

# Case-Based Reasoning

Knut Hinkelmann

Literature: Bergmann, Ralph: Experience Management. Springer-Verlag 2002



# Two ways of Learning from Experience

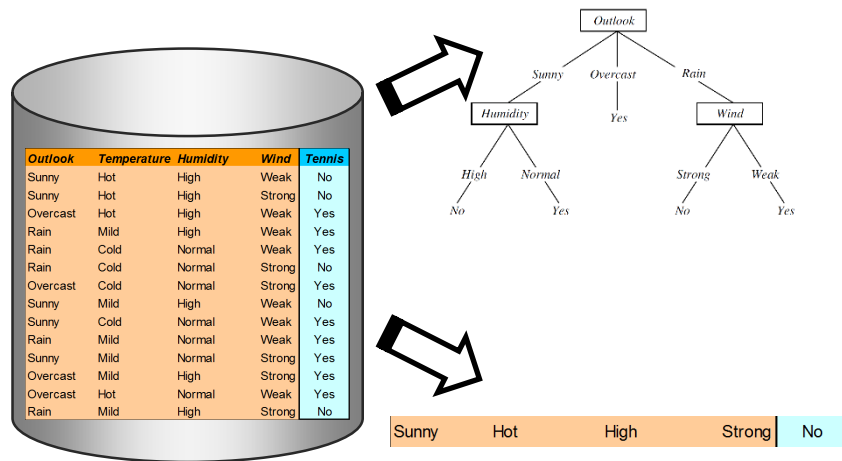
- There are two ways of learning from data

- ◆ **Machine Learning:**

- Learn a set of rules from data
- Apply this model for any new case

- ◆ **Case-Based Reasoning (CBR):**

- For a new situation find the most similar data set and take the conclusion
- If no appropriate data set is found, solve the new case ad hoc and store it



# Humans Use Cases for Problem Solving

## Examples:

- A medical doctor remembers the case history of another patient
- A lawyer argues with similar original precedence
- An architect studies the construction of existing building
- A work scheduler remembers the construction steps of a similar workpiece (Variant)
- A mathematician tries to transfers a known proof to a new problem
- A service technician remembers a similar defect at another device

# Machine Learning vs. Case Based Reasoning

## ■ Machine Learning

- ◆ can deliver good rules, if enough data sets are available

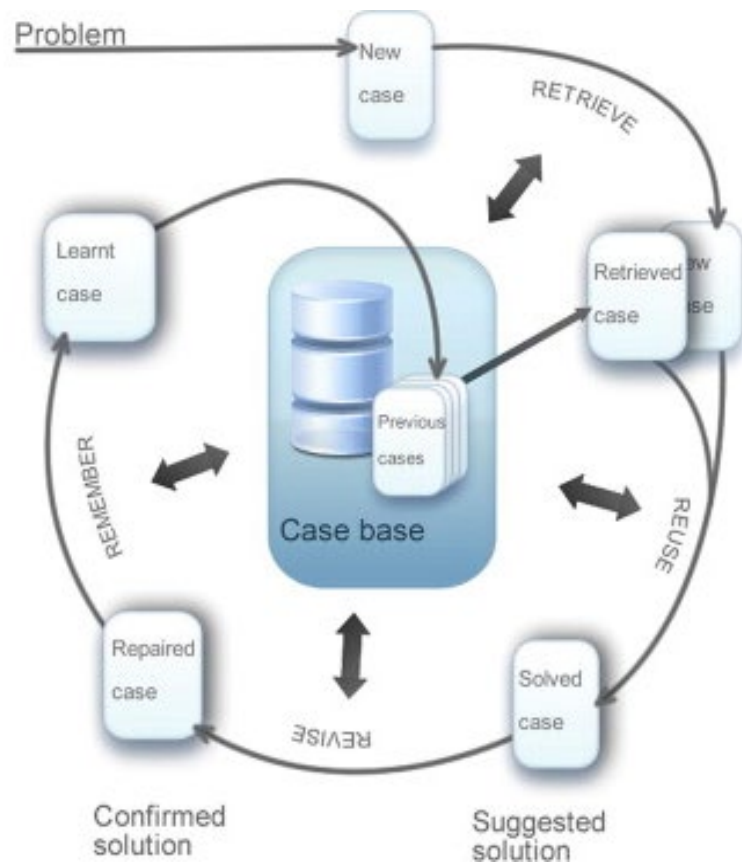
## ■ Case-Based Reasoning

- ◆ If not enough data sets are available
- ◆ If Generalization is not possible
  - e.g. medicine, law
- ◆ If solutions are complex or can only be vaguely described
  - e.g. therapy of a disease, repair of a machine
- ◆ If a domain is evolving
  - e.g. law, architecture



# Case-Based Reasoning

**Assumption:** Similar problems have similar solutions



## General approach:

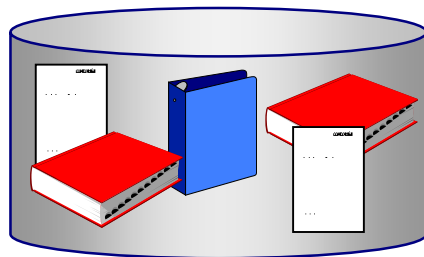
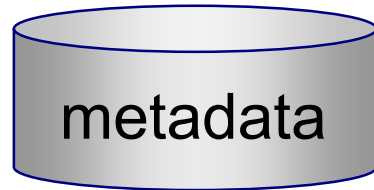
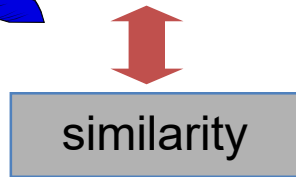
- Experiences are stored as cases
- To solve a new problem ...
  - ... retrieve similar cases
  - ... solution of the most similar case is reused in the new situation
- If there is no similar case (or solution does not work) ...
  - ... derive a new solution
  - ... store it as a new case

Source: A. Aamodt, E. Plaza (1994); AI Communications, IOS Press, Vol. 7: 1, pp. 39-59.

