



Rule-Based Systems: Logic Programming

Remember: our first rule-based system

father(peter,mary)

father(peter,john)

mother(mary,mark)

mother(jane,mary)

father(X,Y) AND father(Y,Z) \rightarrow grandfather(X,Z)

father(X,Y) AND mother(Y,Z) \rightarrow grandfather(X,Z)

mother(X,Y) AND father(Y,Z) \rightarrow grandmother(X,Z)

mother(X,Y) AND mother(Y,Z) \rightarrow grandmother(X,Z)

father(X,Y) AND father(X,Z) \rightarrow sibling(Y,Z)

mother(X,Y) AND mother(X,Z) \rightarrow sibling(Y,Z)

The rules can be used to

- Derive all grandparent and sibling relationships (forward chaining)
- Answer questions about relationships (backward chaining)

Logic Programming

- Logic programming is the use of
 - ◆ logic as a declarative representation language
 - ◆ Backward chaining as inference rule
- Logic Programming is the basis of the programming language PROLOG

Logic Programs – A Sequence of Horn Clauses

- The sentences of a logic program are Horn clauses
 - Facts: H
 - Rules: $H \leftarrow B_1 \wedge B_2 \wedge \dots \wedge B_n$
- A Horn clause without any head H is called a query
 - ◆ Query: $\leftarrow B_1 \wedge B_2 \wedge \dots \wedge B_n$
- Queries are not part of a logic program, they start the inference

Predicates and Literals

- Predicates are the building blocks of clauses
- Predicates have a name and arguments (parameters). Arity is the number of arguments.
- Predicates combine values which “makes sense” together (are true)
- Examples:
 - ◆ person(peter)
 - ◆ married(peter, cindy)
 - ◆ appointment(1.3.2016, holger, “AB1”, “Lecture KE”)
 - ◆ not female(holger)
- Literals are predicates and negated predicates

Variables, Constants, and Data

- Data are classical data which you know from programming languages
 - ◆ 31
 - ◆ “Lecture KE”
 - ◆ 21.3.2015
- Constants (symbols) are values
 - ◆ peter
 - ◆ cindy
- Differences between constants and data
 - ◆ data: rich operations (+, -, ...) and comparisons (=, <, >, <=, ...)
 - ◆ constants: only identity (=), but very quick!
- Variables are placeholders for constants or data
 - ◆ likes(holger, X)

Exercises (1/2)

- Write as a logic programme
 - ◆ john is a person
 - ◆ peter and mary are persons
 - ◆ fhnw is a university
 - ◆ john is matriculated at fhnw
 - ◆ A student is a person who is matriculated at a university.
 - ◆ Is john a student?
 - ◆ Is peter a student?

Exercises (2/2)

- Write as a logic programme
 - ◆ knut is a person
 - ◆ «KEBI» is a class
 - ◆ classes are taught by teachers
 - ◆ john attends to class «KEBI»
 - ◆ students are attending to classes
 - ◆ Is john a student?
 - ◆ knut teaches «KEBI»
 - ◆ Is knut a teacher?

PROLOG

- PROLOG (= PROgramming in LOGic) is a programming language based on Horn clauses
- Syntax:
 - ◆ Prolog uses „:-“ instead of „←“
 - ◆ Literals in the body are separated by comma „ , “ (the comma is equivalent to the logical AND or „^“)
 - ◆ Each clause ends with a period „. “
 - ◆ Variables are either
 - strings starting with capital letter: X, Person
 - strings starting with a underline: x, person

A Logic Programme in PROLOG Syntax

```
father (peter , mary) .  
father (peter , john) .  
  
mother (mary , mark) .  
mother (jane , mary) .
```

```
grandfather (X , Z) :- father (X , Y) , father (Y , Z) .  
grandfather (X , Z) :- father (X , Y) , mother (Y , Z) .  
  
grandmother (X , Z) :- mother (X , Y) , father (Y , Z) .  
grandmother (X , Z) :- mother (X , Y) , mother (Y , Z) .  
  
sibling (Y , Z) :- father (X , Y) , father (X , Z) .  
sibling (Y , Z) :- mother (X , Y) , mother (X , Z) .
```

