Model Checking I
alias
Reactive Systems Verification

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Topics

- Impact of fairness on liveness properties
- Fairness of actions
- Unconditional, Strong and Weak fairness conditions.
- Realizability of fairness.

Material

Reading:

Chapter 3 of the book, pages 126–141.

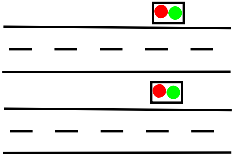
More:

The slides in the following pages are taken from the material of the course “Introduction to Model Checking” held by Prof. Dr. Ir. Joost-Pieter Katoen at Aachen University.

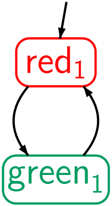
Observation

liveness properties are often violated
although we expect them to hold

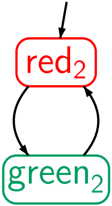
Two independent traffic lights



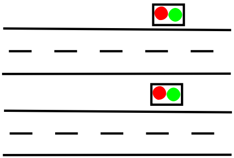
light 1



light 2



Two independent traffic lights



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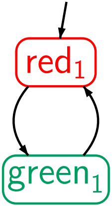
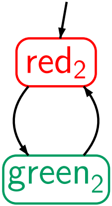
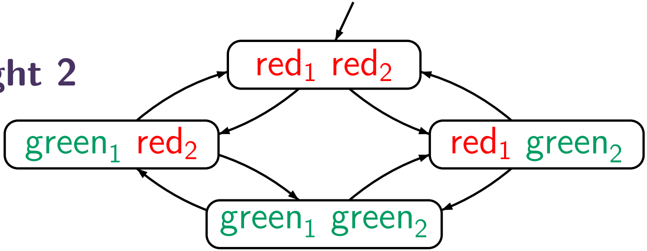


light 2



light 1 ||| light 2



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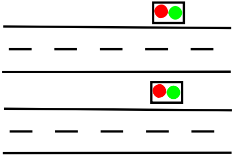

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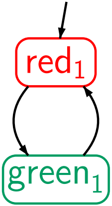
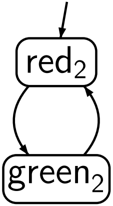
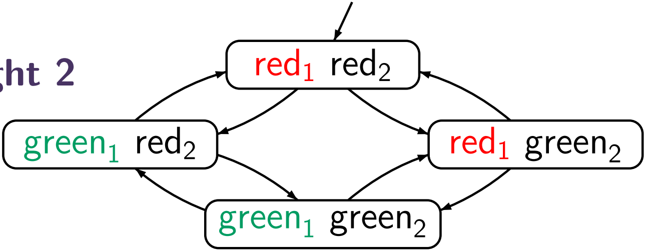


light 2

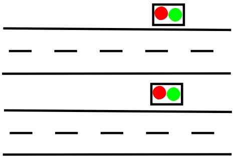


light 1 ||| light 2

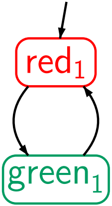


light 1 ||| light 2 $\not\models$ "infinitely often ***green***$_1$"

Two independent traffic lights



light 1



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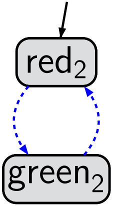
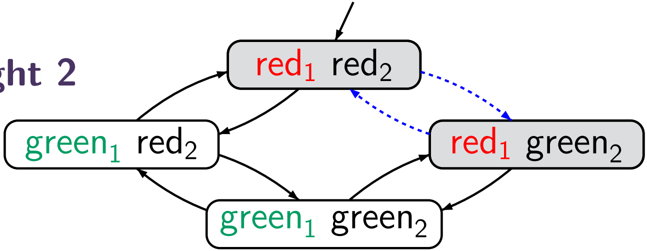
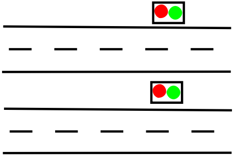


light 1 ||| light 2



light 1 ||| light 2 $\not\models$ "infinitely often ***green*$_1$**"

Two independent traffic lights



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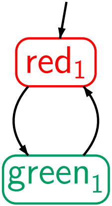
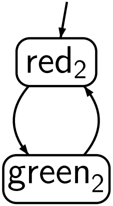
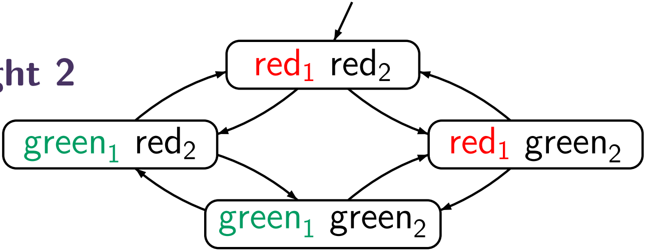


light 2



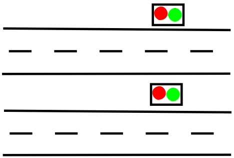
light 1 ||| light 2



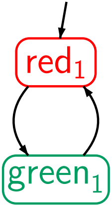
light 1 ||| light 2 $\not\models$ "infinitely often ***green***$_1$"

although **light 1** $\models$ "infinitely often ***green***$_1$"

