Process Discovery: An Introduction

Based on an event log a process model is constructed capturing the behavior in the log

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Process Mining

Problem Statement

Focusing on discovery the control-flow perspective

Definition (General process discovery problem)

Let $\mathcal{L}$ be an event log. A process discovery algorithm is a function that maps $\mathcal{L}$ onto a process model such that the model is representative for the behavior seen in the event log. The challenge is to find such an algorithm.

This definition does not specify what kind of process model should be generated, e.g., a BPMN, EPC, YAWL, or Petri net model

To make things more concrete:

- We define the target to be a Petri net model
- We use a simple event log as input

A simple event log $\mathcal{L}$ is a multi-set of traces over $\mathcal{A}$, i.e., $\mathcal{L} \in \mathbb{B}(\mathcal{A}^*)$
$\mathcal{L}1 = [\langle a, b, c, d\rangle^3, \langle a, c, b, d\rangle^2, \langle a, e, d\rangle]$

The goal is to discover a Petri Net that can replay event log $\mathcal{L}1$
—> Ideally, the Petri Net is a **sound WF-Net**

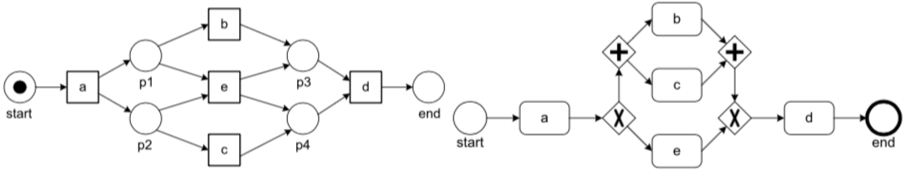
Process discovery algorithm

Definition (Specific process discovery problem)

A process discovery algorithm is a function γ that maps a log $\mathcal{L} \in \mathbb{B}(\mathcal{A}^*)$ onto a marked Petri net $\gamma(\mathcal{L}) = (\mathcal{N}, \mathcal{M})$. Ideally, $\mathcal{N}$ is a sound WF-Net and all traces in $\mathcal{L}$ correspond to possible firing sequences of $(\mathcal{N}, \mathcal{M})$.

Function γ defines a so-called **Play-In technique**

Based on $\mathcal{L}1 = [\langle a, b, c, d\rangle^3, \langle a, c, b, d\rangle^2, \langle a, e, d\rangle]$, a process discovery algorithm γ could discover the following WF-Net



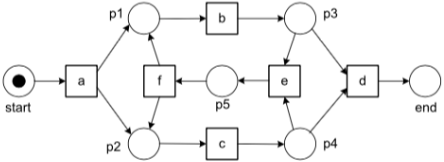
It is easy to see that the WF-Net can indeed replay all traces in the event log

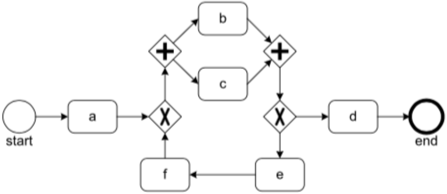
Discovery into practice

$\mathcal{L}2 = [\langle a, b, c, d\rangle^3, \langle a, c, b, d\rangle^4, \langle a, b, c, e, f, b, c, d\rangle^2,$
$\langle a, b, c, e, f, c, b, d\rangle^2, \langle a, c, b, e, f, b, c, d\rangle^2, \langle a, c, b, e, f, b, c, e, f, c, b, d\rangle]$

$\mathcal{L}2$ is a simple event log consisting of 13 cases represented by 6 different traces

Based on event log $\mathcal{L}2$, let's discover the following WF-Net!!!





