Formal Modelling of Software Intensive Systems

CCS

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CCS Basics

Sequential Fragment

- *Nil* process (the only atomic process)
- action prefixing ($a.P$)
- names and recursive definitions ($\triangleq$)
- nondeterministic choice ($+$)

Any finite LTS can be described (up to isomorphism) by using the operations above

Parallelism and Renaming

- parallel composition ($|$) (synchronous communication between two components = handshake synchronization)
- restriction ($P \setminus L$)
- relabelling ($P[f]$)

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Definition of CCS: channels, actions, process names

Let

- $\mathcal{A}$ be a set of channel names (e.g. *tea*, *coffee* are channel names)
- $\mathcal{L} = \mathcal{A} \cup \overline{\mathcal{A}}$ be a set of labels where
    - $\overline{\mathcal{A}} = \{\overline{a} \mid a \in \mathcal{A}\}$
      (elements of $\mathcal{A}$ are called names and those of $\overline{\mathcal{A}}$ are called co-names)
    - by convention $\overline{\overline{a}} = a$
- $Act = \mathcal{L} \cup \{\tau\}$ is the set of actions where
    - τ is the internal or silent action

  (e.g. τ, *tea*, $\overline{coffee}$ are actions)
- $\mathcal{K}$ is a set of process names (constants) (e.g. CM).

Definition of CCS (expressions)

$$\begin{array}{lll|l}
P := & K & | & \text{process constants } (K \in \mathcal{K}) \\
 & \alpha.P & | & \text{prefixing } (\alpha \in Act) \\
 & \sum_{i \in I} P_i & | & \text{summation } (I \text{ is an arbitrary index set}) \\
 & P_1 | P_2 & | & \text{parallel composition} \\
 & P \setminus L & | & \text{restriction } (L \subseteq \mathcal{A}) \\
 & P[f] & & \text{relabelling } (f : Act \to Act) \text{ such that}
\end{array}$$

- $f(\tau) = \tau$
- $f(\overline{a}) = \overline{f(a)}$

The set of all terms generated by the abstract syntax is the set of CCS process expressions (and is denoted by $\mathcal{P}$)

Notation

$$P_1 + P_2 = \sum_{i \in \{1,2\}} P_i \qquad\qquad Nil = \sum_{i \in \emptyset} P_i$$

Precedence

Precedence

1. restriction and relabelling (tightest binding)
2. action prefixing
3. parallel composition
4. summation

Example: $R + a.P|b.Q \smallsetminus L$ means $R + \big((a.P)|(b.(Q \smallsetminus L))\big)$

Definition of CCS (defining equations)

CCS program

A collection of defining equations of the form

$$K \triangleq P$$

where $K \in \mathcal{K}$ is a process constant and $P \in \mathcal{P}$ is a CCS process expression.

- Only one defining equation per process constant.
- Recursion is allowed: e.g. $A \triangleq \overline{a}.A \mid A$.

Structural Operational Semantics for CCS

Structural Operational Semantics (SOS)—G. Plotkin 1981

Small-step operational semantics where the behaviour of a system is inferred using syntax driven rules

Given a collection of CCS defining equations, we define the following LTS $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$:

- $Proc = \mathcal{P}$ (the set of all CCS process expressions)
- $Act = \mathcal{L} \cup \{\tau\}$ (the set of all CCS actions including τ)
- transition relation is given by SOS rules of the form:

$$\text{RULE} \quad \frac{premises}{conclusion} \quad conditions$$

SOS rules for CCS $(\alpha \in Act,\ a \in \mathcal{L})$

$$\text{ACT}\quad \frac{}{\alpha.P \xrightarrow{\alpha} P} \qquad\qquad \text{SUM}_j\quad \frac{P_j \xrightarrow{\alpha} P'_j}{\sum_{i\in I} P_i \xrightarrow{\alpha} P'_j}\ \ j \in I$$

$$\text{COM1}\quad \frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q} \qquad\qquad \text{COM2}\quad \frac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'}$$

$$\text{COM3}\quad \frac{P \xrightarrow{a} P' \qquad Q \xrightarrow{\overline{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

$$\text{RES}\quad \frac{P \xrightarrow{\alpha} P'}{P \smallsetminus L \xrightarrow{\alpha} P' \smallsetminus L}\ \ \alpha, \overline{\alpha} \notin L \qquad\qquad \text{REL}\quad \frac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$$

$$\text{CON}\quad \frac{P \xrightarrow{\alpha} P'}{K \xrightarrow{\alpha} P'}\ \ K \triangleq P$$

Deriving Transitions in CCS

Let $A \triangleq a.A$. Then

$$\big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a] \xrightarrow{c} \big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a].$$

Why?

