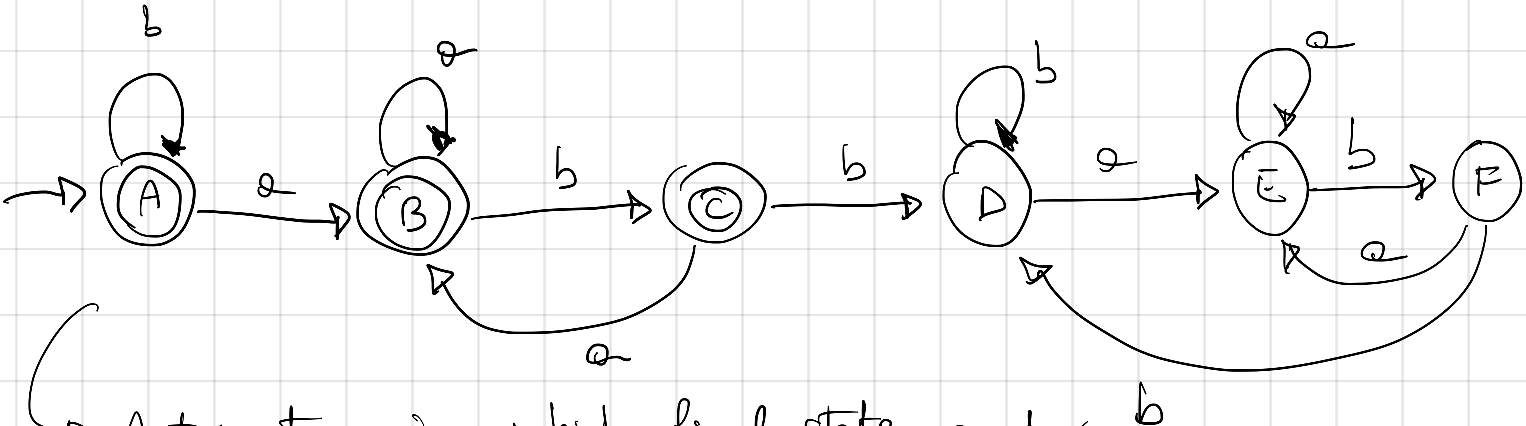


into a DFA

	a	b
A = {0}	{0,1} = B	{0} = A
B = {0,1}	{0,1} = B	{0,2} = C
C = {0,2}	{0,1} = B	{0,3} = D
D = {0,3}	{0,1,3} = E	{0,3} = D
E = {0,1,3}	{0,1,3} = E	{0,2,3} = F
F = {0,2,3}	{0,1,3} = E	{0,3} = D

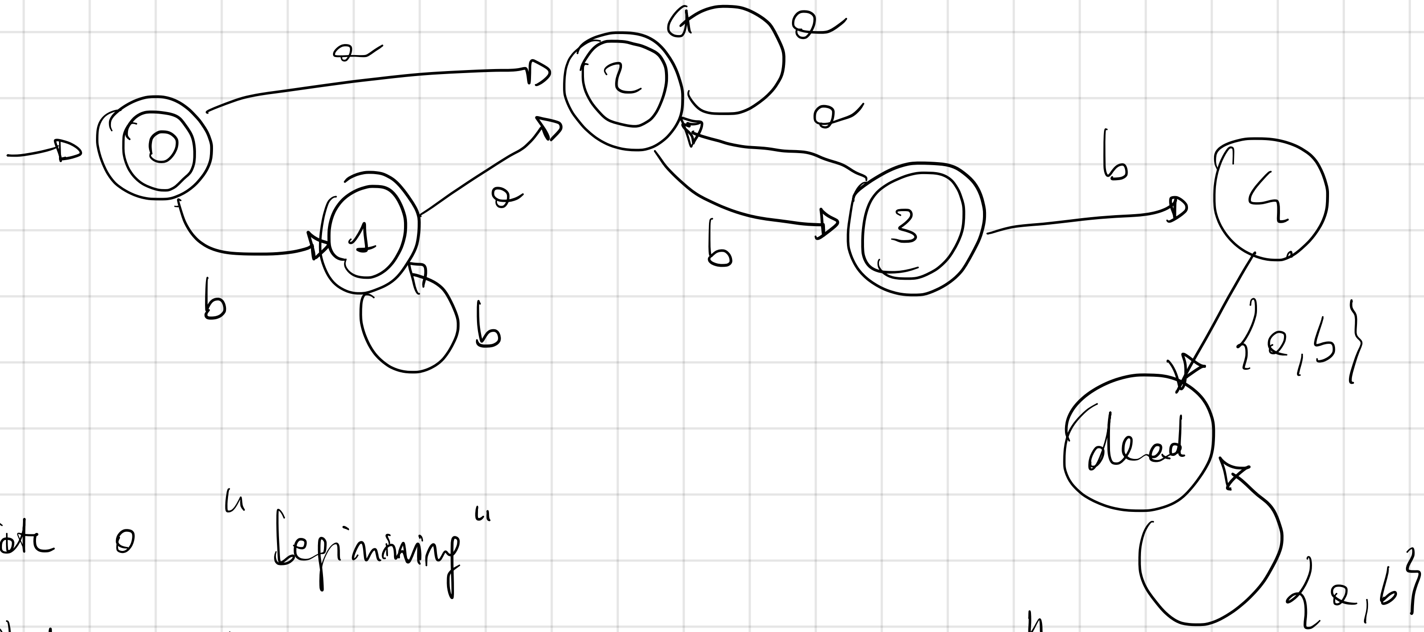
Non-blocking DFA





Automaton in which final states and non-final ones are exchanged. It accepts the complement language.