Quelle: Bergmann

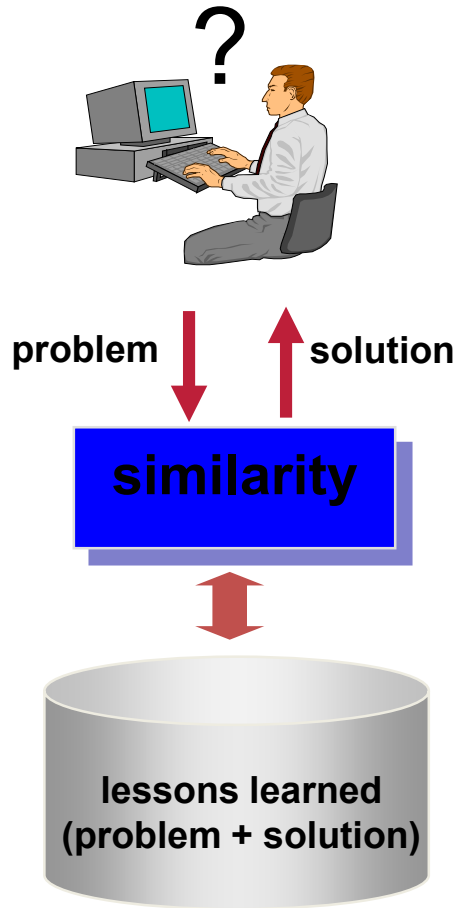


# Application Example: Information Retrieval



- Scenario: A database with offers of used cars
- Assume you look for the following car:
  - ◆ Audi A4, limousine, 125 PS, colour silver, automatic transmission, 2 years, bis 30'000 Fr.
- Problem: The database does not contain a car with exact this equipment
- Objective: The system should suggest those cars that are most similar to the one I look for, e.g..
  - ◆ Audi A4, limousine, 150 PS, blue, 5 gears, 2 years, € 28.000
  - ◆ Audi A4, station waggon, 125 PS, silver, automatic transmission, 3 years, € 26.000
  - ◆ BMW 320, limousine, 138 PS, silver, automatic transmission, 2 years, € 29.500
  - ◆ Volvo S40, limousine, 125 PS, silver, automatic transmission, 18 months, € 29.000
- similarity = relaxing query

# Learning from Experience: A simple Example (Overview)



## ■ Repairing a Car

- ◆ Symptoms are observed (e.g. engine doesn't start) and values are measured (e.g. battery voltage = 6.3V)
- ◆ Goal: Find the cause for the failure (e.g. battery empty) and a repair strategy (e.g. charge battery)

## ■ Case-Based Diagnosis:

- ◆ A case describes a diagnostic situation and contains:
  - description of the symptoms
  - description of the failure and the cause
  - description of a repair strategy
- ◆ A collection of cases is stored in a case base
- ◆ Find case similar to current problem and reuse repair strategy

# A simple Example: What's in a Case

- A case describes one particular diagnostic situation
- A case records several features and their specific values
- Assumption: All cases are independent from each other

C A S E  1	<b>Problem (Symptoms)</b> <ul style="list-style-type: none"> <li>• <i>Problem:</i> Front light doesn't work</li> <li>• <i>Car:</i> VW Golf II, 1.6 L</li> <li>• <i>Year:</i> 1993</li> <li>• <i>Battery voltage:</i> 13,6 V</li> <li>• <i>State of lights:</i> OK</li> <li>• <i>State of light switch:</i> OK</li> </ul>
	<b>Solution</b> <ul style="list-style-type: none"> <li>• <i>Diagnosis:</i> Front light fuse defect</li> <li>• <i>Repair:</i> Replace front light fuse</li> </ul>

Feature

Value

Source: Ralph Bergmann





# Example: A Case Base with 2 Cases

<b>C A S E  1</b>	<b>Problem (Symptoms)</b>
	<ul style="list-style-type: none"> <li>• Problem: Front light doesn't work</li> <li>• Car: VW Golf II, 1.6 L</li> <li>• Year: 1993</li> <li>• Battery voltage: 13,6 V</li> <li>• State of lights: OK</li> <li>• State of light switch: OK</li> </ul>
<b>Solution</b>	<ul style="list-style-type: none"> <li>• Diagnosis: Front light fuse defect</li> <li>• Repair: Replace front light fuse</li> </ul>

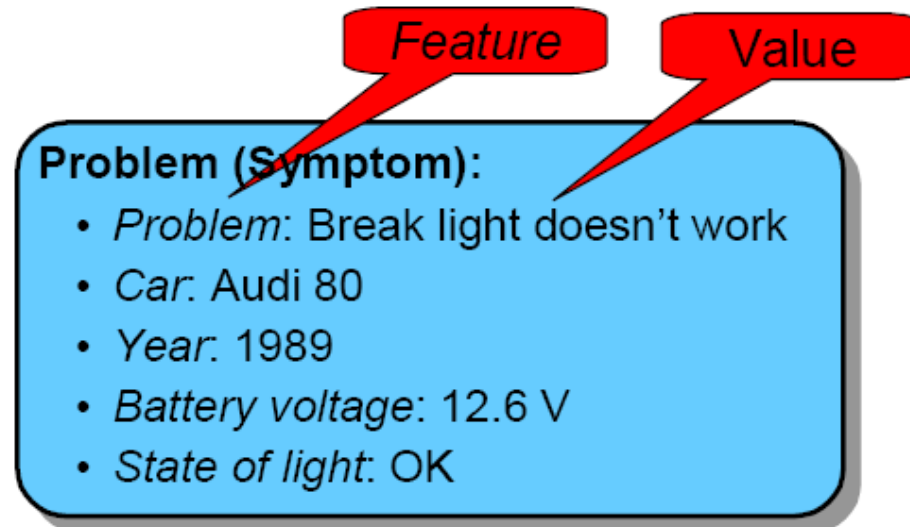
<b>C A S E  2</b>	<b>Problem (Symptoms)</b>
	<ul style="list-style-type: none"> <li>• Problem: Front light doesn't work</li> <li>• Car: Audi A6</li> <li>• Year: 1995</li> <li>• Battery voltage : 12,9 V</li> <li>• State of lights: surface damaged</li> <li>• State of light switch: OK</li> </ul>
<b>Solution</b>	<ul style="list-style-type: none"> <li>• Diagnosis: Bulb defect</li> <li>• Repair: Replace front light</li> </ul>

Source: Ralph Bergmann  
Adapted from Bergmann



## Solving a new Diagnostic Problem

- When a new problem occurs, we make several observations in the current situation
  - ◆ Observations define a new problem
  - ◆ Not all feature values may be known
- Note: The new problem is a case without solution part



Source: Ralph Bergmann

# Compare the New Problem with Each Case and Select the Most Similar Case



**Similarity is the most important concept in CBR !!**

- When are two cases similar?
- How to rank the cases according to their similarity?
- We can assess similarity based on the similarity of each feature
  - ◆ Similarity of each feature depends on the feature **value**.

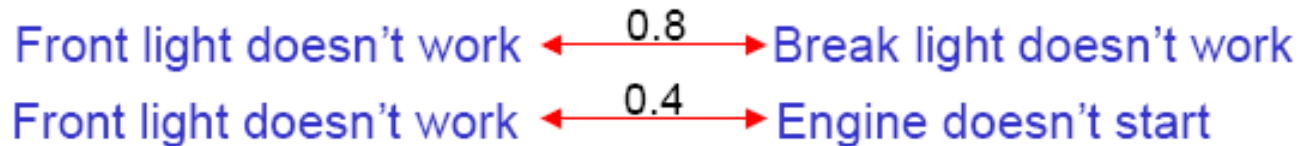
# Similarity Computation

- Assignment of similarities for features values.
- Express degree of similarity by a real number between 0 and 1

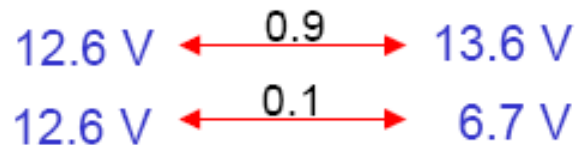
Not similar      Very similar

## Examples:

- Feature: *Problem*



- Feature: *Battery voltage* (similarity depends on the difference)



- Different features have different importance (weights) !