- All Clauses with the same predicate in the head are called the definition of the predicate

Reasoning in Logic Programming

■ INPUT:

- ◆ A logic programme P and
- ◆ a query Q ($?- Q_1, Q_2, \dots, Q_m$)

■ INFERENCE: Backward Chaining

■ OUTPUT:

- ◆ If the query Q does *not* contain variables the answer is
 - yes if Q can be deduced from P
 - no, if Q cannot be deduced from P
- ◆ If the query Q does contain variables the answer is
 - A substitution σ for the variables in Q such $Q\sigma$ can be deduced from P
 - no, if there is no substitution σ such that $Q\sigma$ can be deduced from Q

A Logic Program and Queries

```
father (peter , mary) .  
father (peter , john) .  
mother (mary , mark) .  
mother (jane , mary) .
```

```
grandfather (X , Z) :- father (X , Y) , father (Y , Z) .  
grandfather (X , Z) :- father (X , Y) , mother (Y , Z) .  
grandmother (X , Z) :- mother (X , Y) , father (Y , Z) .  
grandmother (X , Z) :- mother (X , Y) , mother (Y , Z) .  
sibling (Y , Z) :- father (X , Y) , father (X , Z) .  
sibling (Y , Z) :- mother (X , Y) , mother (X , Z) .
```

Queries :

```
?- father (peter , john) .  
?- father (peter , X) .  
?- grandfather (peter , mark) .  
?- grandfather (peter , mary) .  
?- grandfather (peter , S) .  
?- sibling (X , Y) .
```