This WF-Net can indeed replay all traces in the log

Not all firing sequences of $\mathcal{N}2$ correspond to traces in $\mathcal{L}2$, (e.g. the firing sequence $\langle a, c, b, e, f, c, b, d\rangle$ is a firing sequence that is not in the $\mathcal{L}2$ traces

Discovered net are sound WF-Nets

WF-Nets are a natural **subclass of Petri nets** tailored toward the modeling and analysis of **operational processes**

A process model describes the life-cycle of one case

WF-Nets explicitly model the **creation** and the **completion** of the cases:

- The **creation** is modeled by putting a token in the unique **source** place i
- The **completion** is modeled by reaching the state marking the unique **sink** place o

Given a unique source place i and a unique sink place o, the **soundness requirement** follows naturally

$\mathcal{WN}$ is sound iff:

- safeness – places cannot hold multiple tokens at the same time
- proper completion – for any marking $\mathcal{M} \in [\mathcal{WN}, [i]\rangle, o \in \mathcal{M}$ implies $\mathcal{M} = [o]$
- option to complete – for any marking $\mathcal{M} \in [\mathcal{WN}, [i]\rangle, [o] \in [\mathcal{WN}, \mathcal{M}\rangle$
- absence of dead parts – $(\mathcal{WN}, [i])$ contains no dead transitions (i.e., for any $t \in \mathcal{T}$, there is at least a firing sequence enabling t)

Quality criteria

The discovered model should be **representative** for the behavior seen in the event log

- **Fitness** - The discovered model should allow for the behavior seen in the event log
- **Precision** - The discovered model should not allow for behavior completely un-related to what was seen in the event log
- **Generalization** - The discovered model should generalize the example behavior seen in the event log
- **Simplicity** - The discovered model should be as simple as possible

The challenge is to **balance** the four **quality criteria** is needed

- **Precision** is related to the notion of underfitting –> A model having a poor precision is underfitting, i.e., it allows for behavior that is very different from what was seen in the event log
- **Generalization** is related to the notion of overfitting –> An overfitting model does not generalize enough, i.e., it is too specific and too much driven by the event log

A **trade-off** between trade-off between underfitting and overfitting is obvious

A Simple Algorithm for Process Discovery

α-algorithm

The α-algorithm focus on **control flow** such as the ordering of the activities

The α-algorithm is one of the first algorithm suitable to **discovery model including concurrency** (e.g. loops, parallel part, choice) while guarantee certain properties

The α-algorithm should not be seen as a very practical mining technique as it has problems with:

- noise
- infrequent/incomplete behavior
- complex routing constructs

INPUT: a simple event log $\mathcal{L}$ over $\mathcal{A}$
OUTPUT: a marked Petri net $\alpha(\mathcal{L}) = (\mathcal{N}, \mathcal{M})$

The α-algorithm scans the event log for particular **patterns**

We distinguish four **log-based ordering relations** to capture relevant **patterns in the log**

For any log $\mathcal{L}$ over $\mathcal{A}$ and $x, y \in \mathcal{A}$, $x >_L y$ (direct succession), $x \rightarrow_L y$ (casuality), $x \|_L y$ (parallel), $x \#_L y$ (choice) i.e., precisely one of these relations holds for any pair of activities

α-algorithm: ordering relations

- **Direct succession**: $x > y$ iff for some case x is directly followed by y
- **Causality**: $x \rightarrow y$ iff $x > y$ and $y \not> x$
- **Parallel**: $x \| y$ iff $x > y$ and $y > x$
- **Choice**: $x \# y$ iff $x \not> y$ and $y \not> x$

$\mathcal{L}1 = [\langle a, b, c, d \rangle^3, \langle a, c, b, d \rangle^2, \langle a, e, d \rangle]$

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$\mathcal{L}1 = [\langle a, b, c, d \rangle^3, \langle a, c, b, d \rangle^2, \langle a, e, d \rangle]$

$$>_{L_1} = \{(a, b), (a, c), (a, e), (b, c), (c, b), (b, d), (c, d), (e, d)\}$$

$$\rightarrow_{L_1} = \{(a, b), (a, c), (a, e), (b, d), (c, d), (e, d)\}$$

$$\#_{L_1} = \{(a, a), (a, d), (b, b), (b, e), (c, c), (c, e), (d, a), (d, d), (e, b), (e, c), (e, e)\}$$

$$\|_{L_1} = \{(b, c), (c, b)\}$$

Ordering relationship and footprint of $\mathcal{L}1$

$\mathcal{L}1 = [\langle a, b, c, d\rangle^3, \langle a, c, b, d\rangle^2, \langle a, e, d\rangle]$

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$$\|_{L_1} = \{(b,c), (c,b)\}$$