$\mathcal{L} = \{ x \in \{a,b\}^* \mid x \neq y\,abb\,z \text{ for any } y,z \in \{a,b\}^* \}$



State 0 "beginning"

State 1 "I have not seen an a yet"

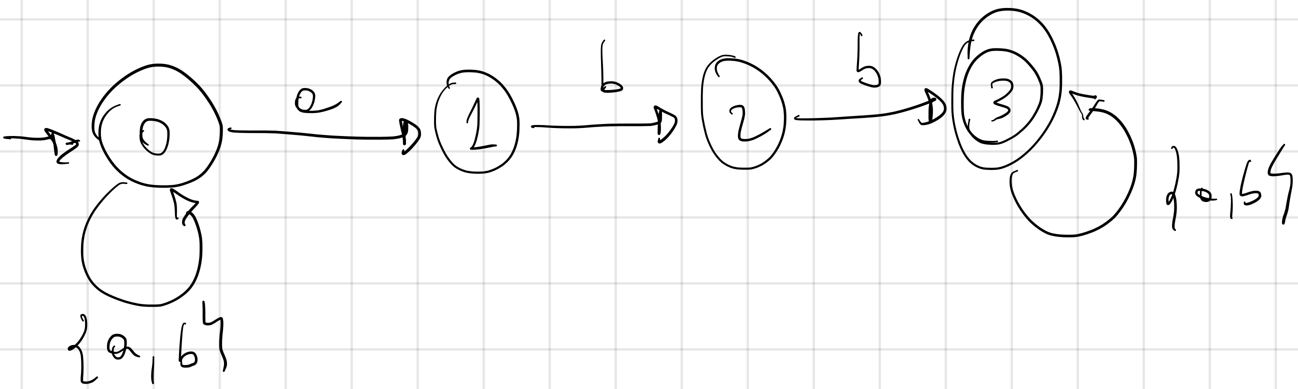
State 2 "Last character was a"

State 3 "Last sequence was ab"

State 4 "The string contains abb"

$\mathcal{L}' = \{ x \in \{a,b\}^* \mid x = y\,abb\,z \text{ for some } y,z \in \{a,b\}^* \}$

$\mathcal{L}'$ is the complement of $\mathcal{L}$. NFA for $\mathcal{L}'$:



Theorem: If $\mathcal{L}$ is a regular language then $\mathcal{L}^c = \Sigma^* - \mathcal{L}$ is a regular language.

Proof: 1) $\mathcal{L}$ is regular then there is a regular expression $r_{\mathcal{L}}$ such that $L(r_{\mathcal{L}}) = \mathcal{L}$

$\uparrow$ the language denoted by $r_{\mathcal{L}}$.

2) $r_{\mathcal{L}} \longrightarrow NFA_{\mathcal{L}}$ 3) $NFA_{\mathcal{L}} \rightarrow DFA_{\mathcal{L}}$

4) if $DFA_{\mathcal{L}}$ is blocking, then add the dead state

5) $DFA'_{\mathcal{L}}$ is $DFA_{\mathcal{L}}$ s.t. the final and non-final states are exchanged

6) Thus, $DFA'_{\mathcal{L}}$ acceps $\mathcal{L}^c$

7) By Kleene theorem, since there is a DFA accepting $\mathcal{L}^c$ then $\mathcal{L}^c$ is REGULAR $\square$

Kleene Theorem

L is regular

iff $\exists$ regexp r s.t. $\mathcal{L}(r) = L$

iff $\exists$ NFA accepting L

iff $\exists$ DFA accepting L

$\mathcal{L}_1$ regular

$\mathcal{L}_2$ regular

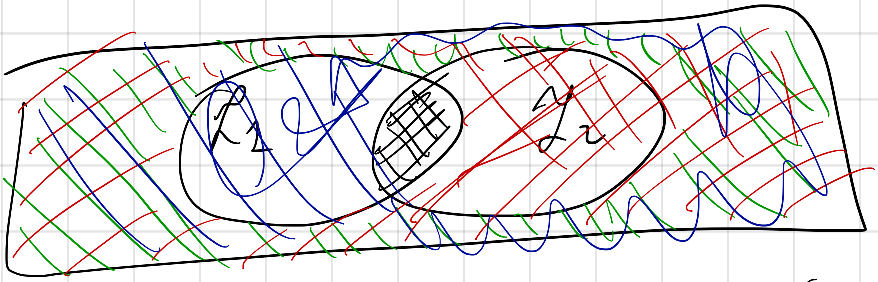
? $\mathcal{L}_1 \cap \mathcal{L}_2$ is regular?

