Formal Languages and Compilers - Exercises I with Solutions

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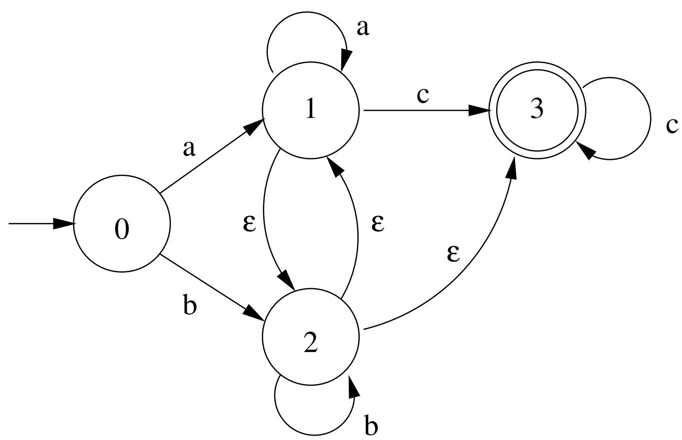
Note Regular expressions are written with the usual precedence order: operator * has precedence on concatenation, which has precedence on $|$. Moreover, the usual shorthands $^+$ and $?$ may be used.

Exercise 1

Write a regular expression denoting the language accepted by the following automaton:



Solution

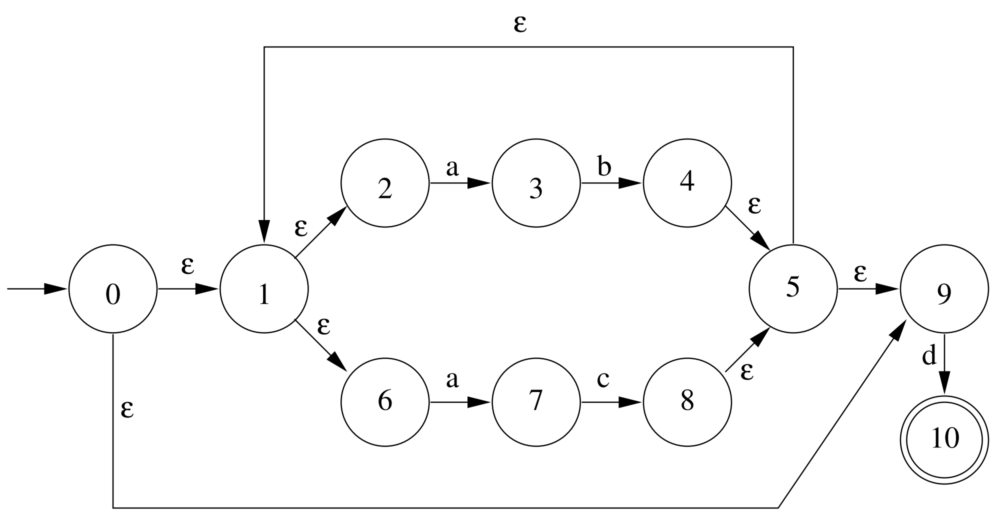
The expression is $(a|b)^+c^*$.

Exercise 2

Use Thompson algorithm to construct an NFA accepting the language denoted by $(ab|ac)^*d$.

Solution

The syntax tree of the regexp is a concatenation between a star and a d, then the star is of a union between two concatenations. Following the inductive definitions of the Thomposon algorithm the following NFA is obtained:

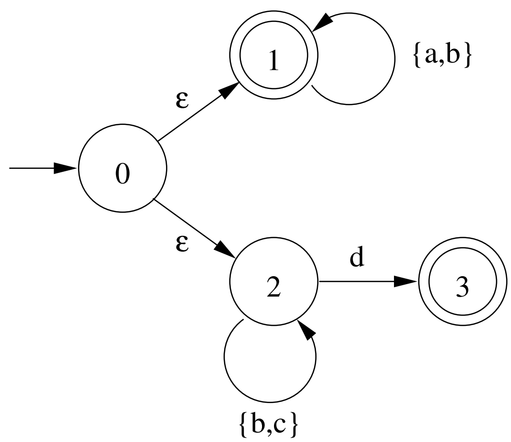


Exercise 3

Write a minimal automaton for the language $(a|b)^* \mid (b|c)^* d$.

Solution

Let us first use non-determinism to easily define an NFA for the language:



Now we can use the subset construction algorithm to find an equivalent DFA. The *move* table of the obtained DFA is the following one where accepting states are $\{A, B, C, E\}$:

State	a	b	c	d
$A = \{0, 1, 2\}$	B	C	D	E
$B = \{1\}$	B	B		
$C = \{1, 2\}$	B	C	D	E
$D = \{2\}$		D	D	E
$E = \{3\}$				

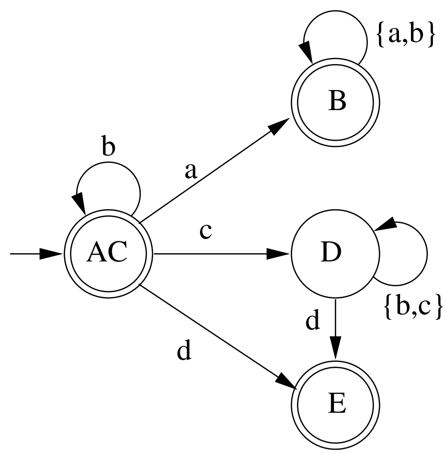
It is quite clear from the table that state A and state C are equivalent, while the rest of the states behave differently. However, for the sake of completeness, let us apply the minimisation algorithm.

