Formal Modelling of Software Intensive Systems

CCS

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A.A. 2015/2016

CCS Basics

Sequential Fragment

- *Nil* process (the only atomic process)
- action prefixing ($a.P$)
- names and recursive definitions ($\triangleq$)
- nondeterministic choice ($+$)

Any finite LTS can be described (up to isomorphism) by using the operations above

Parallelism and Renaming

- parallel composition ($|$) (synchronous communication between two components = handshake synchronization)
- restriction ($P \setminus L$)
- relabelling ($P[f]$)

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Definition of CCS: channels, actions, process names

Let

- $\mathcal{A}$ be a set of channel names (e.g. *tea*, *coffee* are channel names)
- $\mathcal{L} = \mathcal{A} \cup \overline{\mathcal{A}}$ be a set of labels where
  - $\overline{\mathcal{A}} = \{\overline{a} \mid a \in \mathcal{A}\}$
    (elements of $\mathcal{A}$ are called names and those of $\overline{\mathcal{A}}$ are called co-names)
  - by convention $\overline{\overline{a}} = a$
- $Act = \mathcal{L} \cup \{\tau\}$ is the set of actions where
  - τ is the internal or silent action

  (e.g. τ, *tea*, $\overline{coffee}$ are actions)
- $\mathcal{K}$ is a set of process names (constants) (e.g. CM).

Definition of CCS (expressions)

$P :=$	K	$\mid$	process constants ($K \in \mathcal{K}$)
	$\alpha.P$	$\mid$	prefixing ($\alpha \in Act$)
	$\sum_{i \in I} P_i$	$\mid$	summation (I is an arbitrary index set)
	$P_1 \mid P_2$	$\mid$	parallel composition
	$P \setminus L$	$\mid$	restriction ($L \subseteq \mathcal{A}$)
	$P[f]$		relabelling ($f : Act \to Act$) such that • $f(\tau) = \tau$ • $f(\overline{a}) = \overline{f(a)}$

The set of all terms generated by the abstract syntax is the set of CCS process expressions (and is denoted by $\mathcal{P}$)

Notation

$$P_1 + P_2 = \sum_{i \in \{1,2\}} P_i \qquad\qquad Nil = \sum_{i \in \emptyset} P_i$$

Precedence

Precedence

1. restriction and relabelling (tightest binding)
2. action prefixing
3. parallel composition
4. summation

Example: $R + a.P|b.Q \setminus L$ means $R + \big((a.P)|(b.(Q \setminus L))\big)$

Definition of CCS (defining equations)

CCS program

A collection of defining equations of the form

$$K \triangleq P$$

where $K \in \mathcal{K}$ is a process constant and $P \in \mathcal{P}$ is a CCS process expression.

- Only one defining equation per process constant.
- Recursion is allowed: e.g. $A \triangleq \overline{a}.A \mid A$.

Structural Operational Semantics for CCS

Structural Operational Semantics (SOS)—G. Plotkin 1981

Small-step operational semantics where the behaviour of a system is inferred using syntax driven rules

Given a collection of CCS defining equations, we define the following LTS $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$:

- $Proc = \mathcal{P}$ (the set of all CCS process expressions)
- $Act = \mathcal{L} \cup \{\tau\}$ (the set of all CCS actions including τ)
- transition relation is given by SOS rules of the form:

$$\text{RULE} \quad \frac{\textit{premises}}{\textit{conclusion}} \quad \textit{conditions}$$

SOS rules for CCS ($\alpha \in Act$, $a \in \mathcal{L}$)

$$\text{ACT} \quad \frac{}{\alpha.P \xrightarrow{\alpha} P} \qquad\qquad \text{SUM}_j \quad \frac{P_j \xrightarrow{\alpha} P'_j}{\sum_{i \in I} P_i \xrightarrow{\alpha} P'_j} \quad j \in I$$

$$\text{COM1} \quad \frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q} \qquad\qquad \text{COM2} \quad \frac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'}$$

$$\text{COM3} \quad \frac{P \xrightarrow{a} P' \quad\quad Q \xrightarrow{\overline{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

$$\text{RES} \quad \frac{P \xrightarrow{\alpha} P'}{P \setminus L \xrightarrow{\alpha} P' \setminus L} \quad \alpha, \overline{\alpha} \notin L \qquad\qquad \text{REL} \quad \frac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$$

$$\text{CON} \quad \frac{P \xrightarrow{\alpha} P'}{K \xrightarrow{\alpha} P'} \quad K \triangleq P$$

Deriving Transitions in CCS

Let $A \triangleq a.A$. Then

$$\big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a] \xrightarrow{c} \big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a].$$

Why?