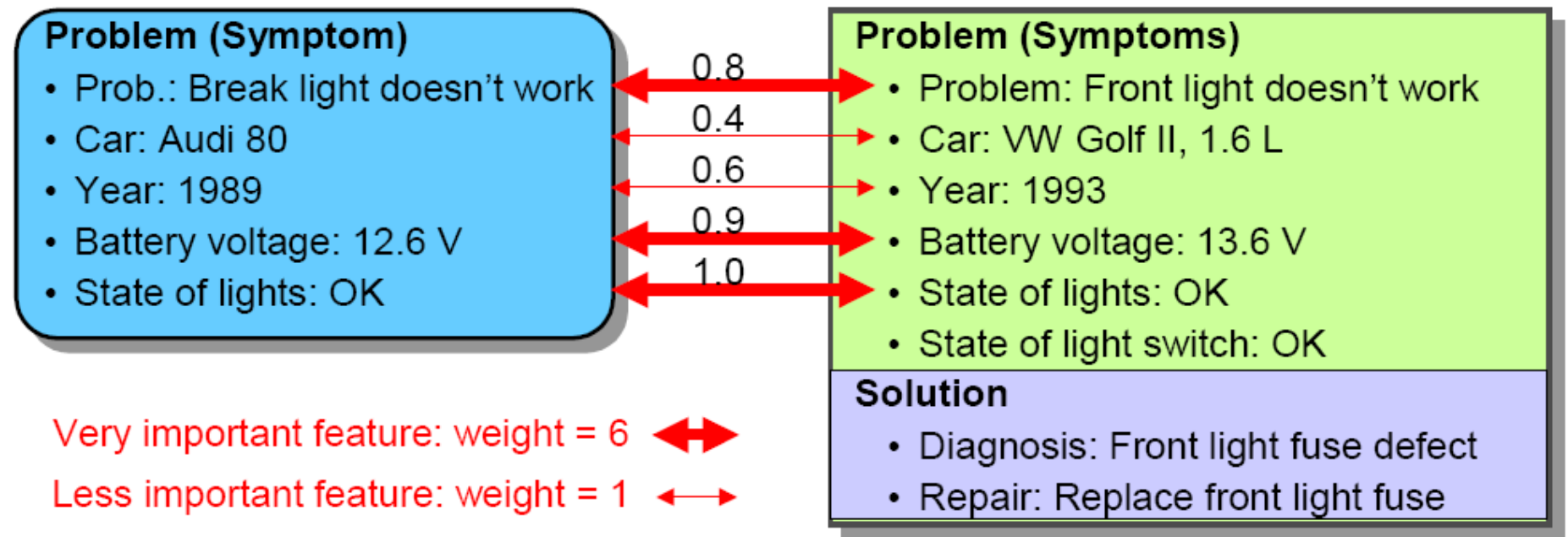


- High importance: Problem, Battery voltage, State of light, ...
- Low importance: Car, Year, ...

Source: Ralph Bergmann



# Compare New Problem and Case 1



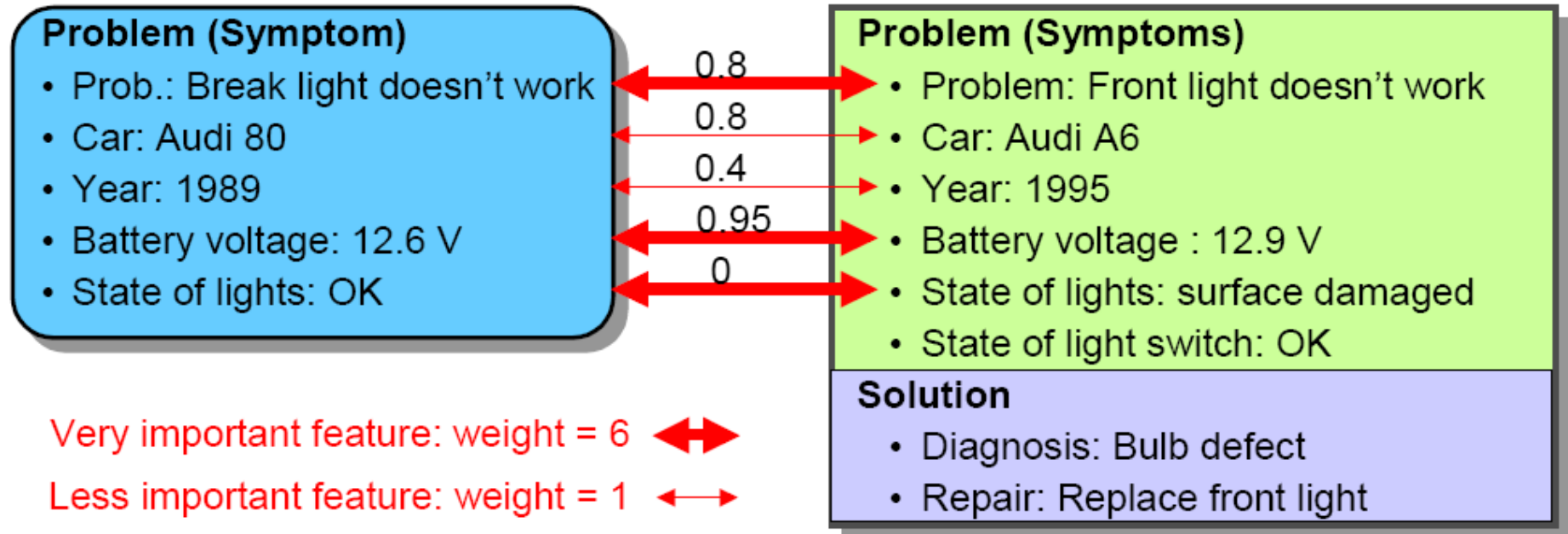
## Similarity Computation by Weighted Average

$$similarity(new, case 1) = 1/20 * [ 6*0.8 + 1*0.4 + 1*0.6 + 6*0.9 + 6* 1.0 ] = 0.86$$

Source: Ralph Bergmann



# Compare New Problem and Case 2



## Similarity Computation by Weighted Average

$$similarity(new, case 2) = 1/20 * [ 6*0.8 + 1*0.8 + 1*0.4 + 6*0.95 + 6*0 ] = 0.585$$