Two independent traffic lights



light 1



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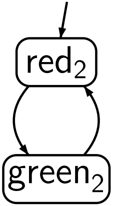

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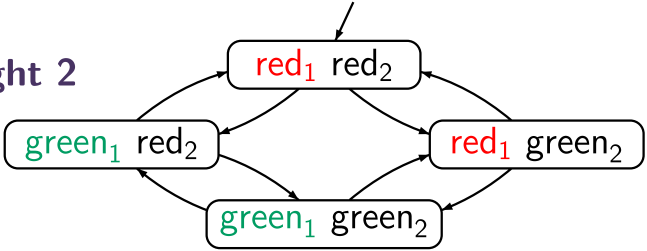
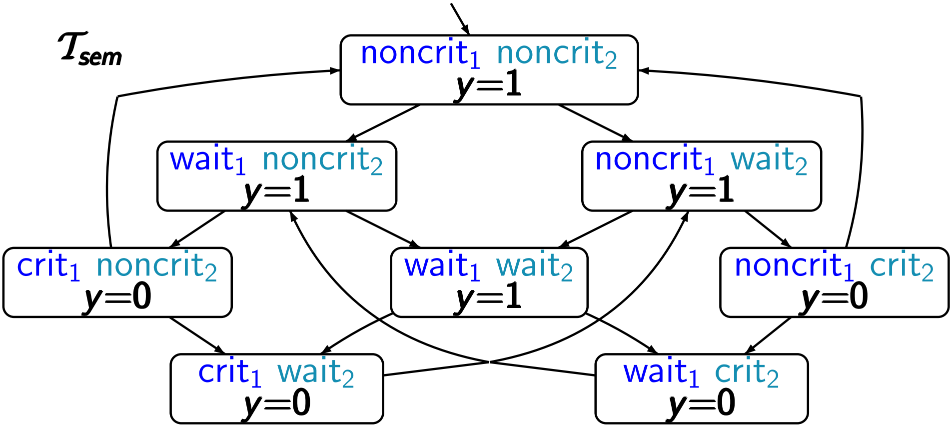


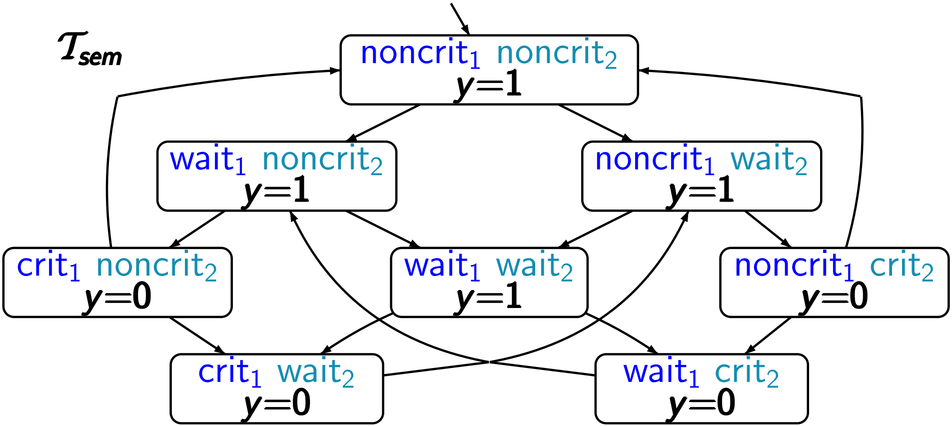
light 1 ||| light 2 $\not\models$ "infinitely often ***green***$_1$"

> interleaving is completely time abstract **!**

Mutual exclusion (semaphore)

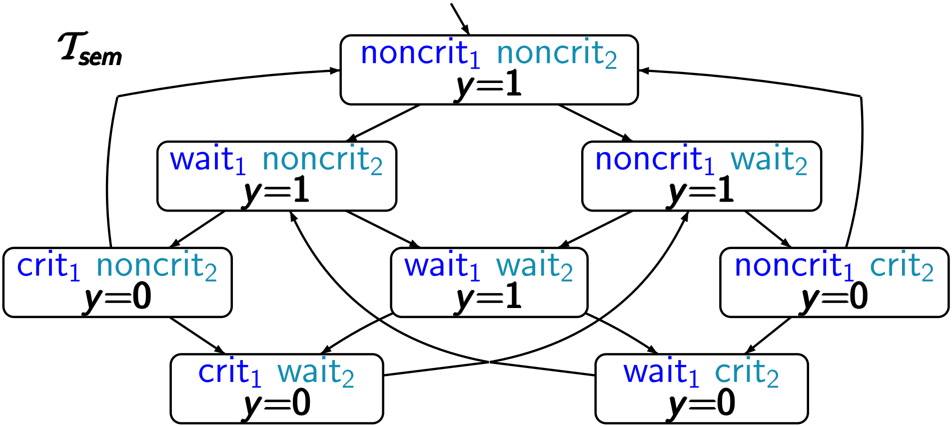


Mutual exclusion (semaphore)