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$$\text{REL}\ \dfrac{\text{COM1}\ \dfrac{}{(A \,|\, \overline{a}.Nil) \,|\, b.Nil \xrightarrow{a} (A \,|\, \overline{a}.Nil) \,|\, b.Nil}}{\big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a] \xrightarrow{c} \big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a]}$$

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$$\text{REL}\ \dfrac{\text{COM1}\ \dfrac{\text{COM1}\ \dfrac{}{A \,|\, \overline{a}.Nil \xrightarrow{a} A \,|\, \overline{a}.Nil}}{(A \,|\, \overline{a}.Nil) \,|\, b.Nil \xrightarrow{a} (A \,|\, \overline{a}.Nil) \,|\, b.Nil}}{\big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a] \xrightarrow{c} \big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a]}$$

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Why?

$$\mathrm{REL}\ \dfrac{\mathrm{COM1}\ \dfrac{\mathrm{COM1}\ \dfrac{\mathrm{CON}\ \dfrac{}{A \xrightarrow{a} A}\ A \triangleq a.A}{A \,|\, \overline{a}.Nil \xrightarrow{a} A \,|\, \overline{a}.Nil}}{(A \,|\, \overline{a}.Nil) \,|\, b.Nil \xrightarrow{a} (A \,|\, \overline{a}.Nil) \,|\, b.Nil}}{\big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a] \xrightarrow{c} \big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a]}$$

Deriving Transitions in CCS

Let $A \triangleq a.A$. Then

$$\big((A \mid \overline{a}.Nil) \mid b.Nil\big)[c/a] \xrightarrow{c} \big((A \mid \overline{a}.Nil) \mid b.Nil\big)[c/a].$$

Why?

$$\dfrac{\dfrac{\dfrac{\dfrac{}{a.A \xrightarrow{a} A}\;\text{ACT}}{A \xrightarrow{a} A}\;\text{CON}\; A \triangleq a.A}{A \mid \overline{a}.Nil \xrightarrow{a} A \mid \overline{a}.Nil}\;\text{COM1}}{(A \mid \overline{a}.Nil) \mid b.Nil \xrightarrow{a} (A \mid \overline{a}.Nil) \mid b.Nil}\;\text{COM1}$$

$$\text{REL}\;\dfrac{(A \mid \overline{a}.Nil) \mid b.Nil \xrightarrow{a} (A \mid \overline{a}.Nil) \mid b.Nil}{\big((A \mid \overline{a}.Nil) \mid b.Nil\big)[c/a] \xrightarrow{c} \big((A \mid \overline{a}.Nil) \mid b.Nil\big)[c/a]}$$

LTS of the Process $a.Nil \mid \overline{a}.Nil$

$$
\begin{tikzcd}
& a.Nil \mid \overline{a}.Nil \arrow[dl, "a"'] \arrow[dd, "\tau"] \arrow[dr, "\overline{a}"] & \\
Nil \mid \overline{a}.Nil \arrow[dr, "\overline{a}"] & & a.Nil \mid Nil \arrow[dl, "a"'] \\
& Nil \mid Nil &
\end{tikzcd}
$$

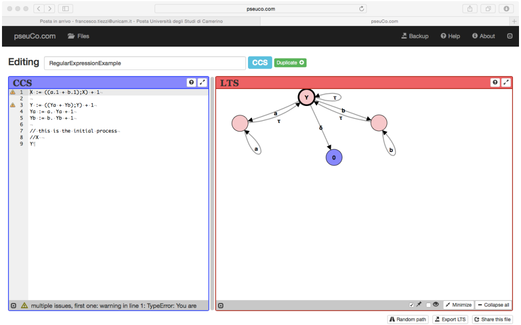
CCS: vending machine example

Examples at the blackboard...

CCS in pseuCo

pseuCo

Web application allowing to create CCS specifications and interactively explore the resulting transition systems



http://pseuco.com

CCS in pseuCo: regular expressions

$(a + b)^*$

```
X := ((a.1 + b.1);X) + 1

// this is the initial process
X
```

$(a^* + b^*)^*$

```
Y := ((Ya + Yb);Y) + 1
Ya := a. Ya + 1
Yb := b. Yb + 1

// this is the initial process
Y
```

Demo!

CCS in pseuCo: regular expressions

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X := ((a.1 + b.1);X) + 1

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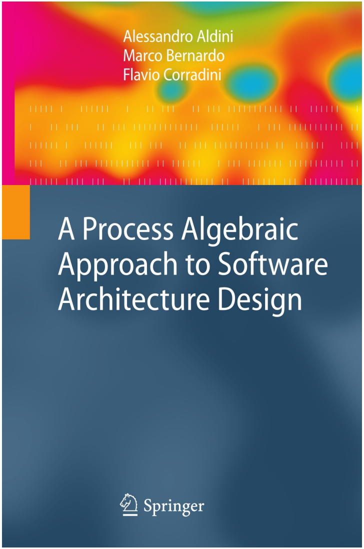
Demo!

Producer-Consumer Example



Producer-Consumer Example

- The system is composed of
  - a producer
  - a finite-capacity buffer
  - a consumer
- The producer **deposits** items into the buffer as long as the buffer capacity is not exceeded
- Stored items can be **withdrawn** by the consumer according to some predefined discipline, like FIFO or LIFO
- Assumptions:
  - The buffer has only two positions
  - Items are all identical, so that the specific discipline that has been adopted for withdrawals is not important from the point of view of an external observer

Demo!

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