YES

In Lex ^ operator

if r is a regexp $^\wedge r$ is a regexp

$$\mathcal{L}(^\wedge r) = \Sigma^* - \mathcal{L}(r)$$



$(\mathcal{L}_1 \cup \mathcal{L}_2)^c$

$\mathcal{L}_1^c$

$\mathcal{L}_2^c$

$(\mathcal{L}_1^c \cup \mathcal{L}_2^c)^c$

Proof: $\mathcal{L}_1 \to r_1$ regexp because $\mathcal{L}_1$ and

$\mathcal{L}_2 \to r_2$ regexp $\mathcal{L}_2$ are regular

Then $^\wedge(^\wedge r_1 \mid {}^\wedge r_2)$ is a regular expression

denoting $(\mathcal{L}_1^c \cup \mathcal{L}_2^c)^c = \mathcal{L}_1 \cap \mathcal{L}_2$.

Thus, $\mathcal{L}_1 \cap \mathcal{L}_2$ is regular $\Box$

Another way :

$\mathcal{L}_1 \to r_1 \to NFA_1 \to DFA_1 = \langle S_1, \Sigma, s_0^1, \delta_1, F_1 \rangle$

$\mathcal{L}_2 \to r_2 \to NFA_2 \to DFA_2 = \langle S_2, \Sigma, s_0^2, \delta_2, F_2 \rangle$

Create an automaton that accepts $\mathcal{L}_1 \cap \mathcal{L}_2$

$$\langle S_1 \times S_2, \Sigma, (s_0^1, s_0^2), \delta, F_1 \times F_2 \rangle$$

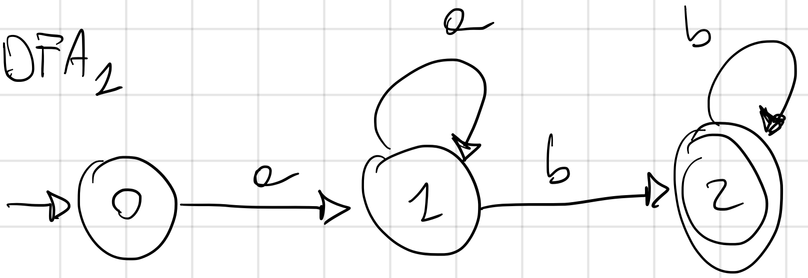
where δ is defined s.t. $\forall s \in S_1, t \in S_2, c \in \Sigma$

if $\big(\delta_1(s,c) = s'$ and $\delta_2(t,c) = t'\big)$ then $\delta((s,t),c) = (s',t')$

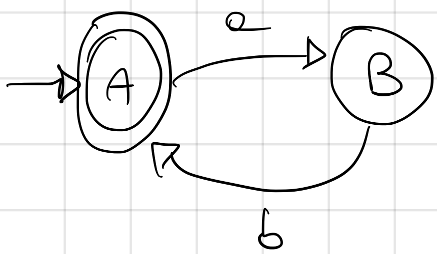
Example $\to$

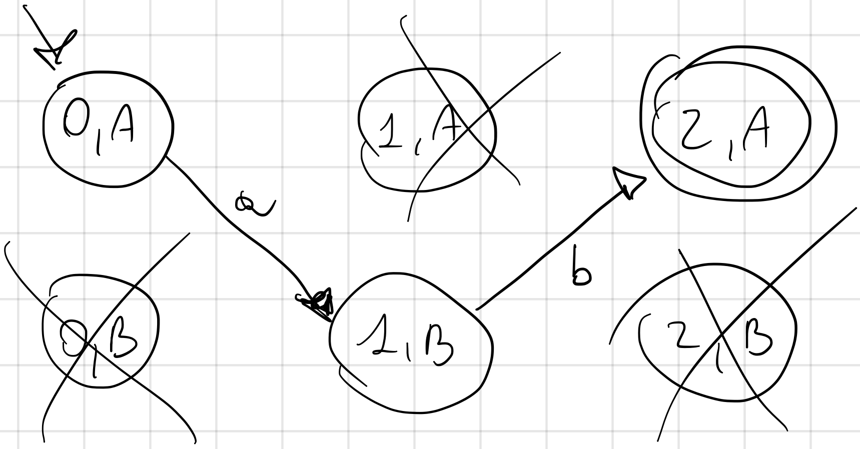
$\mathcal{L}_1 = \{ a^n b^m \mid n > 0,\ m > 0 \}$ | $\mathcal{L}_2 = \{ (ab)^n \mid n \geq 0 \}$

DFA_1



DFA_2





$$\frac{s \xrightarrow{c} s' \quad \text{and} \quad t \xrightarrow{c} t'}{(s,t) \xrightarrow{c} (s',t')}$$

This automaton accepts $\{ab\} = \mathcal{L}_1 \cap \mathcal{L}_2$