Lecture 10: Cuts and Negation

- Theory
  - Explain how to control Prolog's backtracking behaviour with the help of the cut predicate
  - Introduce negation
  - Explain how cut can be packaged into a more structured form, namely "negation as failure"

Lecture 10: Cuts and Negation

- Exercises
  - Exercises of LPN: 10.1, 10.2, 10.3, 10.4
  - Practical session

The Cut

- Backtracking is a characteristic feature of Prolog
- But backtracking can lead to inefficiency:
  - Prolog can waste time and memory exploring possibilities that lead nowhere
  - It would be nice to have some control

The Cut

- The cut predicate (!) offers a way to control backtracking
- The cut has no arguments, so we write (officially): !/0

Example of cut

- The cut is a Prolog predicate, so we can add it to the body of rules:
  - Example:

```
p(X):- b(X), c(X), !, d(X), e(X).
```

- Cut is a goal that always succeeds
- The cut commits Prolog to the choices that were made since the parent goal was called

Explaining the cut

- In order to explain the cut, we will
  - Look at a piece of cut-free Prolog code and see what it does in terms of backtracking
  - Add cuts to this Prolog code
  - Examine the same piece of code with added cuts and look how the cuts affect backtracking

Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

```
?- p(X).
```

Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

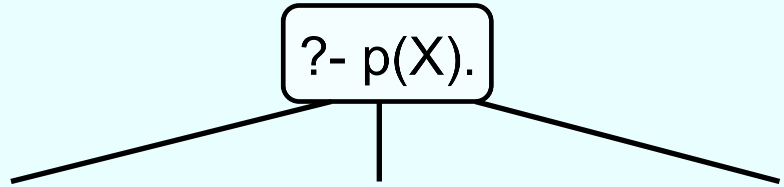
```
?- p(X).
```

```
?- p(X).
```

Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

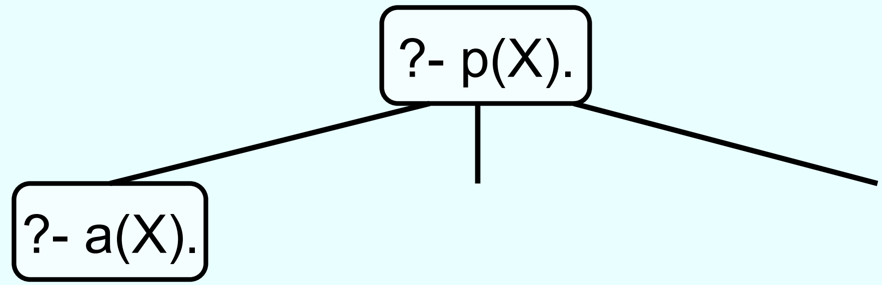
```
?- p(X).
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

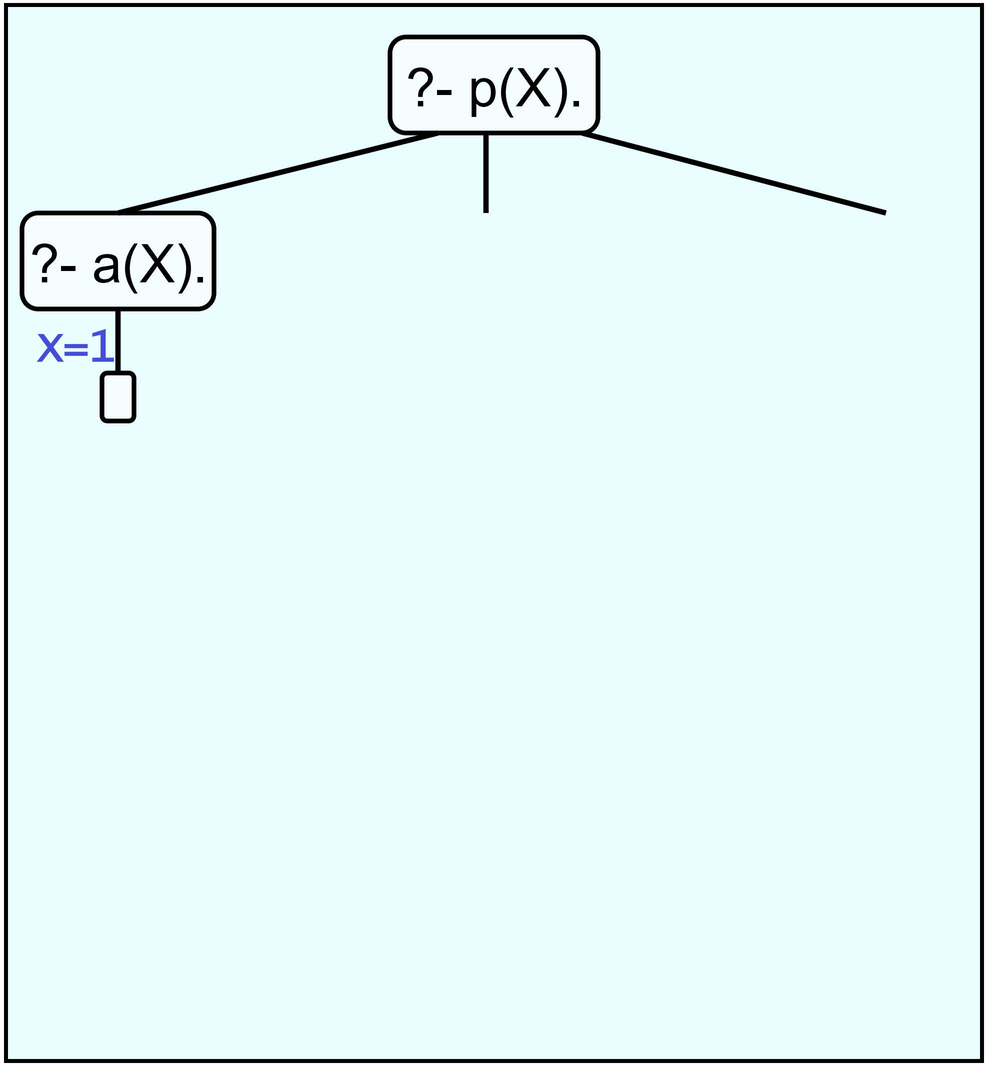
```
?- p(X).
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

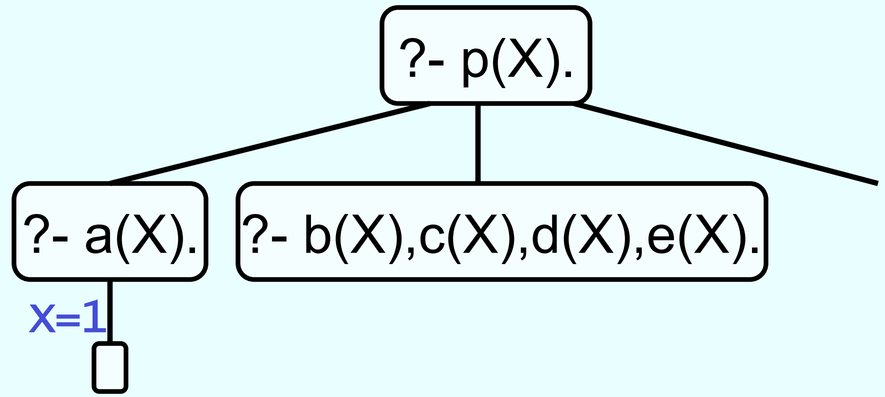
```
?- p(X).
X=1
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

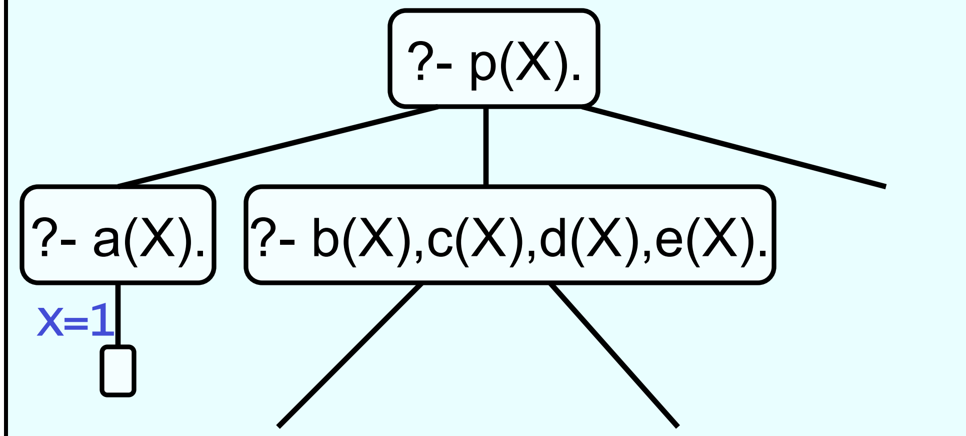
```
?- p(X).
X=1;
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