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$$\#_{L_1} = \{(a,a), (a,d), (b,b), (b,e), (c,c), (c,e), (d,a), (d,d), (e,b), (e,c), (e,e)\}$$

$$\|_{L_1} = \{(b,c), (c,b)\}$$

	a	b	c	d	e
a	$\#_{L_1}$	$\rightarrow_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$	$\rightarrow_{L_1}$
b	$\leftarrow_{L_1}$	$\#_{L_1}$	$\|_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$
c	$\leftarrow_{L_1}$	$\|_{L_1}$	$\#_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$
d	$\#_{L_1}$	$\leftarrow_{L_1}$	$\leftarrow_{L_1}$	$\#_{L_1}$	$\leftarrow_{L_1}$
e	$\leftarrow_{L_1}$	$\#_{L_1}$	$\#_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$

Ordering relationship and footprint of $\mathcal{L}2$

- **Direct succession**: $x > y$ iff for some case x is directly followed by y
- **Causality**: $x \rightarrow y$ iff $x > y$ and $y \not> x$
- **Parallel**: $x \| y$ iff $x > y$ and $y > x$
- **Choice**: $x \# y$ iff $x \not> y$ and $y \not> x$

$$\mathcal{L}2 = [\langle a, b, c, d\rangle^3, \langle a, c, b, d\rangle^4, \langle a, b, c, e, f, b, c, d\rangle^2, \langle a, b, c, e, f, c, b, d\rangle^2, \langle a, c, b, e, f, b, c, d\rangle^2, \langle a, c, b, e, f, b, c, e, f, c, b, d\rangle]$$

PLEASE DEFINE THE ORDERING RELATIONS

(direct succession) $>_L = \{(\ldots, \ldots), \ldots\}$
(casuality) $\rightarrow_L = \{(\ldots, \ldots), \ldots\}$
(parallel) $\|_L = \{(\ldots, \ldots), \ldots\}$
(choice) $\#_L = \{(\ldots, \ldots), \ldots\}$

PLEASE DEFINE THE FOOTPRINT

Ordering relationship and footprint of $\mathcal{L}3$

- **Direct succession**: $x > y$ iff for some case x is directly followed by y
- **Causality**: $x \rightarrow y$ iff $x > y$ and $y \not> x$
- **Parallel**: $x \| y$ iff $x > y$ and $y > x$
- **Choice**: $x \# y$ iff $x \not> y$ and $y \not> x$

$\mathcal{L}3 =$
$[\langle a, b, c, d, e, f, b, d, c, e, g \rangle, \langle a, b, d, c, e, g \rangle^2, \langle a, b, c, d, e, f, b, c, d, e, f, b, d, c, e, g \rangle]$

PLEASE DEFINE THE ORDERING RELATIONS

(direct succession) $>_L = \{(..., ...), ...\}$
(casuality) $\rightarrow_L = \{(..., ...), ...\}$
(parallel) $\|_L = \{(..., ...), ...\}$
(choice) $\#_L = \{(..., ...), ...\}$

PLEASE DEFINE THE FOOTPRINT

Ordering relationship and footprint of $\mathcal{L}4$

- **Direct succession**: $x > y$ iff for some case x is directly followed by y
- **Causality**: $x \rightarrow y$ iff $x > y$ and $y \not> x$
- **Parallel**: $x \| y$ iff $x > y$ and $y > x$
- **Choice**: $x \# y$ iff $x \not> y$ and $y \not> x$

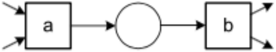
$\mathcal{L}4 = [\langle a, c, d \rangle^{45}, \langle b, c, d \rangle^{42}, \langle a, c, e \rangle^{38}, \langle b, c, e \rangle^{22}]$

PLEASE DEFINE THE ORDERING RELATIONS

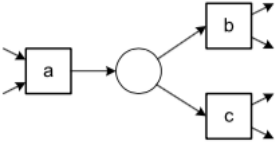
(direct succession) $>_L = \{(..., ...), ...\}$
(casuality) $\rightarrow_L = \{(..., ...), ...\}$
(parallel) $\|_L = \{(..., ...), ...\}$
(choice) $\#_L = \{(..., ...), ...\}$