First, let us complete the DFA by adding a dead state F to which we create a transition for every empty entry in the table.

The first partition to consider is $(ABCE), (DF)$. Consider the group (DF); we have that $move(D, d) = E$ and $move(F, d) = F$. We conclude that the two states are not equivalent because the input d sends the two states in different groups. Thus, the new partition to consider is $(ABCE), (D), (F)$.

The only group that can be refined is $(ABCE)$. We have $move(A, d) = E$, $move(B, d) = F$, $move(C, d) = E$, $move(E, d) = F$. Thus, the new partition is $(AC), (BE), (D), (F)$.

We have already observed that there are no differences between A and C. Let us then consider B and E. We have that $move(B, a) = B$ and $move(E, a) = F$. Thus they must be distinguished. We obtain the following automaton, which is minimal for the language and in which the dead state F is not represented:



Exercise 4

Consider the following grammar:

$$
\begin{array}{lcl}
S & \rightarrow & aSb \mid Ad \mid Bc \\
A & \rightarrow & Aa \mid c \\
B & \rightarrow & ddA \mid dC \\
C & \rightarrow & ac
\end{array}
$$

1. Formalise the language generated by the grammar
2. Is the grammar $LL(k)$ for some k?
3. Construct the table of a top-down non-recursive predictive parser for the language.

Solution

1) The language can be formalised as follows:

$$\{a^n \, c \, a^* \, d \, b^n \mid n \geq 0\} \cup \{a^n \, d \, a \, c \, c \, b^n \mid n \geq 0\} \cup \{a^n \, d \, d \, c \, a^* \, c \, b^n \mid n \geq 0\}$$

2) The grammar is not $LL(k)$ for any k because it has an immediate left recursion in the production: $A \to Aa$.

3) Let us eliminate the left recursion and also factorise the productions of B. We obtain the following grammar:

$$\begin{array}{lcl} S & \to & aSb \mid Ad \mid Bc \\ A & \to & cA' \\ A' & \to & aA' \mid \epsilon \\ B & \to & dB' \\ B' & \to & dA \mid C \\ C & \to & ac \end{array}$$

We have FOLLOW(S) $= \{\$, b\}$, FOLLOW($A$) $= \{d, c\} =$ FOLLOW(A'), FOLLOW(B) $= \{c\} =$ FOLLOW(B') $=$ FOLLOW(C).

This modified grammar is $LL(1)$ and the parsing table is the following:

	a	b	c	d	\$
S	$S \to aSb$		$S \to Ad$	$S \to Bc$	
A			$A \to cA'$		
A'	$A' \to aA'$		$A' \to \epsilon$	$A' \to \epsilon$	
B				$B \to dB'$	
B'	$B' \to C$			$B' \to dA$	
C	$C \to ac$				

Exercise 5

Consider the following grammar:

$$\begin{array}{lcl} S & \to & A \mid Bbb \\ A & \to & aB \\ B & \to & aAb \mid b \end{array}$$

1. Formalise the language generated by the grammar
2. Is the grammar LR(1)?
3. Is the string $aaAb$ a viable prefix? If the answer is yes, enumerate the valid $LR(0)$ items for this prefix.

Solution

1) The language is

$$\{a^{2n+1}\, b^{n+1} \mid n \geq 0\} \cup \{a^{2n}\, b^{n+3} \mid n \geq 0\}$$

2) Let us first construct the collection of $LR(0)$ items. If there are no conflicts then the grammar is $SLR(1)$ and so also $LR(1)$. Let us augment the grammar, as usual, with the production $S' \to S$.

$I_0 =$	$S' \to \cdot S$ $S \to \cdot A$ $S \to \cdot Bbb$ $A \to \cdot aB$ $B \to \cdot aAb$ $B \to \cdot b$	$I_1 = goto(I_0, S) =$	$S' \to S\cdot$
$I_2 = goto(I_0, A) =$	$S \to A\cdot$	$I_3 = goto(I_0, B) =$	$S \to B \cdot bb$
$I_4 = goto(I_0, a) =$	$A \to a \cdot B$ $A \to a \cdot Ab$ $B \to \cdot aAb$ $B \to \cdot b$ $A \to \cdot aB$	$I_5 = goto(I_0, b) =$	$B \to b\cdot$
$I_6 = goto(I_3, b) =$	$B \to Bb \cdot b$	$I_7 = goto(I_4, B) =$	$A \to aB\cdot$
$I_8 = goto(I_4, A) =$	$A \to aA \cdot b$	$I_9 = goto(I_6, b) =$	$B \to Bbb\cdot$
$I_{10} = goto(I_8, b) =$	$B \to aAb\cdot$	$goto(I_4, a) = I_4$ $goto(I_4, b) = I_5$	

We have that FOLLOW(S') = FOLLOW(S) = $\{\$\}$. And also FOLLOW($A$) = FOLLOW($B$) = $\{b, \$\}$.

There are no conflicts in the states, thus the grammar is $SLR(1)$.

3) We can use the fact that the construction of the collection of the $LR(0)$ items corresponds to the definition of a DFA starting in state I_0. All the states are accepting and this automaton recognises all the viable prefixes. Thus, we can test if the string $aaAb$ is accepted. A labelled path for the string on the automaton is $0 \xrightarrow{a} 4 \xrightarrow{a} 4 \xrightarrow{A} 8 \xrightarrow{b} 10$. This means that the string is a viable prefix.

The theory also tells us that the $LR(0)$ items contained in the final state reached with the viable prefix are exactly all the items that are valid for it. Looking at the state I_{10}, the only valid item for the viable prefix is $B \to aAb\cdot$.