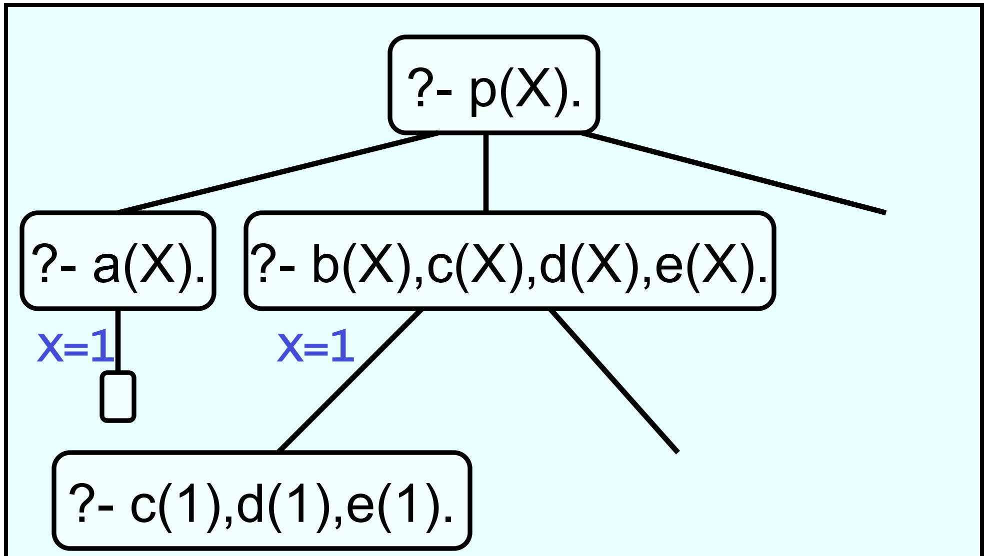
```
?- p(X).
X=1;
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

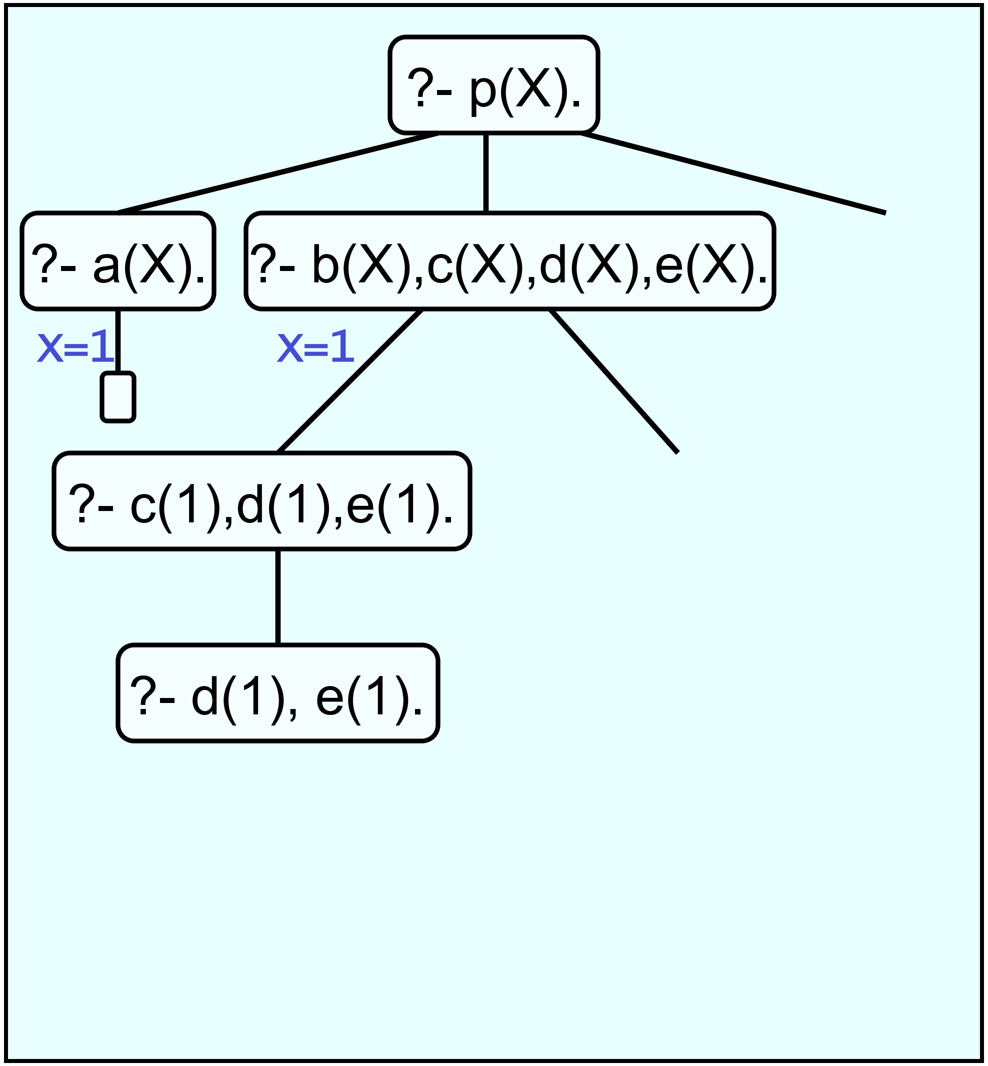
```
?- p(X).
X=1;
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

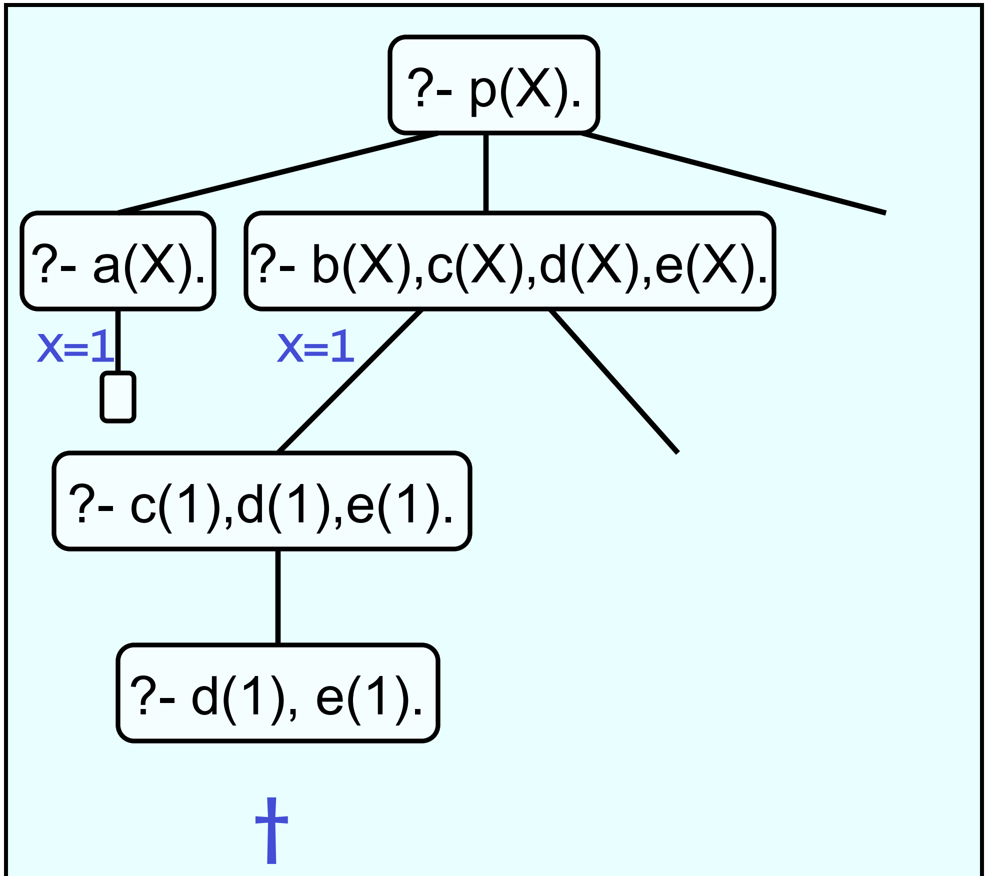
```
?- p(X).
X=1;
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

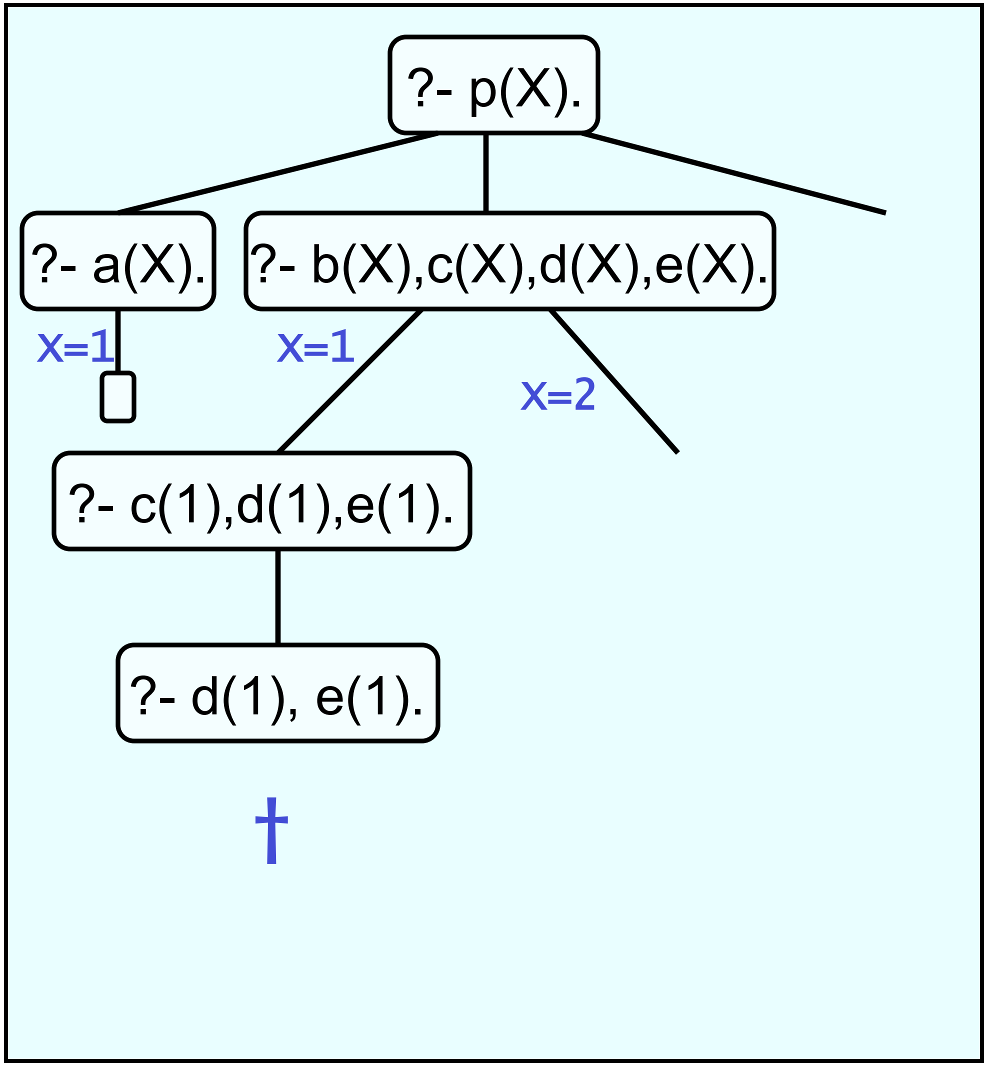
```
?- p(X).
X=1;
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