PLEASE DEFINE THE FOOTPRINT

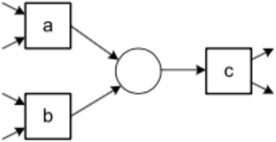
Typical process patterns



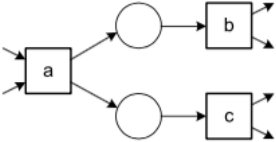
(a) sequence pattern: a→b



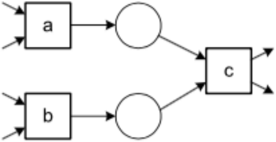
(b) XOR-split pattern:
a→b, a→c, and b#c



(c) XOR-join pattern:
a→c, b→c, and a#b



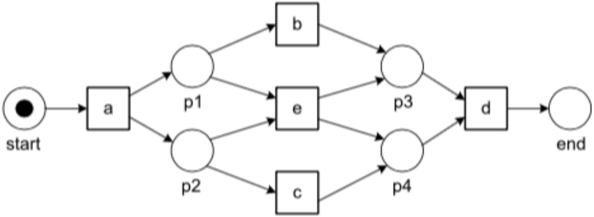
(d) AND-split pattern:
a→b, a→c, and b||c



(e) AND-join pattern:
a→c, b→c, and a||b

α-algorithm: footprint of $\mathcal{L}1$

$\mathcal{L}1 = [\langle a, b, c, d\rangle^3, \langle a, c, b, d\rangle^2, \langle a, e, d\rangle]$



Model and event log have the same footprint!

	a	b	c	d	e
a	$\#_{L_1}$	$\rightarrow_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$	$\rightarrow_{L_1}$
b	$\leftarrow_{L_1}$	$\#_{L_1}$	$\|_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$
c	$\leftarrow_{L_1}$	$\|_{L_1}$	$\#_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$
d	$\#_{L_1}$	$\leftarrow_{L_1}$	$\leftarrow_{L_1}$	$\#_{L_1}$	$\leftarrow_{L_1}$
e	$\leftarrow_{L_1}$	$\#_{L_1}$	$\#_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$

α-algorithm

Let $\mathcal{L}$ be an event log over $\mathcal{T} \subseteq \mathcal{T}$, than $\alpha(\mathcal{L})$ is defined as follows:

1. $\mathcal{T}_{\mathcal{L}} = \{t \in \mathcal{T} | \exists_{\sigma \in \mathcal{L}} t \in \sigma\}$
2. $\mathcal{T}_{\mathcal{I}} = \{t \in \mathcal{T} | \exists_{\sigma \in \mathcal{L}} t = \mathit{first}(\sigma)\}$
3. $\mathcal{T}_{\mathcal{O}} = \{t \in \mathcal{T} | \exists_{\sigma \in \mathcal{L}} t = \mathit{last}(\sigma)\}$
4. $\mathcal{X}_{\mathcal{L}} = \{(\mathcal{A}, \mathcal{B}) | \mathcal{A} \subseteq \mathcal{T}_{\mathcal{L}} \land \mathcal{A} \neq \emptyset \land \mathcal{B} \subseteq \mathcal{T}_{\mathcal{L}} \land \mathcal{B} \neq \emptyset$
$\land \forall_{a \in \mathcal{A}} \forall_{b \in \mathcal{B}} a \rightarrow_L b \land \forall_{a_1, a_2 \in \mathcal{A}} a_1 \#_L a_2 \land \forall_{b_1, b_2 \in \mathcal{B}} b_1 \#_L b_2\}$
5. $\mathcal{Y}_{\mathcal{L}} = \{(\mathcal{A}, \mathcal{B}) \in \mathcal{X}_{\mathcal{L}} | \forall_{(\mathcal{A}', \mathcal{B}') \in \mathcal{X}_{\mathcal{L}}} \mathcal{A} \subseteq \mathcal{A}' \land \mathcal{B} \subseteq \mathcal{B}' \Longrightarrow (\mathcal{A}, \mathcal{B}) = (\mathcal{A}', \mathcal{B}')\}$
6. $\mathcal{P}_{\mathcal{L}} = \{p_{(\mathcal{A}, \mathcal{B})} | (\mathcal{A}, \mathcal{B}) \in \mathcal{Y}_{\mathcal{L}}\} \cup \{i_L, o_L\}$
7. $\mathcal{F}_{\mathcal{L}} = \{(a, p_{(\mathcal{A}, \mathcal{B})}) | (\mathcal{A}, \mathcal{B}) \in \mathcal{Y}_{\mathcal{L}} \land a \in \mathcal{A}\} \cup \{(p_{(\mathcal{A}, \mathcal{B})}, b) | (\mathcal{A}, \mathcal{B}) \in \mathcal{Y}_{\mathcal{L}} \land b \in \mathcal{B}\} \cup \{(i_L, t) | t \in \mathcal{T}_{\mathcal{I}}\} \cup \{(t, o_L) | t \in \mathcal{T}_{\mathcal{O}}\}$
8. $\alpha(\mathcal{L}) = (\mathcal{P}_{\mathcal{L}}, \mathcal{T}_{\mathcal{L}}, \mathcal{F}_{\mathcal{L}})$