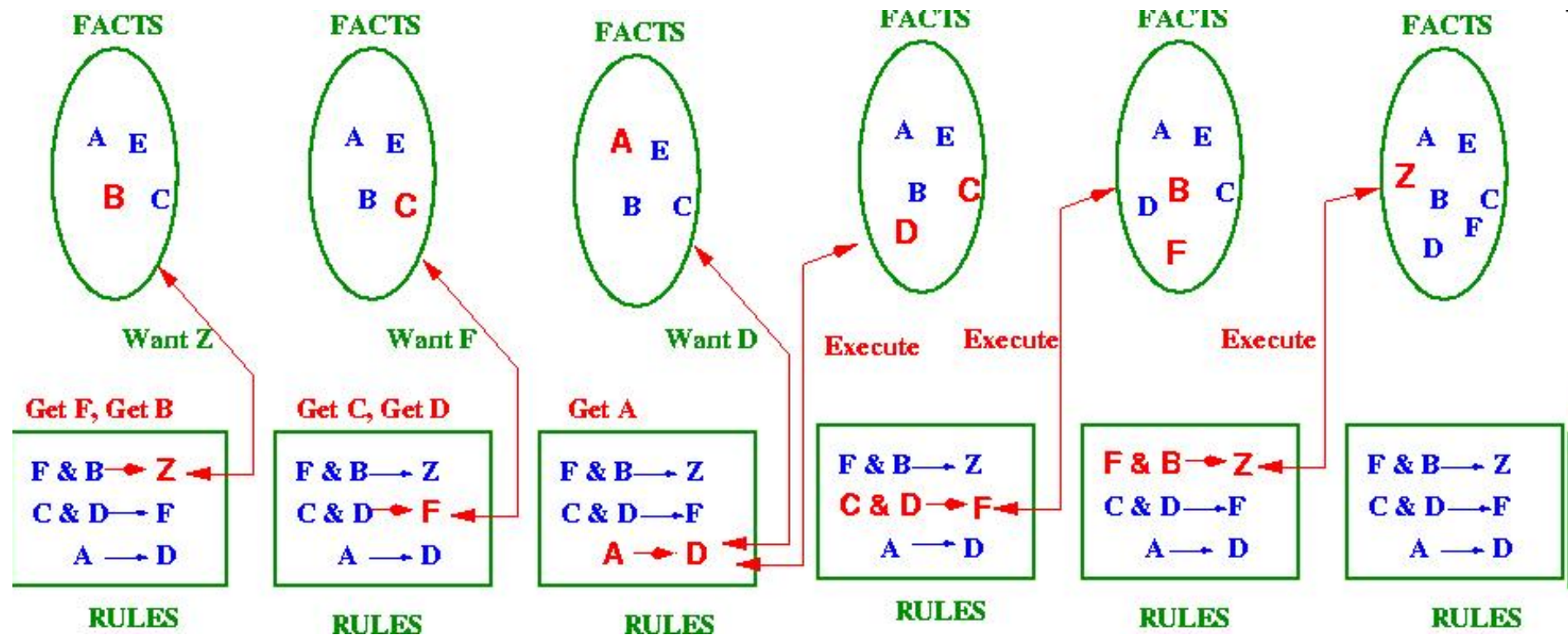
Substitution

- A *substitution* is a finite set of the form $\sigma = \{v_1 / t_1, \dots, v_n / t_n\}$
 - ◆ v_i 's: distinct variables.
 - ◆ t_i 's: terms with $t_i \neq v_i$.
- Applying a substitution σ to an expression E means to replace each occurrence of a variables v_i with the value t_i

■ Example:

$$E = p(X, Y, f(a))$$
$$\sigma = \{X / b, Y / Z\}$$
$$E\sigma = p(b, Z, f(a))$$
$$E = \text{father}(\text{peter}, X)$$
$$\sigma = \{X / \text{mary}\}$$
$$E\sigma = \text{father}(\text{peter}, \text{mary})$$

Illustrating Backward Chaining



Source: Kerber (2004), <http://www.cs.bham.ac.uk/~mmk/Teaching/AI/I2.html>

Inference Procedure for Logic Programming

Let *resolvent* be the query $?- Q_1, \dots, Q_m$

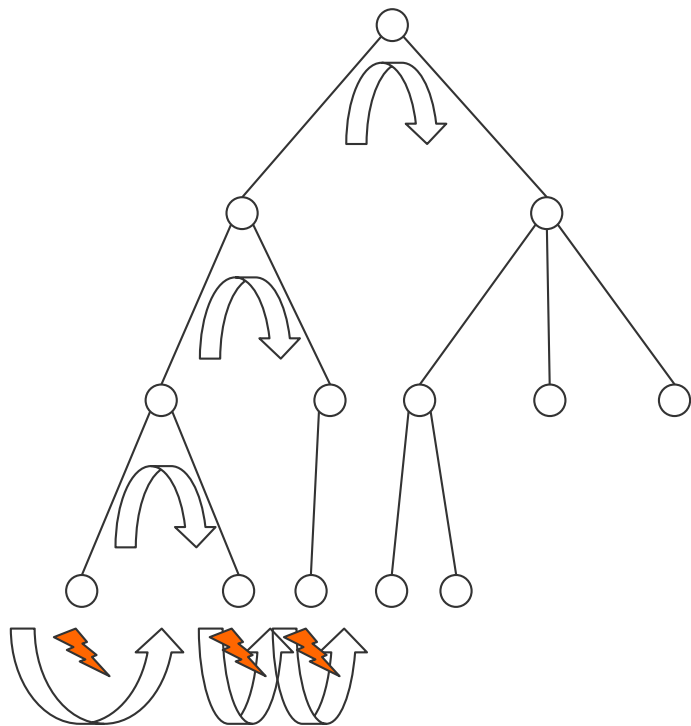
While *resolvent* is not empty do

1. **Choose** a query literal Q_i from *resolvent*.
2. **Choose** a renamed¹ clause $H :- B_1, \dots, B_n$ from P such that Q_i and H unify with an most general **unifier** σ , i.e. $Q_i\sigma = H\sigma$
3. **If** no such Q_i and clause exist, then **backtrack**
4. Remove Q_i from the resolvent
5. **Add** B_1, \dots, B_n to the resolvent
6. Add σ to σ_{all}
7. Apply substitution σ to the *resolvent* and go to 1.

If *resolvent* is empty, **return** σ_{all} , else **return failure**.