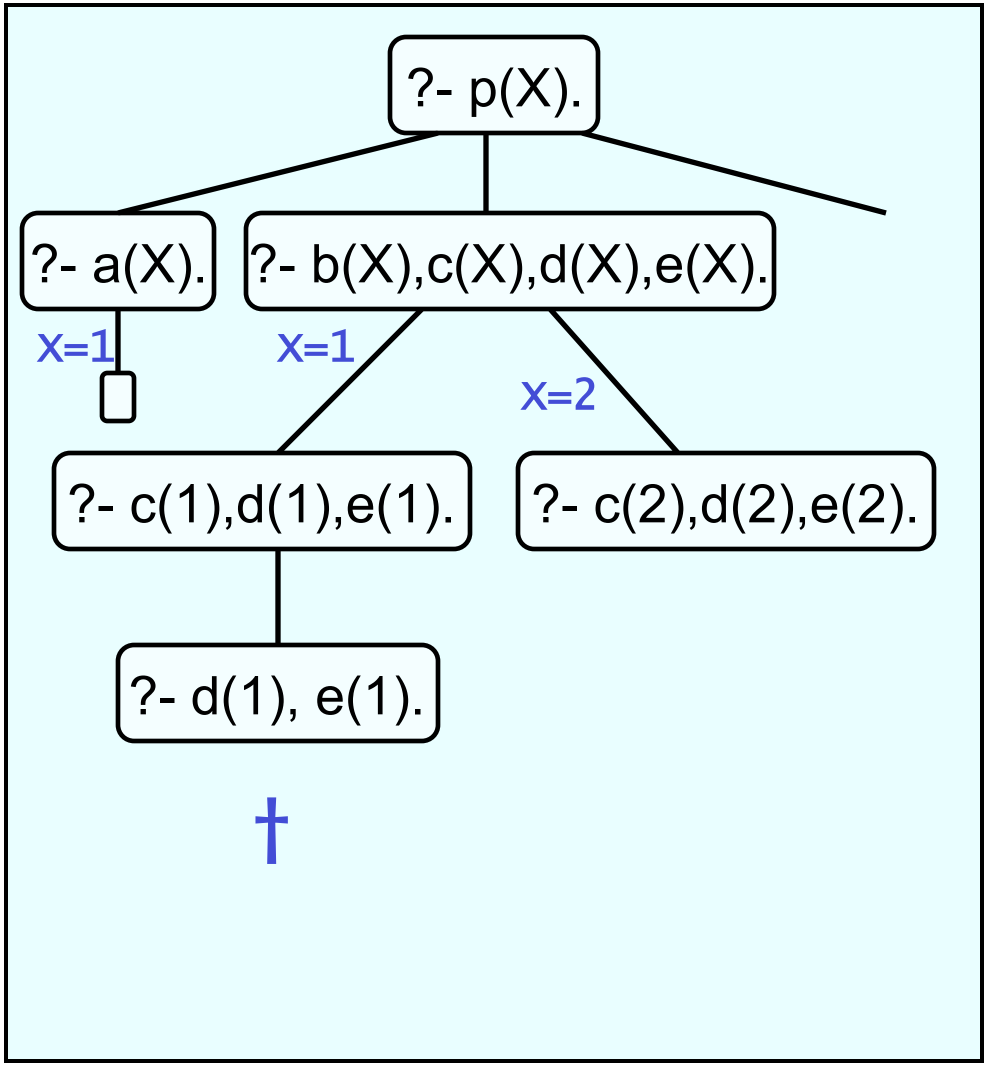
```
?- p(X).
X=1;
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

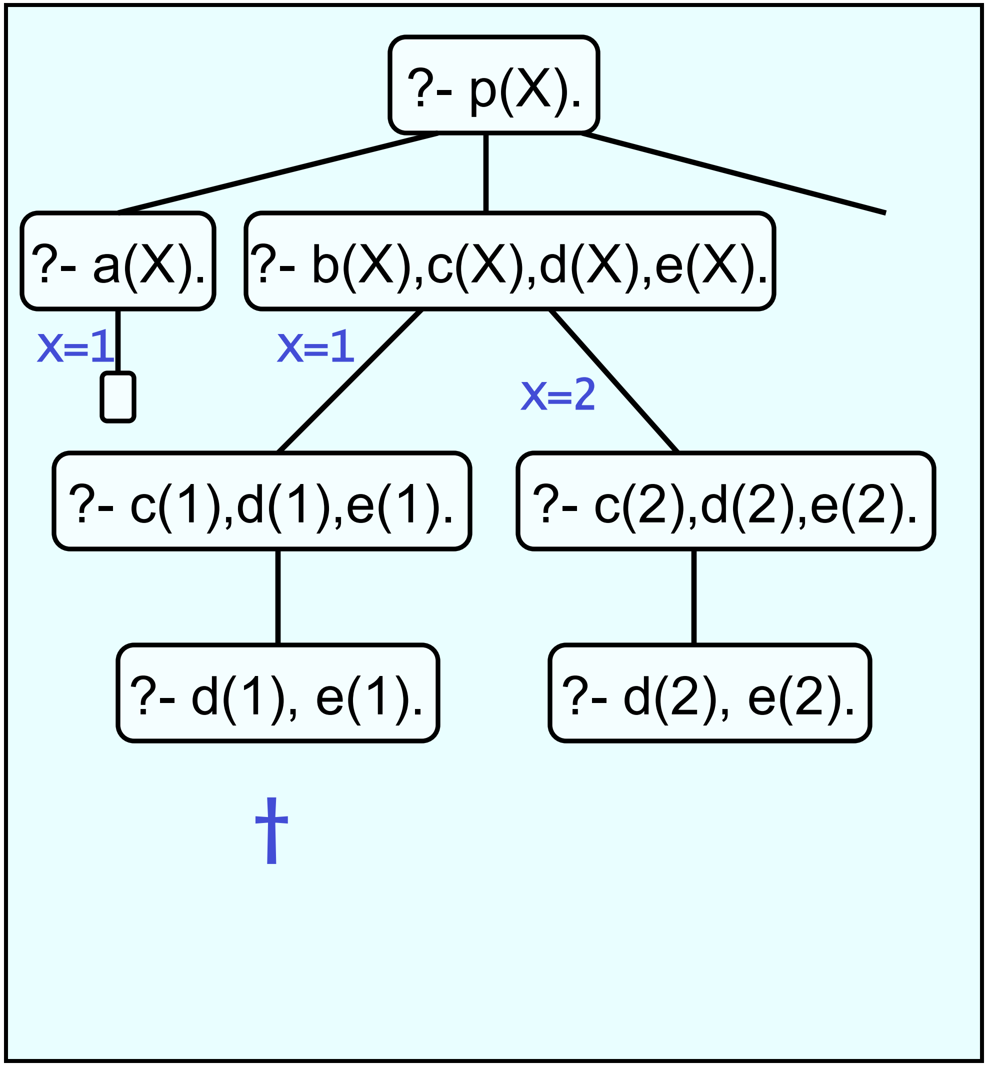
```
?- p(X).
X=1;
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