**Case 1 is more similar: due to feature "State of lights"**

Source: Ralph Bergmann

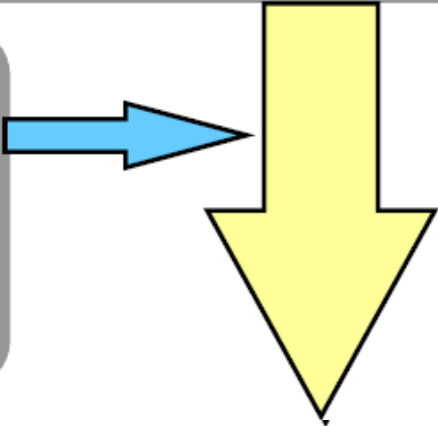


# Reuse Solution of Case 1

<b>C A S E</b>	<b>Problem (Symptoms):</b> <ul style="list-style-type: none"> <li>• <b>Front light</b> doesn't work</li> <li>• ...</li> </ul>
	<b>Solution:</b> <ul style="list-style-type: none"> <li>• Diagnosis: <b>Front light</b> fuse defect</li> <li>• Repair: Replace <b>front light</b> fuse</li> </ul>

**Problem (Symptom):1**

- **Prob.:** **Break light** doesn't work
- Car: Audi 80
- Year: 1989
- Battery voltage: 12,6 V
- state of break light: OK



**Adapt Solution:**  
How do differences in the problem affect the solution?

• **New Solution:**

- Diagnosis: **Break light** fuse defect
- Repair: Replace **break light** fuse

Source: Ralph Bergmann



# Store the New Experience

- If diagnosis is correct: store new case in the memory.

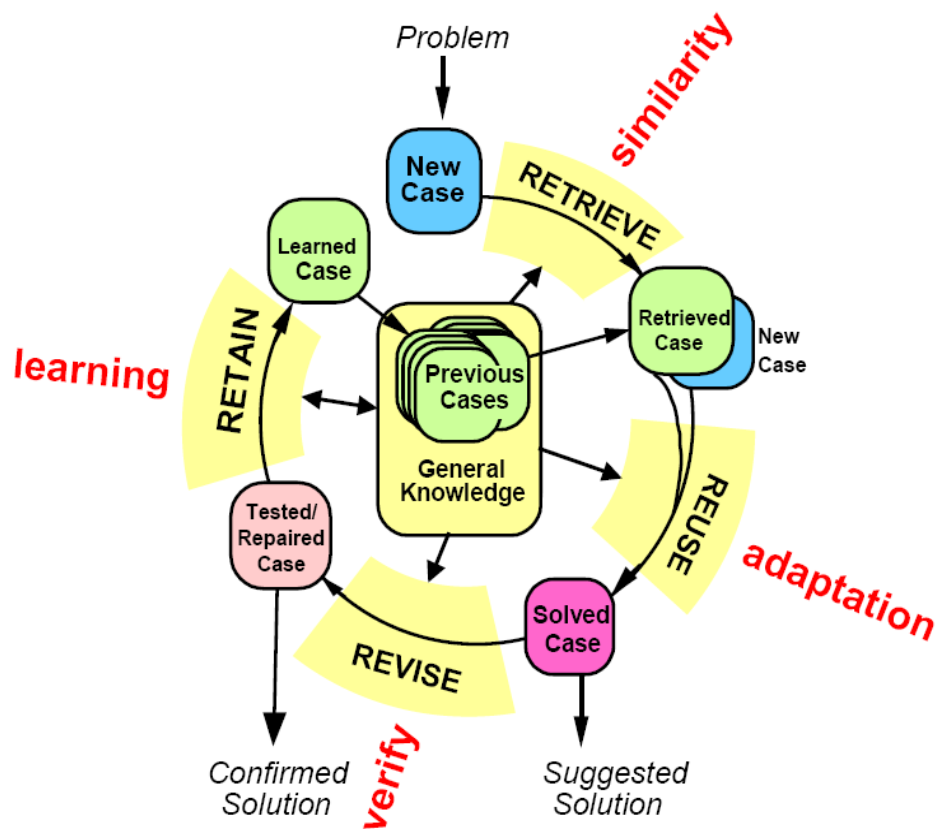
<b>C A S E  3</b>	<b>Problem (Symptoms):</b> <ul style="list-style-type: none"> <li>• Problem: Break light doesn't work</li> <li>• Car: Audi 80</li> <li>• Year: 1989</li> <li>• Battery voltage: 12.6 V</li> <li>• State of break lights: OK</li> <li>• light switch clicking: OK</li> </ul>
	<b>Solution:</b> <ul style="list-style-type: none"> <li>• Diagnosis: break light fuse defect</li> <li>• Repair: replace break light fuse</li> </ul>

Source: Ralph Bergmann





# CBR Cycle



Retrieve ...  
most similar case or cases

Reuse ...  
the information and knowledge in that case to solve the problem

Revise ...  
the proposed solution if necessary

Retain ...  
the parts of this experience likely to be useful for future problem solving

Source: K.-D. Althoff & A. Aamodt: Relating case-based problem solving and learning methods to task and domain characteristics. AI Communications 1996



# Case Representation

## Textual Approach

### Frequently Asked Question 241

**Title:** Order numbers of CPUs with which communications is possible.

**Question:** Which order numbers must the S7-CPU's have to be able to run basic communications with SFC's?

**Answer:** In order to participate in communications via SFC's without a configured connection table, the module concerned must have the correct order number. The following table illustrates which order number your CPU must have to be able to participate in these S7 homogeneous communications.

## Conversational Approach

**Case:** 241

**Title:** Printer does not work in the new release.

**Q1:** *What kind of problem do you have?* Printer Problem

**Q1:** *Does the printer perform a self-test?* Yes

**Q2:** *Does the printer work with other software?* Yes

**Q3:** *Did you just install the software?* Yes

**Q4:** *Did you create a printer definition file?* Yes

**Q5:** *What release did you install?* 4.2

**Problem:** Installation procedure overrides printer definition

**Action:** Reinstall the printer from disk 2.3

## Structural Approach

Reference : AD8009

Price : 2.25

Input offset voltage : 2 mV

Input bias current : 50 uA

Output voltage : 1.2 V

Output current drive : 175 mA

Single supply : No

PSPS : 70 dB

Number of devices per package : single

Available Package(s) : SOIC

(Bergmann 2002, p. 54ff)



# Structural Cases: Attribute Value Representation

- A common approach for structural case representations are attribute value pairs

Example: *Price: 25.000 CHF*

The diagram shows the text "Example: Price: 25.000 CHF". Below "Price" is a yellow callout box with the word "attribute". Below "25.000 CHF" is a yellow callout box with the word "value". Red lines connect the boxes to their respective parts of the text.