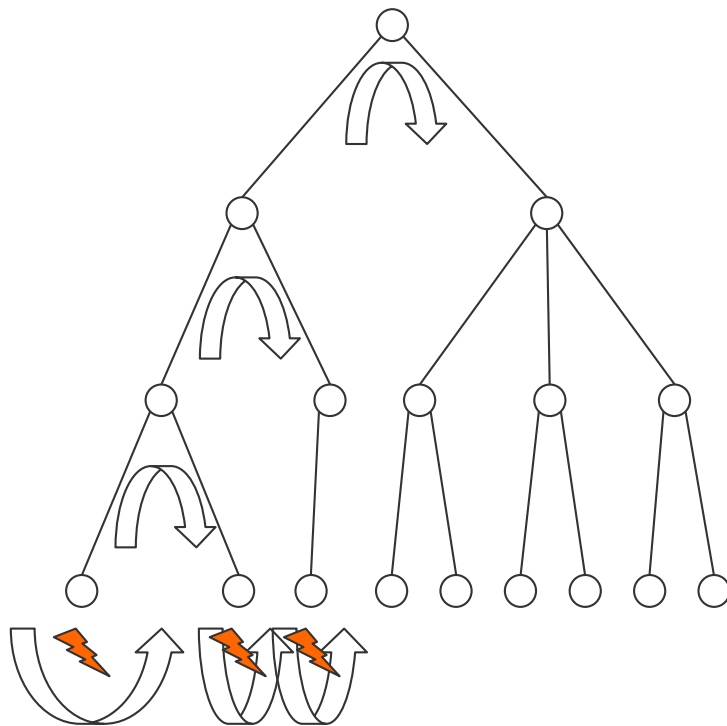
¹ Renaming means that the variables in the clause get new unique identifiers

Backtracking



- Record any decision (**choose**) and its alternative
- If backtracking, then go back to the last decision and try another option
- When backtracking then roll back to the former situation (esp. for *resolvent* and σ_{all})

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Two Choices in the Inference Procedure

There are two choices in Inference Procedure of Prolog:

- Step 1: Choice of a query literal Q_i from the resolvent
 - ◆ The inference procedure could select any literal without affecting the computation: If there exists a successful computation by choosing one literal, then there is a successful computation by choosing any other literal.
 - ◆ Prolog's solution: **leftmost goal; (normally not backtracked)**
- Step 2: Choice of a clause:
 - ◆ This selection is non-deterministic. Depending on the selection
 - ◆ Affects computation: Choosing one clause might lead to success, while choosing some other might lead to failure.
 - ◆ Prolog's solution: **topmost clause**
 - ◆ This means that the order of the clauses matters: clauses are selected in the order of appearance.
 - ◆ **Backtracking:** If a selected clause does not lead to success and there are alternative clauses then the next one is selected.

Adding Goal to Resolvent

- In step 5 of the Inference procedure the literals of the clause are added to the resolvent.
- Depending on whether the literals are added at the beginning or the end of the resolvent, we get two different strategies:
 - ◆ Adding the literals to the beginning of the resolvent gives **depth-first search**.
 - ◆ Adding the literals to the end of the resolvent gives **breadth-first search**.

Prolog 's Solution: Summary

- Choice of a query literal:
→ **leftmost** literal first
- Choice of a clause
→ **Topmost clause first - with backtracking**
- Adding new goal to the resolvent
→ **At the beginning.**

Unification

- Two expressions Q and H unify if there exists a substitution σ for any variables in the expressions so that the expressions are made identical ($Q\sigma = H\sigma$)

Unification Rules

- A constant unifies only with itself
- Two structures unify if and only if
 - ◆ they have the same (function or) predicate symbol and the same number of arguments, and
 - ◆ the corresponding arguments unify recursively
- An unbound variable unifies with anything

Unifier

- A substitution σ is a *unifier* of expressions E and F iff

$$E\sigma = F\sigma$$

- Example: Let E and F be two expressions:

- ◆ $E = f(x, b, g(z))$,
- ◆ $F = f(f(y), y, g(u))$.