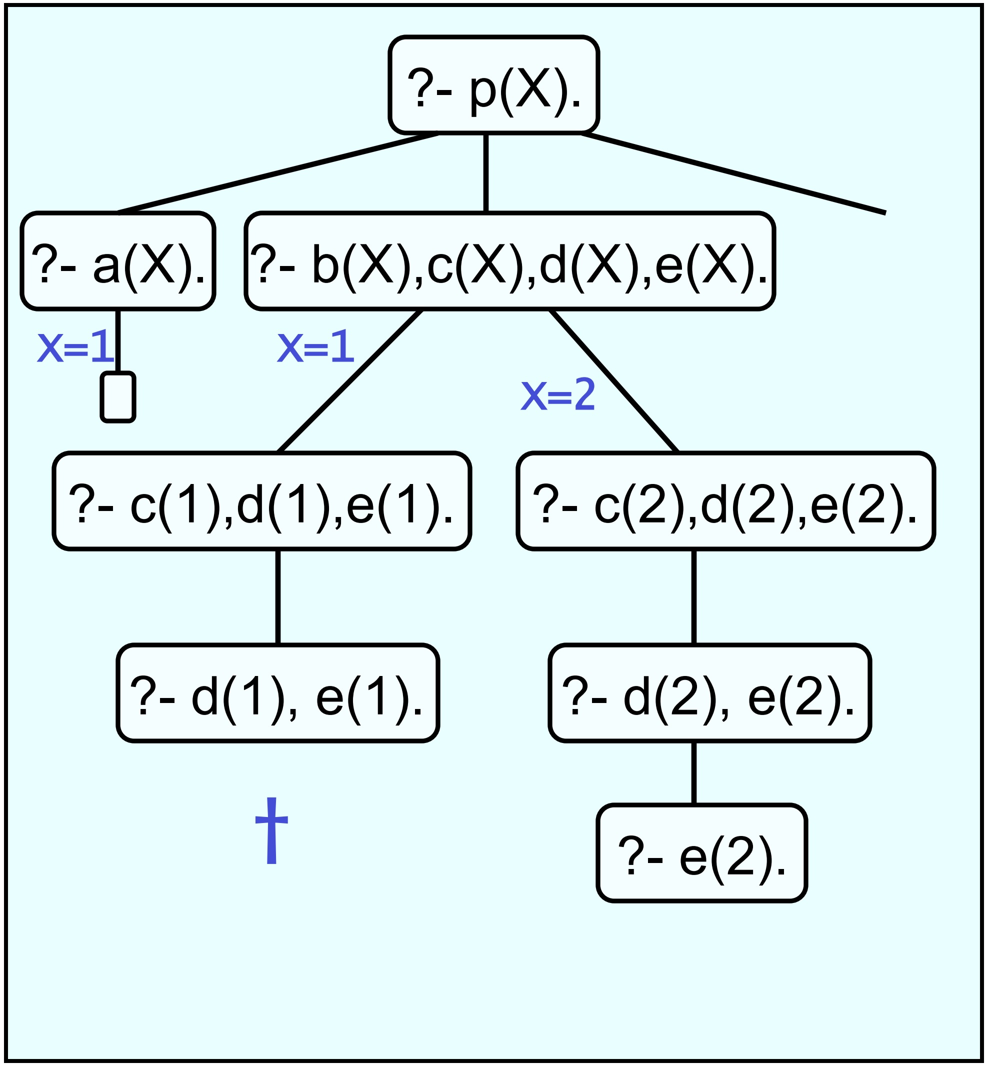
```
?- p(X).
X=1;
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

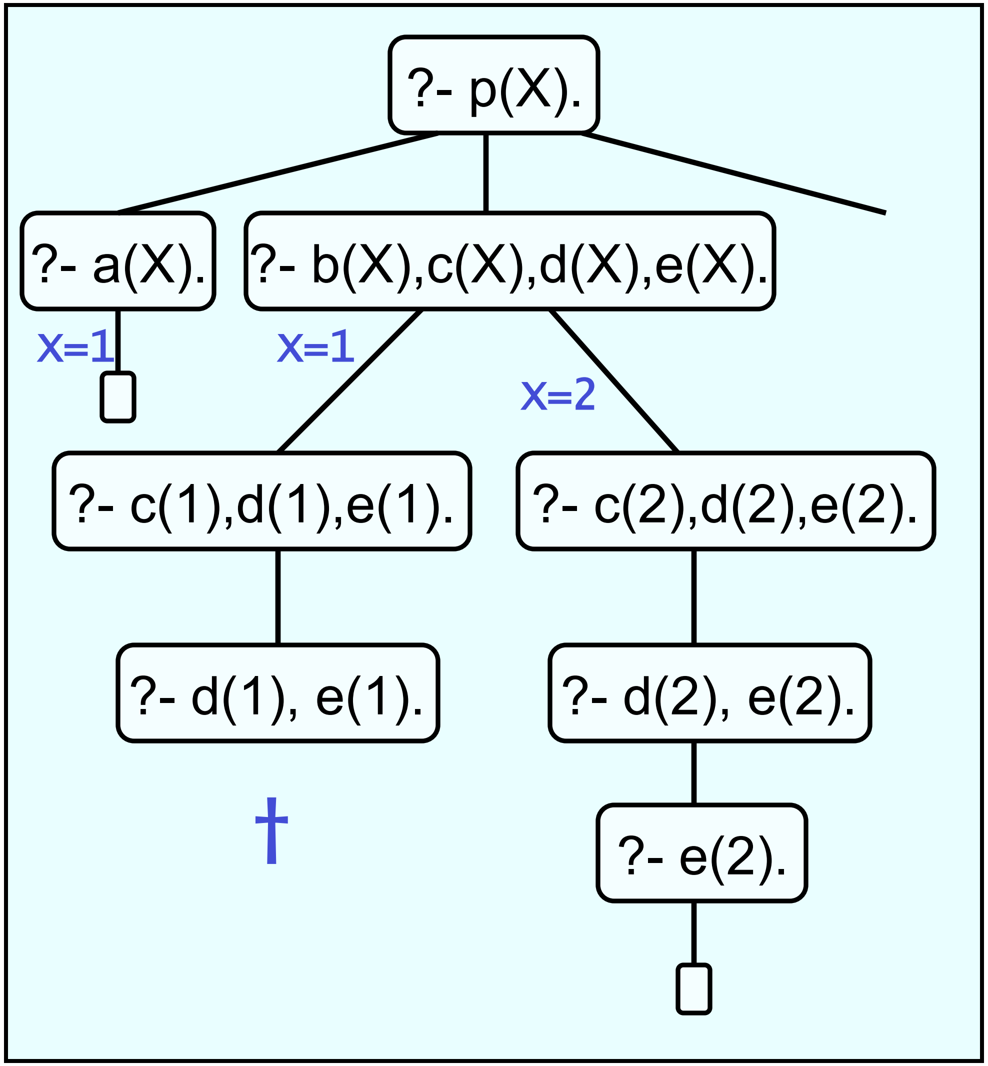
```
?- p(X).
X=1;
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