- As in programming language, types define allowed values for attributes. Examples of types are
  - ◆ *numbers types* like integer, real or intervals
  - ◆ *symbols* defined by
    - enumeration of symbols {red, yellow, green}
    - elements of a knowledge structure, e.g. classification scheme
  - ◆ *text* such as strings or markups
  - ◆ *special types* like URLs

## Example: Searching for Used Car

### car 1:

- ◆ model: Audi A4
- ◆ year: 2014
- ◆ price: 15'000
- ◆ type: station waggon
- ◆ transmission: automatic
- ◆ extra: AC

### car 2:

- ◆ model: Audi A3
- ◆ year: 2015
- ◆ price: 20'000
- ◆ type: limousine
- ◆ transmission: 5 gear
- ◆ extra: CD, handsfree-kit

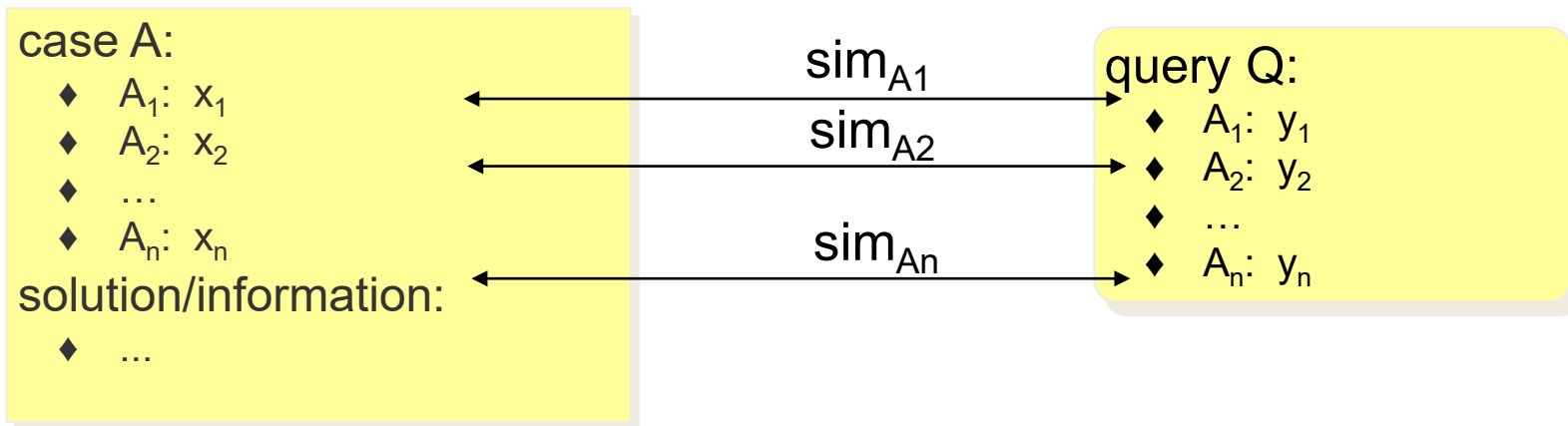
### query:

- ◆ model: VW Golf
- ◆ year: 2016
- ◆ price: 18'000
- ◆ type: station waggon
- ◆ transmission: 4 gear
- ◆ extra: AC, handsfree-kit

# Similarity Calculation for Attribute-Value Pairs

Cases resp. meta-data are represented by  $n$  attributes  $A_1, \dots, A_n$

- ◆ each attribute  $A_i$  has type  $T_i$



**Local similarity:** for each attribute a similarity function is defined

- ◆  $sim_{A_i}(x_i, y_i): T_i \times T_i \rightarrow [0..1]$
- ◆ local similarity measures depend on the type of the attribute

**Global similarity:** combining values for local similarity

- ◆  $sim(A, Q) = F(sim_{A_1}(x_1, y_1), sim_{A_2}(x_1, y_1), \dots, sim_{A_n}(x_n, y_n))$
- ◆  $F: [0..1]^n \rightarrow [0..1]$  is called an **aggregation function**

# Similarity Measure

**Definition:** A *similarity measure* is a function  $sim: M \times M \rightarrow [0, 1]$

- This definition restricts similarity to a number in the intervall  $[0, 1]$ .
  - ◆ It allows to express the most similar (1) und the least similar (0) situation
  - ◆ It also allows to express degrees of similarity: if  $sim(x, y) > sim(x, z)$ , then x is more similar to y than to z

(Bergmann 2002, p. 96)



# CBR vs. Database Query

- Remark: Query answering in SQL can be seen as a special case of a similarity measure where the value range is a set  $\{0,1\}$ 
  - ◆  $\text{sim}(x,y) = 1$  (equal)      or
  - ◆  $\text{sim}(x,y) = 0$  (unequal)

Database System	CBR System
<ul style="list-style-type: none"> <li>• simple search <math>\Rightarrow</math> "all or nothing"</li> </ul>	<ul style="list-style-type: none"> <li>• using same database but search for most similar cases</li> </ul>
<ul style="list-style-type: none"> <li>• often too many hits (underspecification) or no hits at all (overspecification)</li> </ul>	<ul style="list-style-type: none"> <li>• system can be told to show only, e.g., 10 cases by descending order</li> </ul>
<ul style="list-style-type: none"> <li>• no specific domain knowledge used for the search</li> </ul>	<ul style="list-style-type: none"> <li>• considers domain knowledge for search by using similarity measures, e.g., spatial or geographical relations</li> </ul>
<ul style="list-style-type: none"> <li>• pure database applications cannot be used for online consulting</li> </ul>	<ul style="list-style-type: none"> <li>• online consulting is the power of a CBR system</li> </ul>

(Bergmann 2002, p. 96)



# Usual Properties for Similarity Measures

- A similarity measure is called *reflexive* if

$$sim(x,x) = 1$$

holds for all  $x$  <sup>1)</sup>

- If additionally  $sim(x,y) = 1$  implies that  $x = y$ , then the similarity measure is called *strong reflexive*

- A similarity measure is called *symmetric* if for all  $x,y$  it holds that

$$sim(x,y) = sim(y,x)$$

Symmetric: it is not important for the similarity computation which attribute value belongs to the query and which one belongs to the case

Assymmetric: it is important which value belongs to the query and which one belongs to the case; a major example is the attribute *price*.

(Bergmann 2002, p. 101f)

1) This means that for every value  $x$  of  $M$ ,  $x$  is maximally similar to itself





# Meanings of Similarity.

Similarity ...

... always refers to a specific aspect

- ◆ Example: Two cars are similar if they
  - are of the same brand
  - have similar maximum speed

... is not necessarily transitive

- ◆ Example: For integers we can say that
  - 2 is similar to 4
  - 4 is similar to 6
  - ...
  - 99998 is similar to 100.000

But: Is 2 similar to 100.000?

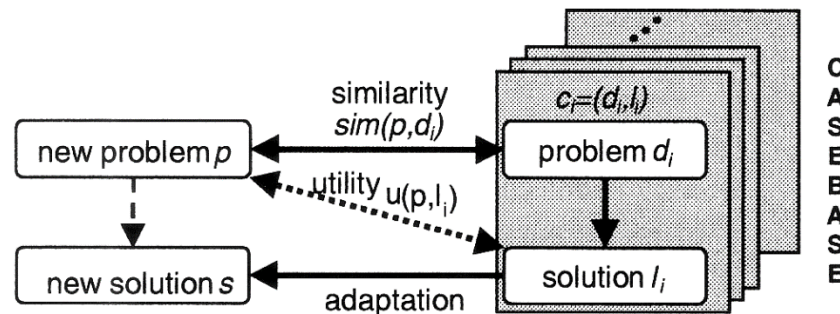
... is not necessarily symmetric

- ◆ if I look for a limousine, I probably could accept a station wagon
- ◆ if I need a station wagon because of the space, a limousine might be less acceptable for me



# Approximating Utility with Similarity

- Assumption
  - ◆ Similar problems have similar solutions
  - ◆ The solution of a problem is also useful for similar problems
- Utility means the usefulness of a solution for a problem
  - ◆ Utility is an **a posteriori** criteria: It can be assessed **after** the problem was solved.
  - ◆ Utility corresponds to relevance in information retrieval
- Utility of a case  $c$  for a problem  $p$  is approximated by the similarity between the problem  $p$  and problem  $d$  contained in case  $c$ 
  - ◆ Similarity is an **a priori** criteria: It must be assessed **before** problem solving.