Then $\sigma = \{x / f(b), y / b, z / u\}$ is a unifier of E and F :

- ◆ $E\sigma = f(f(b), b, g(u))$,
- ◆ $F\sigma = f(f(b), b, g(u))$

- A unifier σ of E and F is *most general* iff is more general than any other unifier of E and F , i.e. for any other unifier ρ there exists a unifier τ such that $\rho = \tau \circ \sigma$

Multiple Answers to a Query

- The inference procedure of Prolog computes one solution.
- The user can force the system to compute the next solution by typing a „;“ (typing „;“ is interpreted by the system as a fail and thus backtracking is started to compute an alternative solution)
- Example:

```
father (peter , mary) .  
father (peter , john) .  
father (peter , paul) .
```

```
sibling (Y , Z) :- father (X , Y) , father (X , Z) .  
sibling (Y , Z) :- mother (X , Y) , mother (X , Z) .
```

```
?- sibling (X , Y) .  
X=mary , Y=mary ;  
X=mary , Y=john ;  
X=mary , Y=paul ;  
X=john , Y=mary
```

Negation as Failure

- Prolog allows a form of negation that is called negation as failure

- A negated query

not Q

is considered proved if the system fails to prove Q

- Thus, the clause

alive(X) :- not dead(X)

can be read as „Everyone is alive if not provably dead“

Declarative Reading vs Procedural Reading

- Logic Program: Finite set of clauses.
 - ◆ $H :- B_1, \dots, B_n \quad n \geq 0$
 - ◆ Example:
 - $\text{mortal}(X) :- \text{human}(X).$
- **Declarative reading:**
 - ◆ H is implied by the conjunction of the B_i 's.
 - ◆ Example: If someone is human then he/she is mortal.
- **Procedural reading (backward chaining):**
 - ◆ To answer the query $?-H$, answer the conjunctive query $?-B_1, \dots, B_n$
 - ◆ Example: To prove that someone is mortal, prove whether he/she is a human
- All clauses with the same head predicate are
 - ◆ A definition (in declarative reading)
 - ◆ A procedure (in procedural reading)

The Cut Operator

- Under procedural reading, a logic program consists of a set of procedure
- Each procedure consists of a sequence of alternatives
- The inference procedure of Prolog computes all possible alternatives for a query
- The cut operator (written as „!“) prevents backtracking. It is a special literal that is always true but that stops all other alternatives from being applied.

```
sibling(Y,Z) :- father(X,Y) , ! , father(X,Z) .  
sibling(Y,Z) :- mother(X,Y) , mother(X,Z) .
```

Defining Negation as Failure with the Cut Operator

- The cut operator can be used to define negation as failure

```
not(Q) :- Q, !, fail.  
not(Q) .
```

- If Q can be proved then the query $\text{not}(Q)$ fails.
- If Q cannot be proved, the second clause is applied which always succeeds.
- If Q can be proved the second clause must not be applied. This is assured by the cut: If Q can be proved, then the cut prevents backtracking.

Built-in Arithmetic

- In Prolog there is a set of built-in functions for arithmetics. To apply these function there exists a special predicate „is“:

`X is Y` is true when X is equal to the value of Y.

- Built-in functions include: +, -, *, /, //, mod, (// performs integer division)
 - ◆ Using these functions we can compute a value for terms involving numbers.
- Example:
 - ◆ `?- X is 7+1.`

Will give the answer `X = 8`

- The `is` Predicate works as follows:
 - ◆ First evaluate the right-hand argument (after the „is“)
 - ◆ The result is unified with the left-hand argument.
 - ◆ The values of all the variables on the right-hand side of `is` must be known for evaluation to succeed.

Comparison

Equality:

Pred	Description	Variable Substitution	Arithmetic Computation
=	unifiable	yes	no
is	is value of	first	second
==	same value	no	yes
==	identical	no	no

Other Comparisons:

$X > Y$	The value of X is greater than the value of Y
$X \geq Y$	The value of X is greater than or equal to the value of Y
$X < Y$	The value of X is less than the value of Y
$X \leq Y$	The value of X is less than or equal to the value of Y
$X \neq Y$	The values of X and Y are unequal