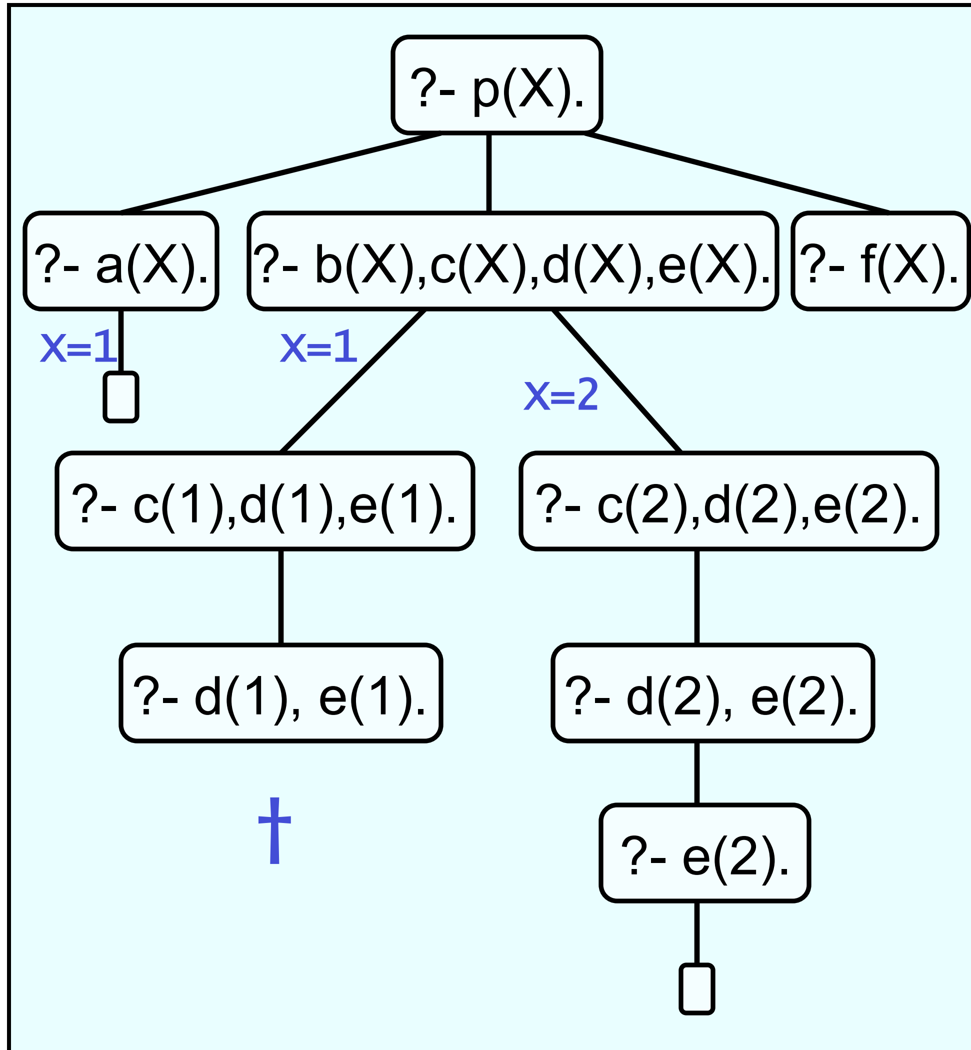
```
?- p(X).
X=1;
X=2
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

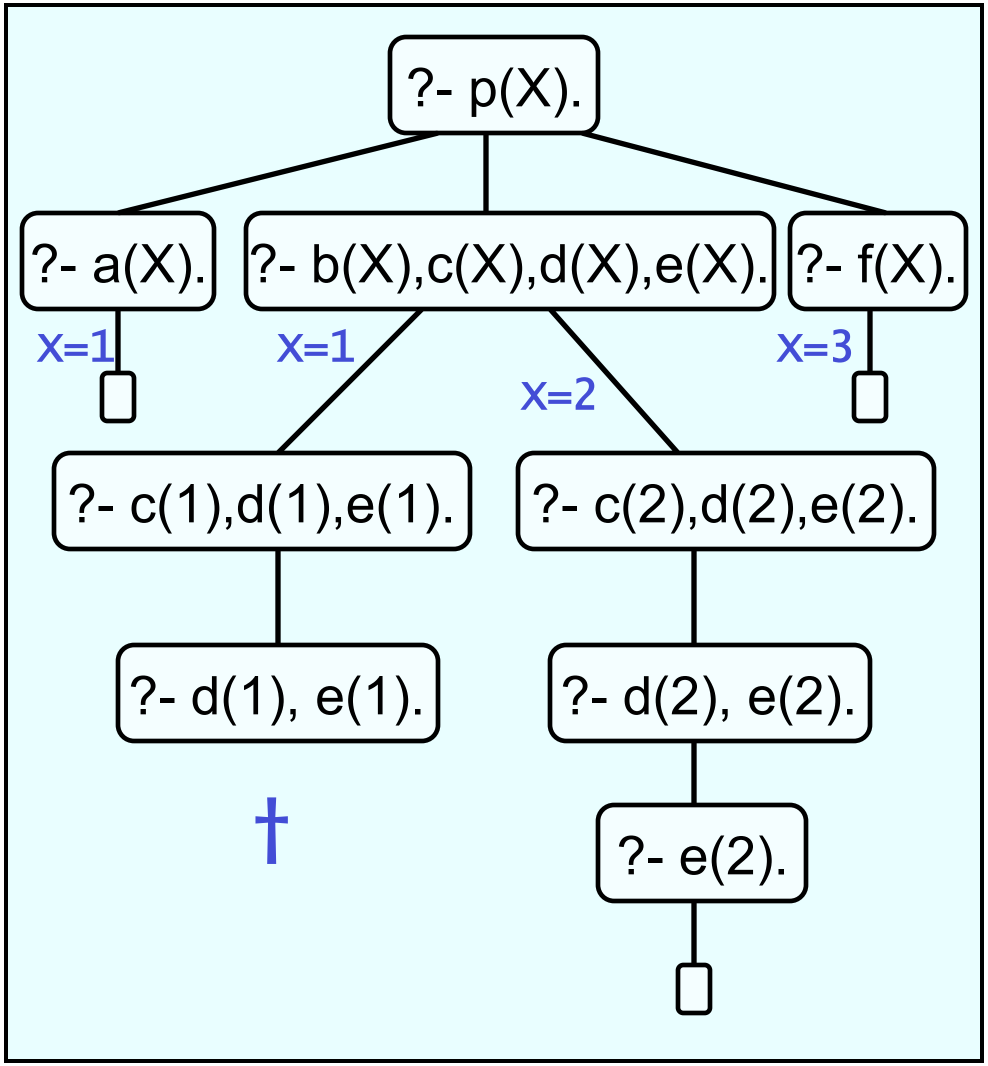
```
?- p(X).
X=1;
X=2;
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

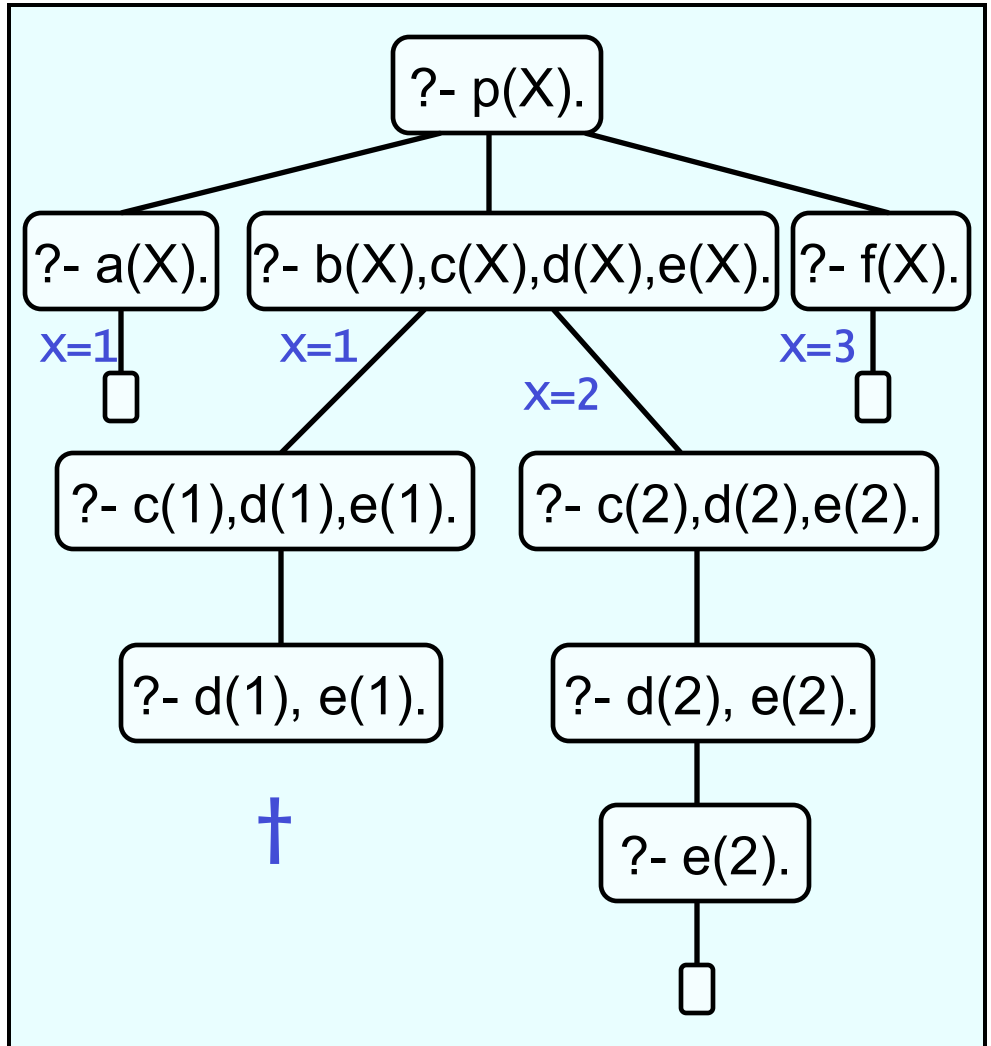
```
?- p(X).
X=1;
X=2;
X=3
```



Example: cut-free code

```
p(X):- a(X).
p(X):- b(X), c(X), d(X), e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

```
?- p(X).
X=1;
X=2;
X=3;
no
```



Adding a cut

- Suppose we insert a cut in the second clause:

```
p(X):- b(X), c(X), !, d(X), e(X).
```

- If we now pose the same query we will get the following response:

```
?- p(X).
X=1;
no
```

Example: cut

```
p(X):- a(X).
p(X):- b(X),c(X),!,d(X),e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

```
?- p(X).
```

Example: cut

```
p(X):- a(X).
p(X):- b(X),c(X),!,d(X),e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

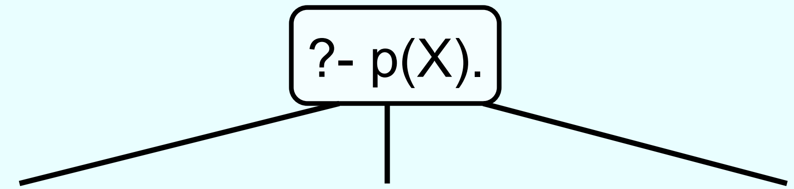
```
?- p(X).
```

```
?- p(X).
```

Example: cut

```
p(X):- a(X).
p(X):- b(X),c(X),!,d(X),e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