(Bergmann 2002, p. 94)



# Local Similarity Measure for Numeric Attributes

- For numeric attributes, similarity is computed as a function of distance  $d$ :

$$\text{sim}_A(x,y) = f(|d(x,y)|)$$

- Typical distance functions are :

- ◆ standard linear distance (= difference)  $d(x,y) = |x - y|$

- ◆ logarithmic distance  $d(x,y) = |\log(x) - \log(y)|$

(logarithmic distance is used for exponentially scaled value ranges. i.e. if the value range for the attribute spans several orders of magnitude)

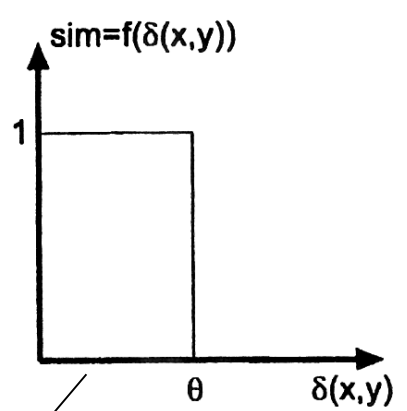
- Examples of similarity measure for numeric attributes:

linear function

non-linear function

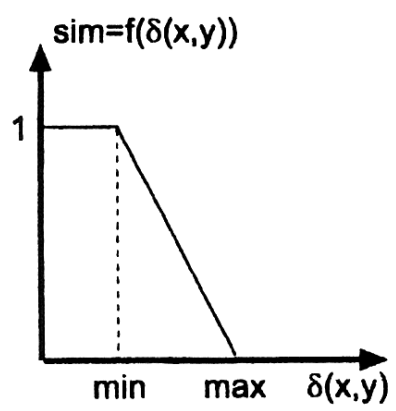
# Further Functions for Numeric Similarity Measures

**Threshold Function**



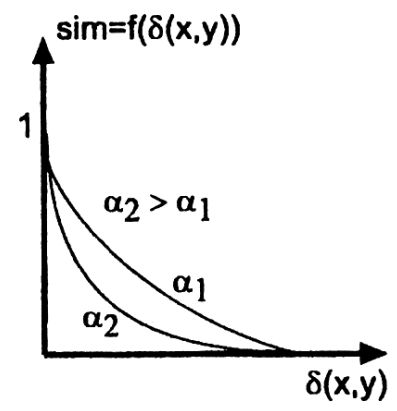
$$f(d) = \begin{cases} 1 & : d < \theta \\ 0 & : d \geq \theta \end{cases}$$

**Linear Function**



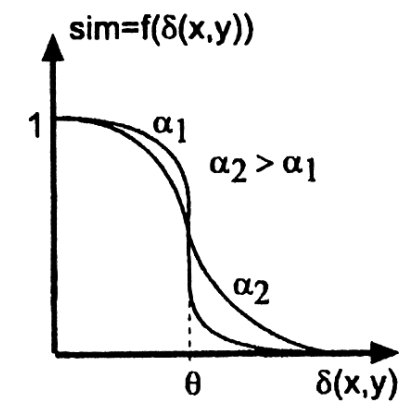
$$f(d) = \begin{cases} 1 & : d < min \\ \frac{max-d}{max-min} & : min \leq d \leq max \\ 0 & : d > max \end{cases}$$

**Exponential Function**



$$f(d) = e^{d \cdot \alpha}$$

**Sigmoid Function**



$$f(d) = \frac{1}{e^{\frac{d-\theta}{\alpha}} + 1}$$

(Bergmann 2002, p. 108f)



# Symmetry for Numeric Values

## ■ Symmetric:

- ◆ it is not important for the similarity computation which attribute value belongs to the query and which one belongs to the case
- ◆ Example: for numeric values use the absolute distance  $d = |\text{query value} - \text{case value}|$ 
  - $\text{sim}(x,y) = f(d(x,y))$

## ■ Asymmetric:

- ◆ it is important which value belongs to the query and which one belongs to the case; a major example is the attribute *price*.
- ◆ similarity can be computed with two different similarity functions

$$\text{sim}(x,y) = \begin{cases} f_1(d(x,y)) & \text{if } x > y \\ 1 & \text{if } x = y \\ f_2(d(x,y)) & \text{if } x < y \end{cases}$$

# Local Similarity for Ordered Symbols

- For symbolic attributes we can distinguish approaches depending on whether there is an order defined on the symbols or not.
- Example for ordered symbols: qualitative values, e.g. {small, medium, large}
  - ◆  $\text{small} < \text{medium} < \text{large}$
- With such an order defined, we can determine the similarity by using the ordinal number of the symbols, e.g.
  - ◆ small            --> 1
  - ◆ medium        --> 2
  - ◆ large           --> 3and applying similarity measure for numeric attributes

# Local Similarity for unordered Symbols

- If there is no obvious ordering on the set of attribute values and no ordering can be defined, we can apply the tabular approach

◆  $\text{sim}_A(x,y) = s[x,y]$

		cases y		
$s[x,y]$		limousine	convertible	station
query x	limousine	1	0.3	0.7
	convertible	0.4	1	0.2
	station	0.5	0.2	1

*similarity of x and y*

- Reflexive similarity measure: diagonal values are 1
- Symmetric similarity measure:  
upper triangular matrix = lower triangular matrix