Do not be scared! :)

α-algorithm

Let $\mathcal{L}$ be an event log over $T \subseteq \mathcal{T}$, than $\alpha(\mathcal{L})$ is defined as follows:

1. $T_{\mathcal{L}} = \{t \in T | \exists_{\sigma \in \mathcal{L}} t \in \sigma\}$

$T_{\mathcal{L}}$ is the set of activities do appear in the log, these will correspond to the transitions of the generated WF-Net

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$\mathcal{T}_{\mathcal{L}}$ is the set of activities do appear in the log, these will correspond to the transitions of the generated WF-Net

2. $\mathcal{T}_{\mathcal{I}} = \{t \in \mathcal{T} | \exists_{\sigma \in \mathcal{L}} t = \mathit{first}(\sigma)\}$

$\mathcal{T}_{\mathcal{I}}$ is the set of start activities, i.e., all activities that appear first in some trace such as $\langle t_1, ..., t_n \rangle, ... \langle t'_1, ..., t'_m \rangle$

α-algorithm

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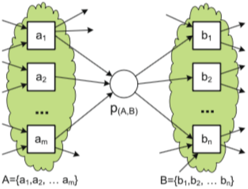
3. $\mathcal{T}_{\mathcal{O}} = \{t \in \mathcal{T} | \exists_{\sigma \in \mathcal{L}} t = \mathit{last}(\sigma)\}$

$\mathcal{T}_{\mathcal{O}}$ is the set of end activities, i.e., all activities that appear last in some trace, such as $\langle t_1, ... t_n \rangle, ...\langle t'_1, ..., t'_m \rangle$

Place p(A,B) connects the transitions in set A to the transitions in set B



4. Calculate pairs (A, B)

$$\mathcal{X}_{\mathcal{L}} = \{(\mathcal{A},\mathcal{B}) | \mathcal{A} \subseteq \mathcal{T}_{\mathcal{L}} \wedge \mathcal{A} \neq \emptyset \wedge \mathcal{B} \subseteq \mathcal{T}_{\mathcal{L}} \wedge \mathcal{B} \neq \emptyset$$
$$\wedge \forall_{a \in \mathcal{A}} \forall_{b \in \mathcal{B}} \, a \rightarrow_L b$$
$$\wedge \forall_{a_1, a_2 \in \mathcal{A}} \, a_1 \#_L a_2$$
$$\wedge \forall_{b_1, b_2 \in \mathcal{B}} \, b_1 \#_L b_2 \}$$

5. Delete non maximal pairs (A, B)

$$\mathcal{Y}_{\mathcal{L}} = \{(\mathcal{A},\mathcal{B}) \in \mathcal{X}_{\mathcal{L}} | \forall_{(\mathcal{A}',\mathcal{B}') \in \mathcal{X}_{\mathcal{L}}} \mathcal{A} \subseteq \mathcal{A}' \wedge \mathcal{B} \subseteq \mathcal{B}' \implies (\mathcal{A},\mathcal{B}) = (\mathcal{A}',\mathcal{B}')\}$$

6. Determine place $p_{(\mathcal{A},\mathcal{B})}$ from pairs (A, B)

$$\mathcal{P}_{\mathcal{L}} = \{p_{(\mathcal{A},\mathcal{B})} | (\mathcal{A},\mathcal{B}) \in \mathcal{Y}_{\mathcal{L}}\} \cup \{i_L, o_L\}$$