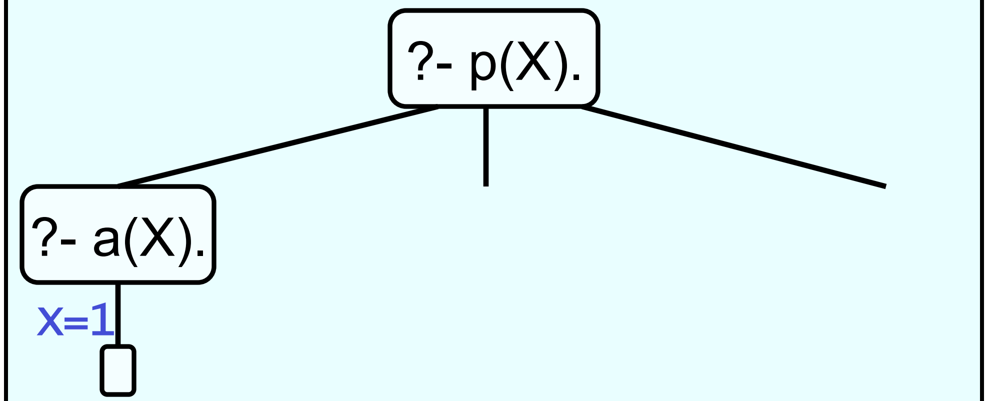
```
?- p(X).
```



Example: cut

```
p(X):- a(X).
p(X):- b(X),c(X),!,d(X),e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

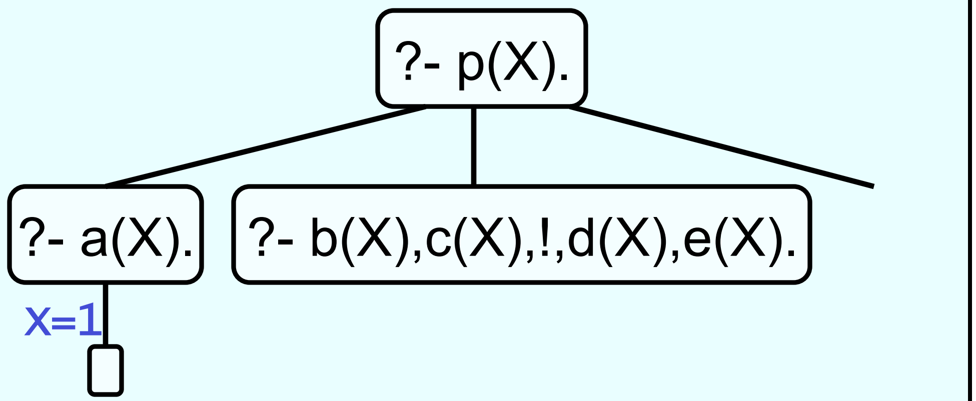
```
?- p(X).
X=1
```



Example: cut

```
p(X):- a(X).
p(X):- b(X),c(X),!,d(X),e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

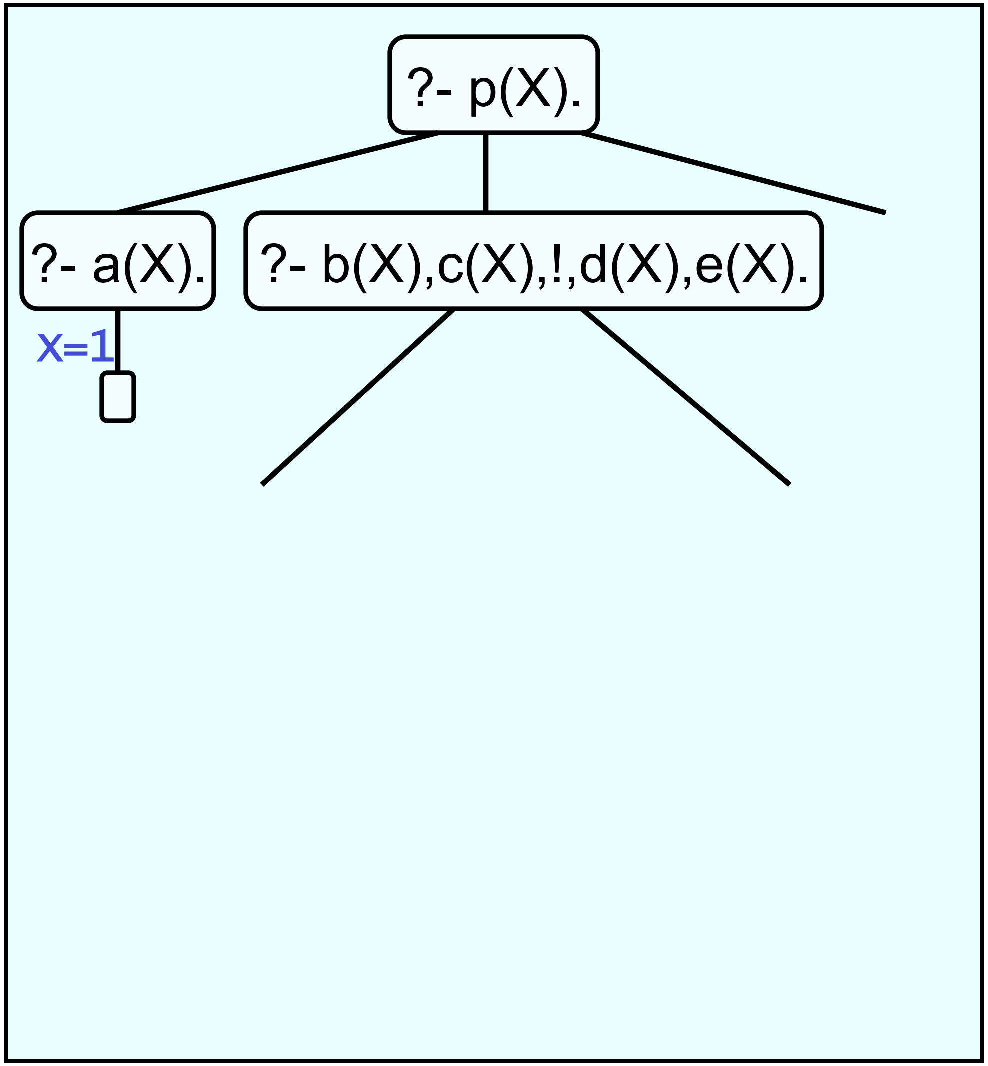
```
?- p(X).
X=1;
```



Example: cut

```
p(X):- a(X).
p(X):- b(X),c(X),!,d(X),e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

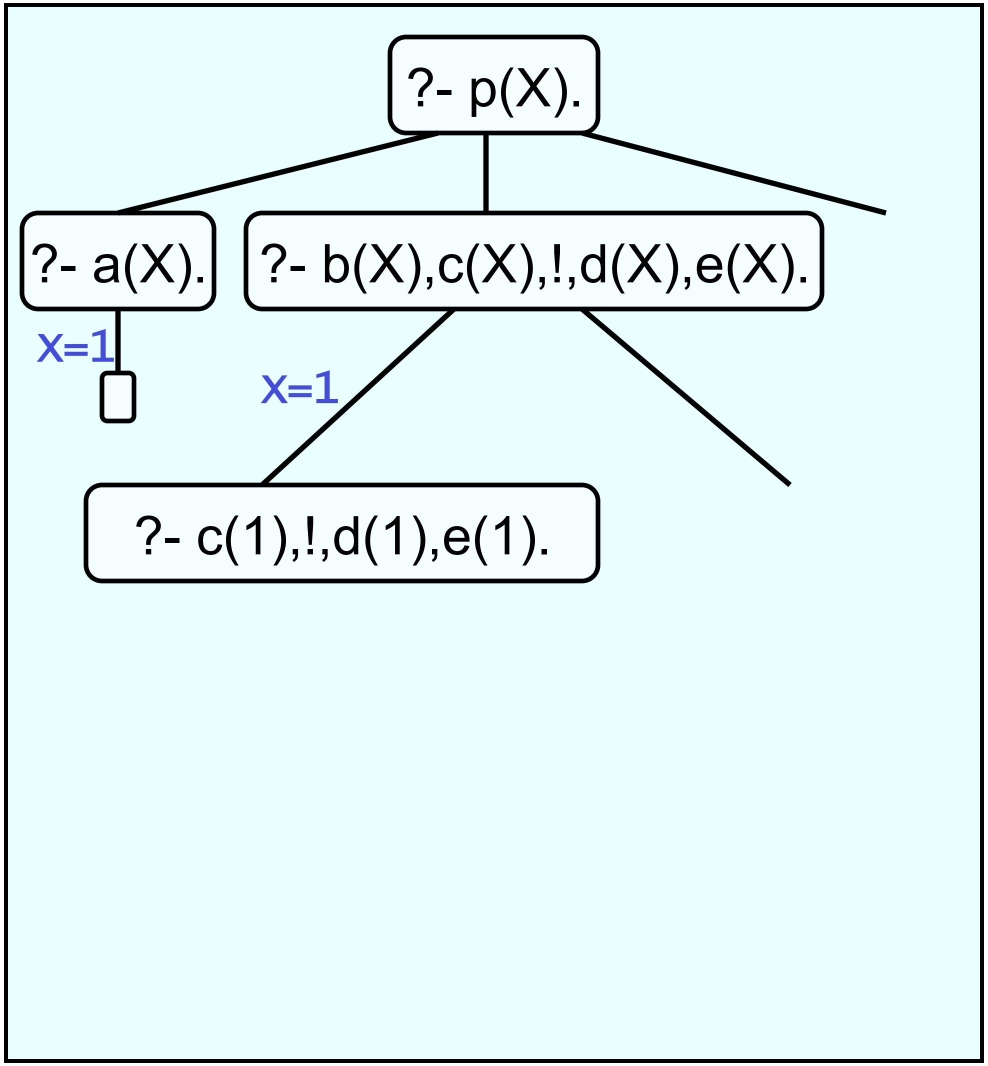
```
?- p(X).
X=1;
```



Example: cut