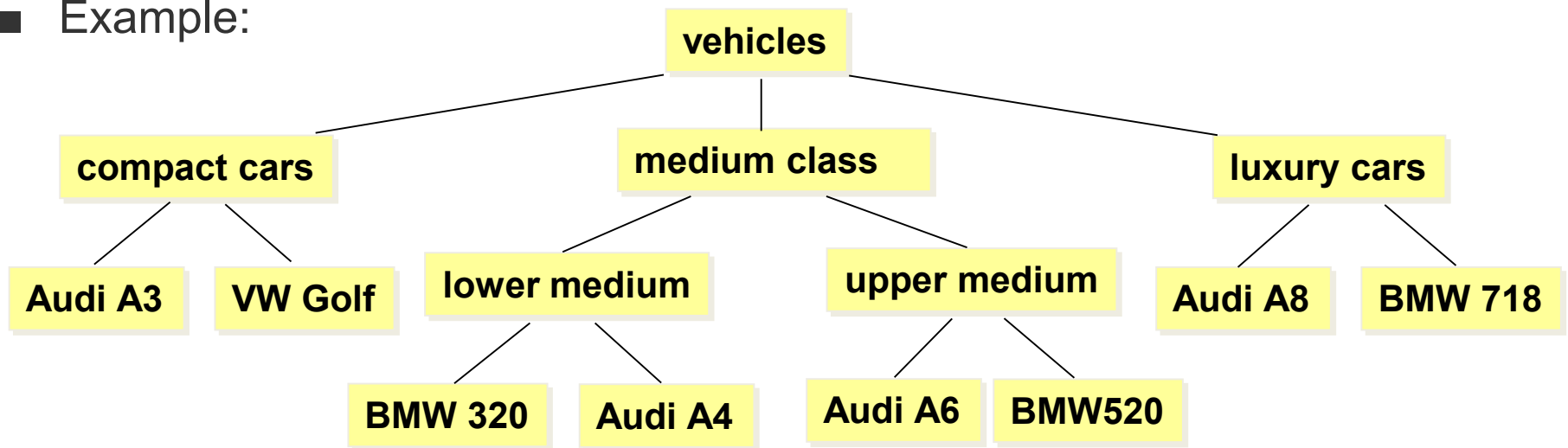
cp. (Bergmann 2002, p. 110)



# Taxonomically Ordered Symbolic Types

- A special variant of symbolic types are taxonomies. A taxonomy is a tree in which the nodes represent symbolic values
- A taxonomy represents an additional relationship between the symbols
  - ◆ Leaf nodes represent concrete objects of the real world
  - ◆ Inner nodes represent classes of real world objects.
  - ◆ An inner node k stands for the set of real world objects represented by leaf nodes below it

■ Example:



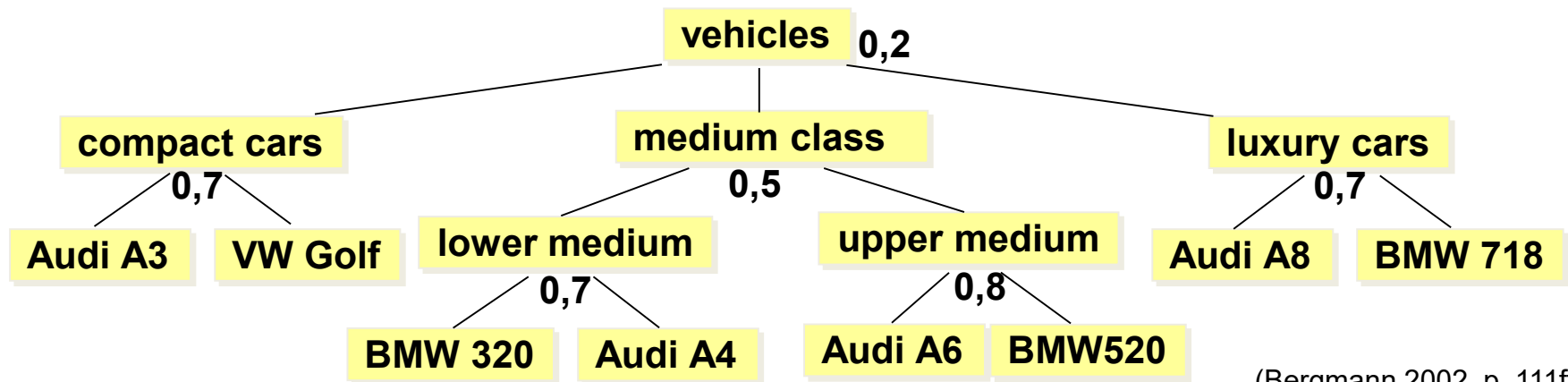
(Bergmann 2002, p. 111ff)





# Similarity Measure for Taxonomies

- Inner nodes cluster real-world objects that have some properties in common.
- The deeper we decent in the taxonomy, the more features do the objects have in common
- Similarity measures in a taxonomy
  - ◆ Every inner node  $K_i$  is annotated with a similarity value  $S_i$
  - ◆ The deeper the nodes in the hierarchy, the larger the similarity value can become
  - ◆ Meaning: All elements belonging to a class have the similarity measure assigned to the class



(Bergmann 2002, p. 111ff)



# Computing Similarity Measure between Leaf Nodes of a Taxonomy

- Leaf nodes represent concrete element objects
- Similarity of two leaf nodes is the similarity value of the lowest common predecessor

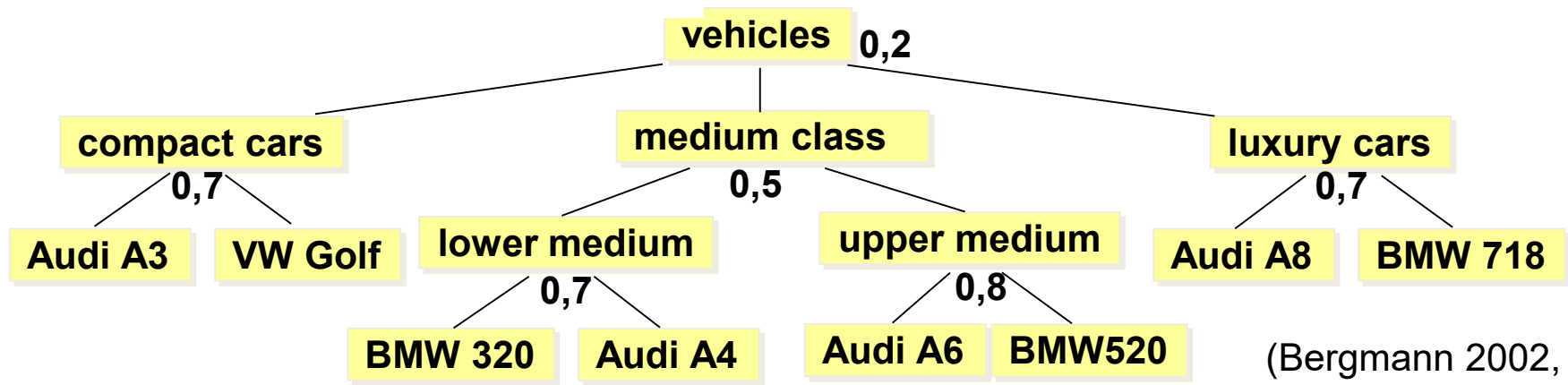
$$sim(K_1, K_2) = \begin{cases} 1 & : K_1 = K_2 \\ S_{\langle K_1, K_2 \rangle} & : K_1 \neq K_2 \end{cases}$$

Example:

sim(BMW320, AudiA4) =

sim(BMW320, AudiA6) =

sim(BMW320, AudiA8) =



(Bergmann 2002, p. 111ff)

# Semantics and Similarity of Inner Nodes

An inner node can have different meanings

- Any value in the query: The inner node in the query stands for any value below this node.
  - ◆ Example: A person is looking for a compact car but does not care whether it is an Audi A3 or a VW Golf
  - ◆ Sample query: „compact car“
- Any value in the Case: Typically cases have concrete values, but it could also include inner nodes, which stand for any value below the node
  - ◆ Assume a car dealer specifies that he sells any compact cars
- Uncertainty: The use of an inner node K means that we do not know the exact value for this attribute, but we know that it must be a concrete value below this node
  - ◆ Assume a car dealer specifies that he sells a single compact car without saying whether it is a Audi A3 or a VW Golf