4. Calculate pairs (A, B)

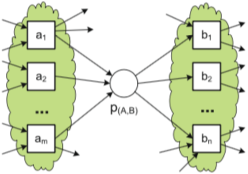
$$\mathcal{X}_{\mathcal{L}} = \{(\mathcal{A},\mathcal{B}) | \mathcal{A} \subseteq \mathcal{T}_{\mathcal{L}} \wedge \mathcal{A} \neq \emptyset \wedge \mathcal{B} \subseteq \mathcal{T}_{\mathcal{L}} \wedge \mathcal{B} \neq \emptyset \\ \wedge \forall_{a \in \mathcal{A}} \forall_{b \in \mathcal{B}} a \rightarrow_L b \\ \wedge \forall_{a_1, a_2 \in \mathcal{A}} a_1 \#_L a_2 \\ \wedge \forall_{b_1, b_2 \in \mathcal{B}} b_1 \#_L b_2 \}$$

We have to find two sets of activities, A and B, and these activities should have the following properties.

- If we take any activity in the set A and we take any activity in the set B, there should always be a direct succession between these two activities. So there should be at least one position in the log where the element of A is followed by the element of B and that should hold for all combinations.
- If I take two activities in the set A, they should never follow one another. If I take two activities in the set B, they should also never follow one another. Even if we take the same activity, it should never follow itself.

How to identify $(\mathcal{A}, \mathcal{B}) \in \mathcal{X}_{\mathcal{L}}$?

Loking at the footprint matrix we can recognize this structure because we are looking for a set a and b where things never follow one another. And we are looking for these other connections where any element of a is directly followed by any element of b, but never the other way around.



	a_1	a_2	...	a_m	b_1	b_2	...	b_n
a_1	#	#	...	#	$\rightarrow$	$\rightarrow$	...	$\rightarrow$
a_2	#	#	...	#	$\rightarrow$	$\rightarrow$	...	$\rightarrow$
...	...	...	...	...	...	...	...	...
a_m	#	#	...	#	$\rightarrow$	$\rightarrow$	...	$\rightarrow$
b_1	$\leftarrow$	$\leftarrow$	...	$\leftarrow$	#	#	...	#
b_2	$\leftarrow$	$\leftarrow$	...	$\leftarrow$	#	#	...	#
...	...	...	...	...	...	...	...	...
b_n	$\leftarrow$	$\leftarrow$	...	$\leftarrow$	#	#	...	#

5. Delete non maximal pairs (A, B)

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Delete the element that are contained in others

6. Determine place $p_{(\mathcal{A},\mathcal{B})}$ from pairs (A, B)

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$\mathcal{P}_{\mathcal{L}} = \{p_{(\mathcal{A},\mathcal{B})} | (\mathcal{A},\mathcal{B}) \in \mathcal{Y}_{\mathcal{L}}\} \cup \{i_L, o_L\}$

All the maximal pairs that we have just discovered in step 5. are places and we add an initial place i_L and a final place o_L

Final Steps

7. $\mathcal{F}_{\mathcal{L}} = \{(a, p_{(\mathcal{A},\mathcal{B})}) | (\mathcal{A}, \mathcal{B}) \in \mathcal{Y}_{\mathcal{L}} \wedge a \in \mathcal{A}\} \cup$
$\{(p_{(\mathcal{A},\mathcal{B})}, b) | (\mathcal{A}, \mathcal{B}) \in \mathcal{Y}_{\mathcal{L}} \wedge b \in \mathcal{B}\} \cup$
$\{(i_L, t) | t \in \mathcal{T}_{\mathcal{I}}\} \cup$
$\{(t, o_L) | t \in \mathcal{T}_{\mathcal{O}}\}$

We already have the transitions and the places. Here you see the arcs. So here, you can see all connections from the initial place, I, to all the initial transitions in $\mathcal{T}_{\mathcal{I}}$. From all the transitions in the set $\mathcal{T}_{\mathcal{O}}$. So the transitions corresponding to the activities that happen at the end. And all internal places, and internal places are represented by sets $\mathcal{A}$ and $\mathcal{B}$ and the connections are made accordingly.