```
p(X):- a(X).
p(X):- b(X),c(X),!,d(X),e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

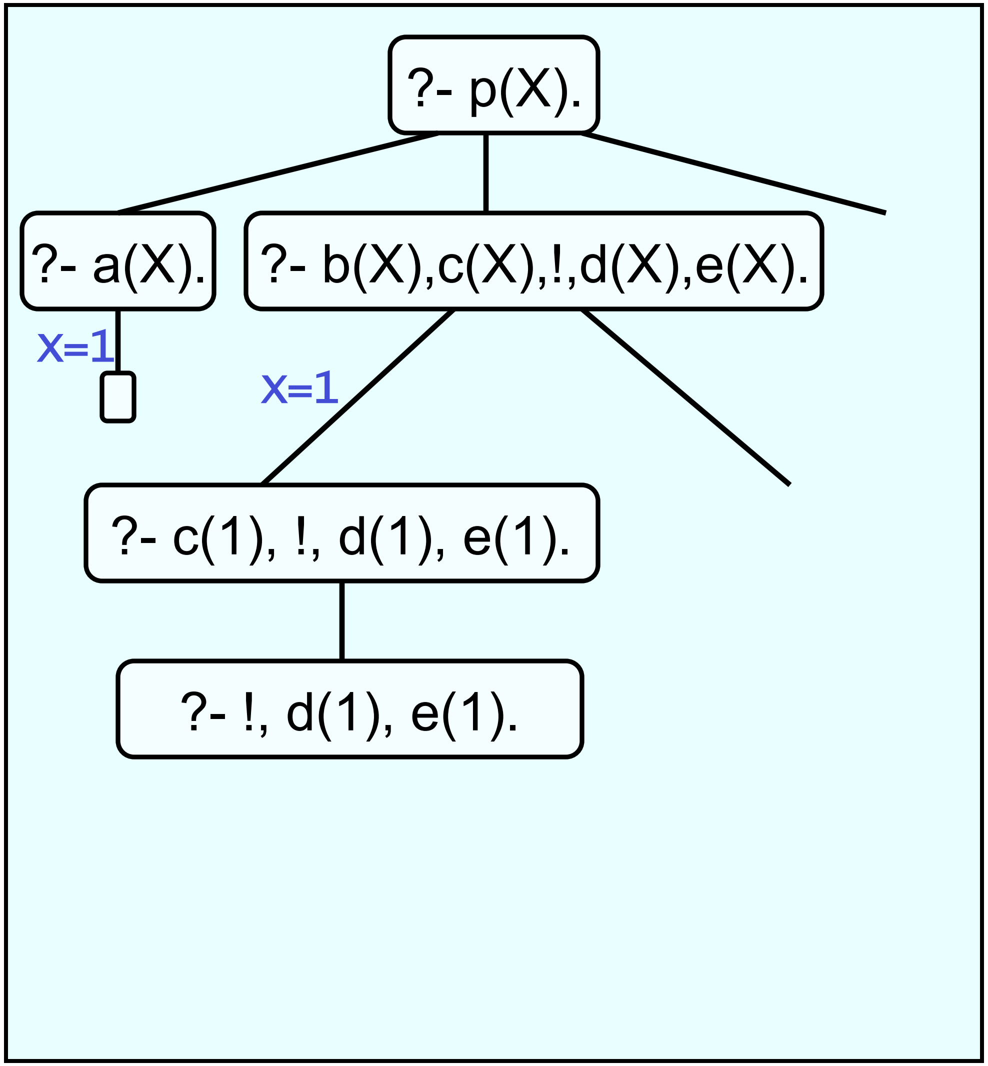
```
?- p(X).
X=1;
```



Example: cut

```
p(X):- a(X).
p(X):- b(X),c(X),!,d(X),e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

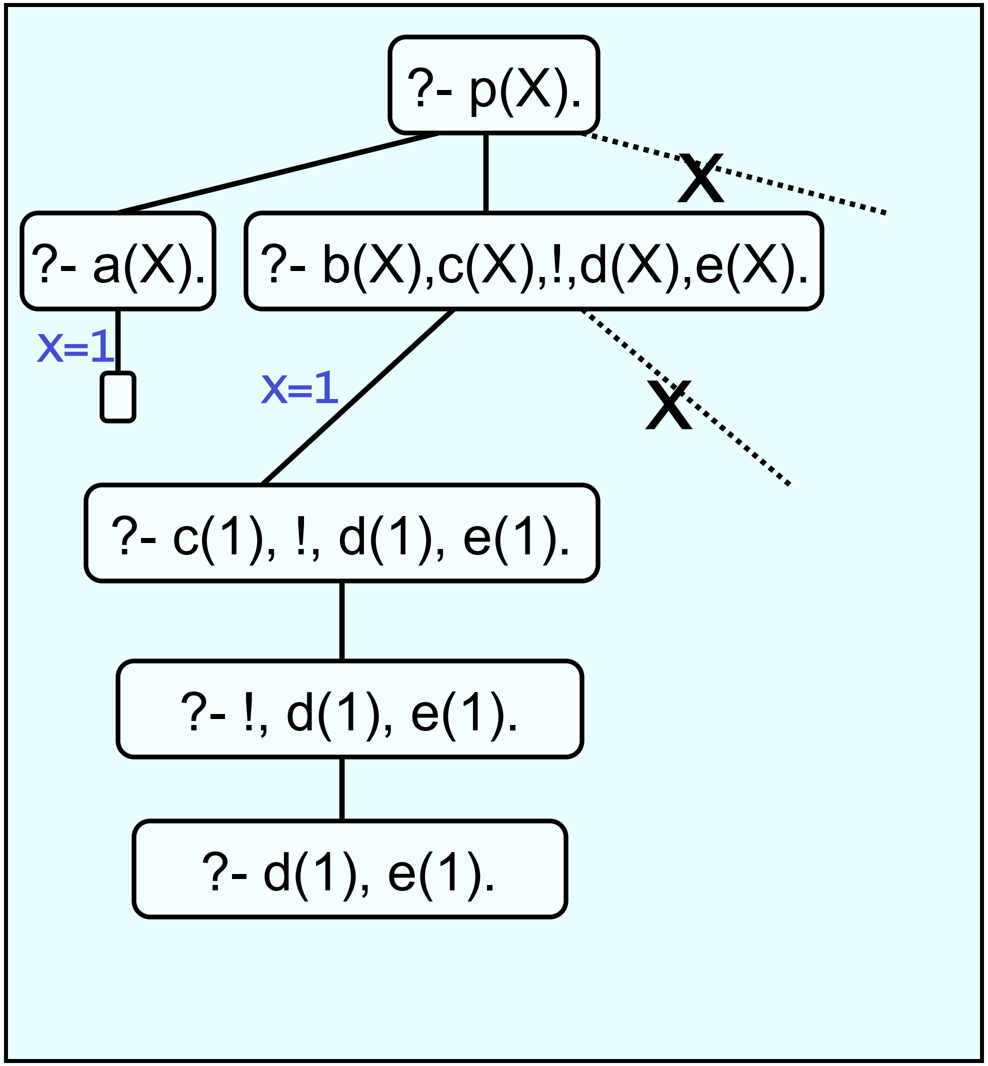
```
?- p(X).
X=1;
```



Example: cut

```
p(X):- a(X).
p(X):- b(X),c(X),!,d(X),e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