(Bergmann 2002, p. 111ff)



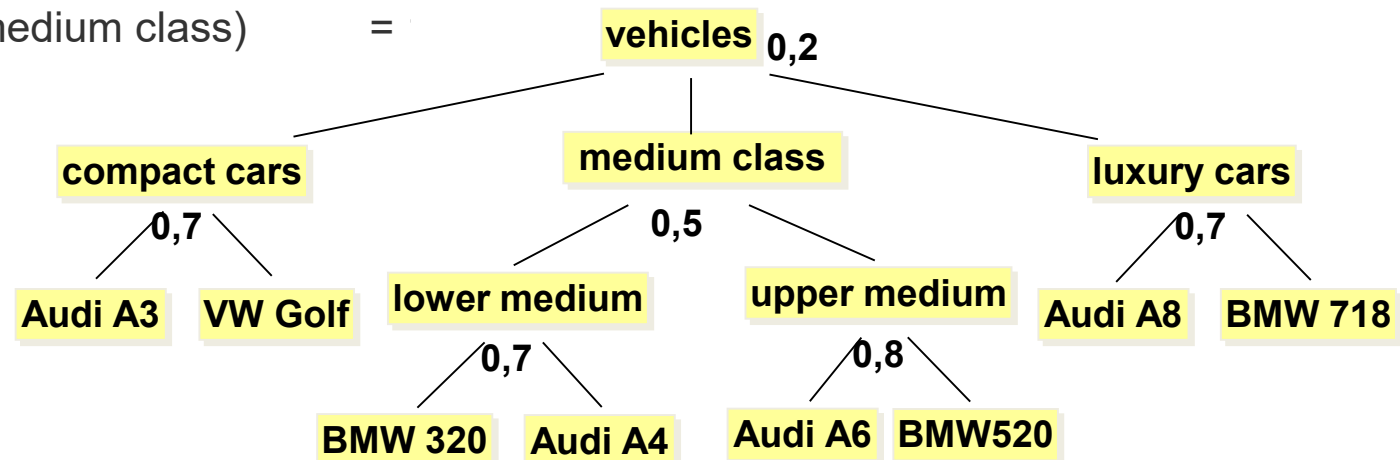
# Similarity with inner Nodes of a Taxonomy: Any Value (1/2)

We are looking for the highest possible similarity  $\text{sim}(Q,C)$  <sup>1)</sup>

- ◆  $\text{sim}(\text{compact car}, \text{AudiA3}) = \dots$
- ◆  $\text{sim}(\text{medium class}, \text{AudiA4}) = \dots$
- ◆  $\text{sim}(\text{medium class}, \text{AudiA3}) = \dots$

Also possible for an inner node in a case:

- ◆  $\text{sim}(\text{AudiA4}, \text{medium class}) = \dots$
- ◆  $\text{sim}(\text{AudiA3}, \text{medium class}) = \dots$
- ◆  $\text{sim}(\text{medium class}, \text{compact car}) = \dots$
- ◆  $\text{sim}(\text{lower medium}, \text{medium class}) = \dots$



<sup>1)</sup> The first parameter of  $\text{sim}(Q,C)$  is the query, the second parameter is the case description



# Similarity with inner Nodes of a Taxonomy: Any Values (2/2)

- Inner node in the query or in the case
  - ◆ All leaf nodes below the inner node have similarity 1
  - ◆ For all other nodes: Take the similarity of the lowest common predecessor.

$$\begin{aligned}
 sim_{A_i}(Q, C) &= \max\{sim(q, c) | q \in L_Q, c \in L_C\} \\
 &= \begin{cases} 1 & : C < Q \text{ or } Q < C \\ S_{\langle Q, C \rangle} & : \text{otherwise} \end{cases}
 \end{aligned}$$

**Notation:**

Q	Query
C	Case
$L_Q, L_C$	leaf nodes below Q or C
$C < Q$	C is below Q in the taxonomy (Q is predecessor of C)
$S_{\langle Q, C \rangle}$	similarity of the lowest common predecessor of Q and C

(Bergmann 2002, p. 118)



# Multiple Attribute Values

- Attributes can contain multiple values

- ◆  $A_{\text{query}} = \{a_1, \dots, a_n\}$

- ◆  $A_{\text{case}} = \{b_1, \dots, b_m\}$

- Similarity measure for sets:

- ◆ Compute all pairs of similarity measures  $sim_A(a_i, b_j)$

- ◆ Aggregate the local similarity

$$sim_A(A_{\text{query}}, A_{\text{case}})$$

$$= MF(sim_A(a_1, b_1), \dots, sim_A(a_1, b_m), \dots, sim_A(a_n, b_1), \dots, sim_A(a_n, b_m))$$

- There are various possible approaches for the aggregate function MF, e.g.

- ◆ minimum

- ◆ maximum

- ◆ average

# Unknown Attribute Values

- It often occurs that attribute values are not known (NULL):
- Strategies to deal with unknown values
  - ◆ **optimistic strategy:** Assume that unknown values are most similar:  $\text{sim}(\text{NULL}, x) = 1$
  - ◆ **pessimistic strategy:** Assume that unknown values are least similar:  $\text{sim}(\text{NULL}, x) = 0$ .
  - ◆ **strategy of expected value:** Use an expected value, e.g. based on probability or average
  - ◆ ignore the attributes

## Global Similarity

- Global similarity measures are defined by applying an aggregation function  $F : [0..1]^n \rightarrow [0..1]$  to the local similarity values.

- ◆ Input: Local similarity measures  $\text{sim}_{A_i}(x_i, y_i)$  for each attribute  $A_i$
- ◆ Global similarity:

$$\text{sim}(x,y) = F(\text{sim}_{A_1}(x_1, y_1), \dots, \text{sim}_{A_n}(x_n, y_n))$$

- Possible properties for  $F$ 
  - $F$  is monotone in each argument
  - $F(0, \dots, 0) = 0$
  - $F(1, \dots, 1) = 1$



# Basic Aggregation Functions

■ Weighted Average:  $F(s_1, \dots, s_n) = \sum_{i=1}^n w_i \cdot s_i$  with  $\sum_{i=1}^n w_i = 1$

■ Generalized weighted average:  $F(s_1, \dots, s_n) = \sqrt[\alpha]{\sum_{i=1}^n w_i \cdot s_i^\alpha}$  with  $\alpha \in \mathbb{R}^+$  und  $\sum_{i=1}^n w_i = 1$

■ Maximum:  $F(s_1, \dots, s_n) = \max_{i=1}^n (w_i \cdot s_i)$

■ Minimum:  $F(s_1, \dots, s_n) = \min_{i=1}^n (w_i \cdot s_i)$

(Bergmann 2002, p. 120f)