8. $\alpha(\mathcal{L}) = (\mathcal{P}_{\mathcal{L}}, \mathcal{T}_{\mathcal{L}}, \mathcal{F}_{\mathcal{L}})$

α-algorithm application considering $\mathcal{L}1$

$\mathcal{L}1 = [\langle a, b, c, d\rangle^3, \langle a, c, b, d\rangle^2, \langle a, e, d\rangle]$

	a	b	c	d	e
a	$\#_{L_1}$	$\rightarrow_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$	$\rightarrow_{L_1}$
b	$\leftarrow_{L_1}$	$\#_{L_1}$	$\Vert_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$
c	$\leftarrow_{L_1}$	$\Vert_{L_1}$	$\#_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$
d	$\#_{L_1}$	$\leftarrow_{L_1}$	$\leftarrow_{L_1}$	$\#_{L_1}$	$\leftarrow_{L_1}$
e	$\leftarrow_{L_1}$	$\#_{L_1}$	$\#_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$

$\mathcal{T}_{\mathcal{L}} =$

$\mathcal{T}_{\mathcal{I}} =$

$\mathcal{T}_{\mathcal{O}} =$

$\mathcal{X}_{\mathcal{L}} =$

$\mathcal{Y}_{\mathcal{L}} =$

$\mathcal{P}_{\mathcal{L}} =$

$\mathcal{F}_{\mathcal{L}} =$

α-algorithm application considering $\mathcal{L}2$

$\mathcal{L}2 = [\langle a, b, c, d\rangle^3, \langle a, c, b, d\rangle^4, \langle a, b, c, e, f, b, c, d\rangle^2,$
$\langle a, b, c, e, f, c, b, d\rangle^2, \langle a, c, b, e, f, b, c, d\rangle^2, \langle a, c, b, e, f, b, c, e, f, c, b, d\rangle]$

$\mathcal{T}_{\mathcal{L}} =$
$\mathcal{T}_{\mathcal{I}} =$
$\mathcal{T}_{\mathcal{O}} =$
$\mathcal{X}_{\mathcal{L}} =$
$\mathcal{Y}_{\mathcal{L}} =$
$\mathcal{P}_{\mathcal{L}} =$
$\mathcal{F}_{\mathcal{L}} =$

α-algorithm application considering $\mathcal{L}3$

$\mathcal{L}3 =$
$[\langle a, b, c, d, e, f, b, d, c, e, g\rangle, \langle a, b, d, c, e, g\rangle^2, \langle a, b, c, d, e, f, b, c, d, e, f, b, d, c, e, g\rangle]$

$\mathcal{T}_{\mathcal{L}} =$
$\mathcal{T}_{\mathcal{I}} =$
$\mathcal{T}_{\mathcal{O}} =$
$\mathcal{X}_{\mathcal{L}} =$
$\mathcal{Y}_{\mathcal{L}} =$
$\mathcal{P}_{\mathcal{L}} =$
$\mathcal{F}_{\mathcal{L}} =$

α-algorithm application considering $\mathcal{L}4$

$\mathcal{L}4 = [\langle a, c, d \rangle^{45}, \langle b, c, d \rangle^{42}, \langle a, c, e \rangle^{38}, \langle b, c, e \rangle^{22}]$

$\mathcal{T}_{\mathcal{L}} =$
$\mathcal{T}_{\mathcal{I}} =$
$\mathcal{T}_{\mathcal{O}} =$
$\mathcal{X}_{\mathcal{L}} =$
$\mathcal{Y}_{\mathcal{L}} =$
$\mathcal{P}_{\mathcal{L}} =$
$\mathcal{F}_{\mathcal{L}} =$

α-algorithm application considering $\mathcal{L}5$

$\mathcal{L}5 =$
$[\langle a, b, e, f\rangle^2, \langle a, b, e, c, d, b, f\rangle^3, \langle a, b, c, e, d, b, f\rangle^2, \langle a, b, e, f\rangle^2, \langle a, b, e, c, d, b, f\rangle^3,$
$\langle a, b, c, e, d, b, f\rangle^2, \langle a, b, c, d, e, b, f\rangle^4, \langle a, e, b, c, d, b, f\rangle^3]$

$T_{\mathcal{L}} =$
$T_{\mathcal{I}} =$
$T_{\mathcal{O}} =$
$X_{\mathcal{L}} =$
$Y_{\mathcal{L}} =$
$P_{\mathcal{L}} =$
$F_{\mathcal{L}} =$