Exercise 3

Define an automaton that recognizes all and only those words on the alphabet Sigma = {a,b,c} such that if "a" occurs then the total number of its occurrences is even.

Exercise 4

Define an automaton that recognizes all and only those words on the alphabet Sigma = {a,b,c} such that if "b" occurs then the total number of its occurrences is odd.

Exercise 5

Define an automaton that recognizes all and only those words on the alphabet Sigma = {a,b,c} such that if "a" occurs then the total number of its occurrences is even and if "b" occurs then the total number of its occurrences is odd.

Solutions

Exercise 3

Let's use states that record the information "even number of a already seen" and "odd number of a already seen". In particular:
0 is traversed as long as no "a" has been seen or whenever the occurrence of a new "a" makes the total number of "a" seen an even number
1 is traversed whenever the first "a" is seen or whenever the occurrence of a new "a" makes the total number of "a" seen an odd number

The automaton is A = <{0,1}, {a,b,c}, Move, {0}, {0}>

where Move is described by the following table:

	a	b	c
0	1	0	0
1	0	1	1

The defined automaton is deterministic.

Exercise 4

Exercise 4 is similar to Exercise 3. It is sufficient to repeat the same reasoning with "b" instead of "a" and indicate 1 as the only final state.

Exercise 5

Let’s use states that can distinguish between even and odd number of “a”s (denoted by 2(a) and 1(a) respectively) and zero, odd and even number of “b”s (denoted by 0(b), 1(b) and 2(b) respectively). There is an asymmetry due to the fact that a word can be accepted if there are no occurrences of “b” or if the number of occurrences of “b” is odd, while for the number of “a”s the case zero occurrences coincides with the case even occurrences. The states of the automaton are listed in the following where after each state there is the description of when the state should be traversed.

0: 2(a)0(b)
1: 1(a)0(b)
2: 1(a)1(b)
3: 1(a)2(b)
4: 2(a)1(b)
5: 2(a)2(b)

The automaton is A = <{0,1,2,3,4,5}, {a,b,c}, Move, {0}, {0,4}>

where Move is described by the following table:

	a	b	c
0	1	4	0
1	0	2	1
2	4	3	2
3	5	2	3
4	2	5	4
5	3	4	5

The defined automaton is deterministic.