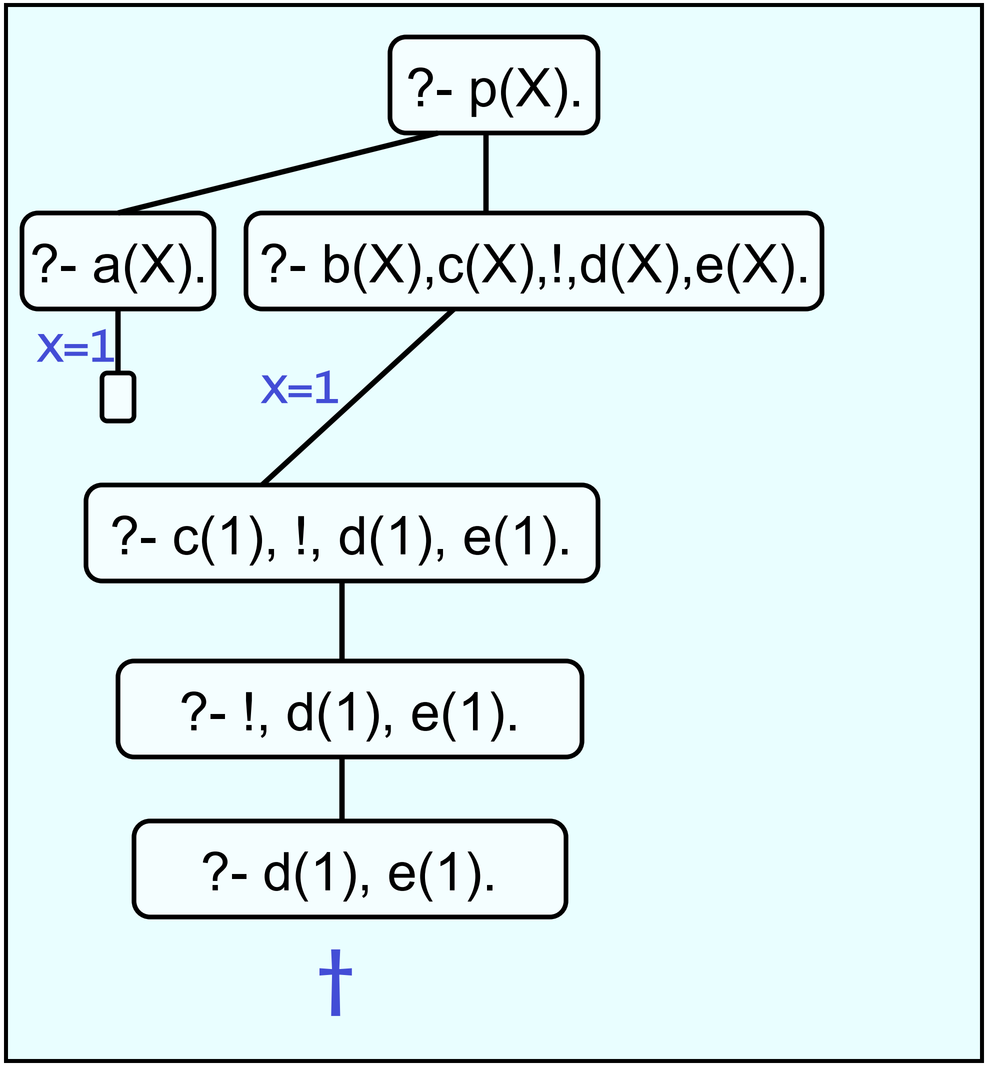
```
?- p(X).
X=1;
```



Example: cut

```
p(X):- a(X).
p(X):- b(X),c(X),!,d(X),e(X).
p(X):- f(X).
a(1).
b(1).   b(2).
c(1).   c(2).
d(2).
e(2).
f(3).
```

```
?- p(X).
X=1;
no
```



What the cut does

- The cut only commits us to choices made since the parent goal was unified with the left-hand side of the clause containing the cut
- For example, in a rule of the form

  q:- p_1, … , p_m, !, r_1, … , r_n.

  when we reach the cut it commits us:
  - to this particular clause of q
  - to the choices made by p_1, … , p_m
  - NOT to choices made by r_1, … , r_n

Using Cut

- Consider the following predicate max/3 that succeeds if the third argument is the maximum of the first two

```
max(X,Y,Y):- X =< Y.
max(X,Y,X):- X > Y.
```

Using Cut

- Consider the following predicate max/3 that succeeds if the third argument is the maximum of the first two

```
max(X,Y,Y):- X =< Y.
max(X,Y,X):- X > Y.
```

```
?- max(2,3,3).
yes

?- max(7,3,7).
yes
```

Using Cut

- Consider the following predicate max/3 that succeeds if the third argument is the maximum of the first two

```
max(X,Y,Y):- X =< Y.
max(X,Y,X):- X > Y.
```

```
?- max(2,3,2).
no

?- max(2,3,5).
no
```

The max/3 predicate

- What is the problem?
- There is a potential inefficiency
  - Suppose it is called with ?- max(3,4,Y).
  - It will correctly unify Y with 4
  - But when asked for more solutions, it will try to satisfy the second clause. This is completely pointless!

```
max(X,Y,Y):- X =< Y.
max(X,Y,X):- X > Y.
```

max/3 with cut

- With the help of cut this is easy to fix

```
max(X,Y,Y):- X =< Y, !.
max(X,Y,X):- X > Y.
```

- Note how this works:
  - If the X =< Y succeeds, the cut commits us to this choice, and the second clause of max/3 is not considered
  - If the X =< Y fails, Prolog goes on to the second clause

Green and Red Cuts

Green Cuts

- Cuts that do not change the meaning of a predicate are called **<u>green cuts</u>**
- The cut in max/3 is an example of a green cut:
  - the new code gives exactly the same answers as the old version,
  - but it is more efficient

Another max/3 with cut

- Why not remove the body of the second clause? After all, it is redundant.

```
max(X,Y,Y):- X =< Y, !.
max(X,Y,X).
```

- How good is it?

Another max/3 with cut

- Why not remove the body of the second clause? After all, it is redundant.

```
max(X,Y,Y):- X =< Y, !.
max(X,Y,X).
```

- How good is it?
  - okay

```
?- max(200,300,X).
X=300
yes
```

Another max/3 with cut

- Why not remove the body of the second clause? After all, it is redundant.

```
max(X,Y,Y):- X =< Y, !.
max(X,Y,X).
```

- How good is it?
  – okay

```
?- max(400,300,X).
X=400
yes
```

Another max/3 with cut

- Why not remove the body of the second clause? After all, it is redundant.

```
max(X,Y,Y):- X =< Y, !.
max(X,Y,X).
```

- How good is it?
  - oops….

```
?- max(200,300,200).
yes
```

Revised max/3 with cut

- Unification after crossing the cut

```
max(X,Y,Z):- X =< Y, !, Y=Z.
max(X,Y,X).
```

- This does work

```
?- max(200,300,200).
no
```

Red Cuts

- Cuts that change the meaning of a predicate are called **red cuts**
- The cut in the revised max/3 is an example of a red cut:
  - If we take out the cut, we don't get an equivalent program
- Programs containing red cuts
  - Are not fully declarative
  - Can be hard to read
  - Can lead to subtle programming mistakes

Another build-in predicate: fail/0

- As the name suggests, this is a goal that will immediately fail when Prolog tries to proof it
- That may not sound too useful
- But remember:
  when Prolog fails, it tries to backtrack

Big Kahuna Burger

Vincent and burgers

```
enjoys(vincent,X):- bigKahunaBurger(X), !, fail.
enjoys(vincent,X):- burger(X).

burger(X):-  bigMac(X).
burger(X):-  bigKahunaBurger(X).
burger(X):-  whopper(X).

bigMac(a).
bigKahunaBurger(b).
bigMac(c).
whopper(d).
```

The cut fail combination allows us to code exceptions

Vincent and burgers

```
enjoys(vincent,X):- bigKahunaBurger(X), !, fail.
enjoys(vincent,X):- burger(X).

burger(X):-  bigMac(X).
burger(X):-  bigKahunaBurger(X).
burger(X):-  whopper(X).

bigMac(a).
bigKahunaBurger(b).
bigMac(c).
whopper(d).
```

```
?- enjoys(vincent,a).
yes
```

Vincent and burgers

```
enjoys(vincent,X):- bigKahunaBurger(X), !, fail.
enjoys(vincent,X):- burger(X).

burger(X):-  bigMac(X).
burger(X):-  bigKahunaBurger(X).
burger(X):-  whopper(X).

bigMac(a).
bigKahunaBurger(b).
bigMac(c).
whopper(d).
```

```
?- enjoys(vincent,b).
no
```

Vincent and burgers

```
enjoys(vincent,X):- bigKahunaBurger(X), !, fail.
enjoys(vincent,X):- burger(X).

burger(X):-  bigMac(X).
burger(X):-  bigKahunaBurger(X).
burger(X):-  whopper(X).

bigMac(a).
bigKahunaBurger(b).
bigMac(c).
whopper(d).
```

```
?- enjoys(vincent,c).
yes
```

Vincent and burgers

```
enjoys(vincent,X):- bigKahunaBurger(X), !, fail.
enjoys(vincent,X):- burger(X).

burger(X):-  bigMac(X).
burger(X):-  bigKahunaBurger(X).
burger(X):-  whopper(X).

bigMac(a).
bigKahunaBurger(b).
bigMac(c).
whopper(d).
```

```
?- enjoys(vincent,d).
yes
```

Negation as Failure

- The cut-fail combination seems to be offering us some form of negation
- It is called **negation as failure**, and defined as follows:

```
neg(Goal):- Goal, !, fail.
neg(Goal).
```

Vincent and burgers revisited

```
enjoys(vincent,X):- burger(X),
                    neg(bigKahunaBurger(X)).

burger(X):-  bigMac(X).
burger(X):-  bigKahunaBurger(X).
burger(X):-  whopper(X).

bigMac(a).
bigKahunaBurger(b).
bigMac(c).
whopper(d).
```

Vincent and burgers revisited

```
enjoys(vincent,X):- burger(X),
                    neg(bigKahunaBurger(X)).

burger(X):-  bigMac(X).
burger(X):-  bigKahunaBurger(X).
burger(X):-  whopper(X).

bigMac(a).
bigKahunaBurger(b).
bigMac(c).
whopper(d).
```

```
?- enjoys(vincent,X).
X=a;
X=c;
X=d;
no
```

Another build-in predicate: \+

- Because negation as failure is so often used, there is no need to define it
- In standard Prolog the prefix operator \+ means negation as failure
- So we could define Vincent's preferences as follows:

```
enjoys(vincent,X):-  burger(X),
                     \+ bigKahunaBurger(X).
```

```
?- enjoys(vincent,X).
X=a;
X=c;
X=d;
no
```

Negation as failure and logic

- Negation as failure is *not* logical negation
- Changing the order of the goals in the Vincent and burgers program gives a different behaviour:

```
enjoys(vincent,X):- \+ bigKahunaBurger(X),
                       burger(X).
```

```
?- enjoys(vincent,X).
no
```