Deriving Transitions in CCS

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$$\text{REL}\ \dfrac{\text{COM1}\ \dfrac{}{(A \,|\, \overline{a}.Nil) \,|\, b.Nil \xrightarrow{a} (A \,|\, \overline{a}.Nil) \,|\, b.Nil}}{\big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a] \xrightarrow{c} \big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a]}$$

Deriving Transitions in CCS

Let $A \triangleq a.A$. Then

$$\big((A \mid \overline{a}.Nil) \mid b.Nil\big)[c/a] \xrightarrow{c} \big((A \mid \overline{a}.Nil) \mid b.Nil\big)[c/a].$$

Why?

$$\text{REL}\ \dfrac{\text{COM1}\ \dfrac{\text{COM1}\ \dfrac{}{A \mid \overline{a}.Nil \xrightarrow{a} A \mid \overline{a}.Nil}}{(A \mid \overline{a}.Nil) \mid b.Nil \xrightarrow{a} (A \mid \overline{a}.Nil) \mid b.Nil}}{\big((A \mid \overline{a}.Nil) \mid b.Nil\big)[c/a] \xrightarrow{c} \big((A \mid \overline{a}.Nil) \mid b.Nil\big)[c/a]}$$

Deriving Transitions in CCS

Let $A \triangleq a.A$. Then

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Deriving Transitions in CCS

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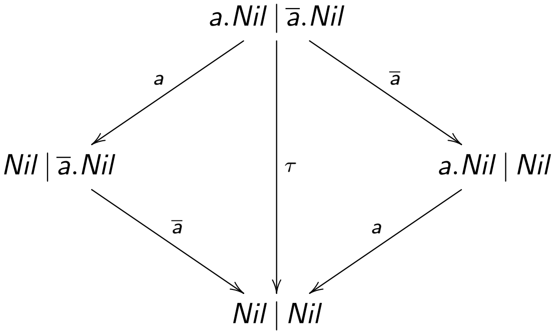
$$\text{REL}\ \dfrac{\text{COM1}\ \dfrac{\text{COM1}\ \dfrac{\text{CON}\ \dfrac{\text{ACT}\ \dfrac{}{a.A \xrightarrow{a} A}}{A \xrightarrow{a} A}\, A \triangleq a.A}{A \,|\, \overline{a}.Nil \xrightarrow{a} A \,|\, \overline{a}.Nil}}{(A \,|\, \overline{a}.Nil) \,|\, b.Nil \xrightarrow{a} (A \,|\, \overline{a}.Nil) \,|\, b.Nil}}{\big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a] \xrightarrow{c} \big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a]}$$

LTS of the Process $a.Nil \mid \overline{a}.Nil$



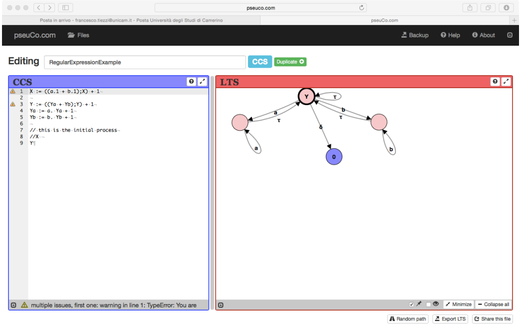
CCS: vending machine example

Examples at the blackboard...

CCS in pseuCo

pseuCo

Web application allowing to create CCS specifications and interactively explore the resulting transition systems



```
X := ((a.1 + b.1);X) + 1

Y := ((Ya + Yb);Y) + 1
Ya := a. Ya + 1
Yb := b. Yb + 1

// this is the initial process
//X
Y
```

multiple issues, first one: warning in line 1: TypeError: You are

http://pseuco.com

CCS in pseuCo: regular expressions

$(a + b)^*$

```
X := ((a.1 + b.1);X) + 1

// this is the initial process
X
```

$(a^* + b^*)^*$

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Y := ((Ya + Yb);Y) + 1
Ya := a. Ya + 1
Yb := b. Yb + 1

// this is the initial process
Y
```

Demo!

CCS in pseuCo: regular expressions

$(a + b)^*$

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X := ((a.1 + b.1);X) + 1

// this is the initial process
X
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$(a^* + b^*)^*$

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// this is the initial process
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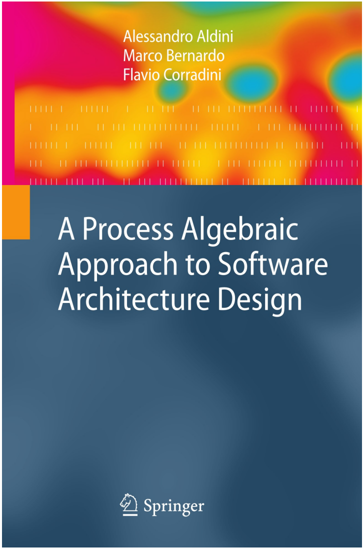
Demo!

Producer-Consumer Example


Alessandro Aldini
Marco Bernardo
Flavio Corradini
A Process Algebraic Approach to Software Architecture Design
Springer

Producer-Consumer Example

- The system is composed of
  - a producer
  - a finite-capacity buffer
  - a consumer
- The producer **deposits** items into the buffer as long as the buffer capacity is not exceeded
- Stored items can be **withdrawn** by the consumer according to some predefined discipline, like FIFO or LIFO
- Assumptions:
  - The buffer has only two positions
  - Items are all identical, so that the specific discipline that has been adopted for withdrawals is not important from the point of view of an external observer

Demo!

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