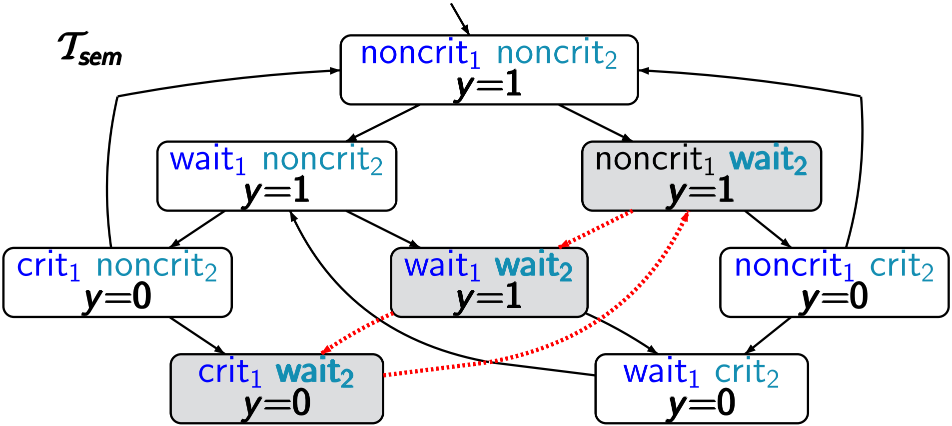
liveness property $\widehat{=}$ "each waiting process will eventually enter its critical section"

Mutual exclusion (semaphore)



$\mathcal{T}_{sem} \not\models$ "each waiting process will eventually enter its critical section"

Mutual exclusion (semaphore)



$\mathcal{T}_{sem} \not\models$ "each waiting process will eventually enter its critical section"

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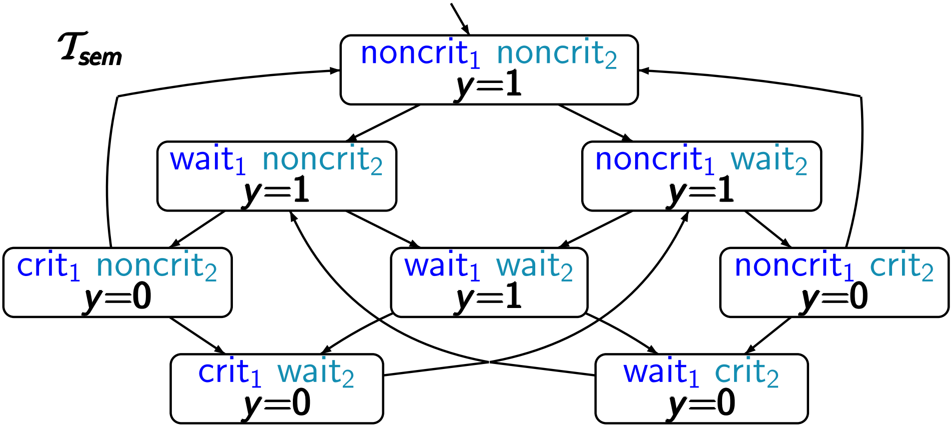


$\mathcal{T}_{sem} \not\models$ "each waiting process will eventually enter its critical section"

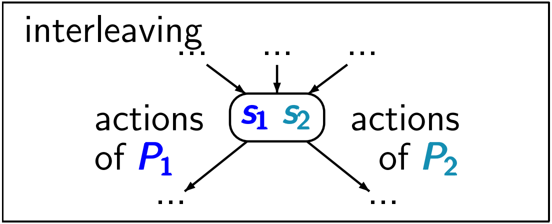
level of abstraction is too coarse !

Process fairness

LF2.6-5

Process fairness

two independent non-communicating processes $P_1 \mid\mid\mid P_2$



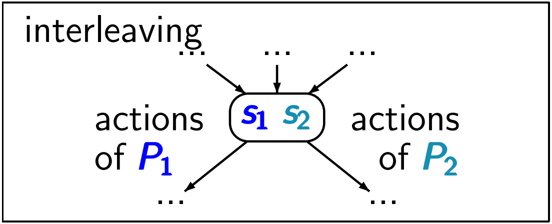
possible interleavings:

$P_1\ P_2\ P_2\ P_1\ P_1\ P_1\ P_2\ P_1\ P_2\ P_2\ P_2\ P_1\ P_1\ \ldots$

$P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ \ldots$

Process fairness

two independent non-communicating processes $P_1 \mid\mid\mid P_2$

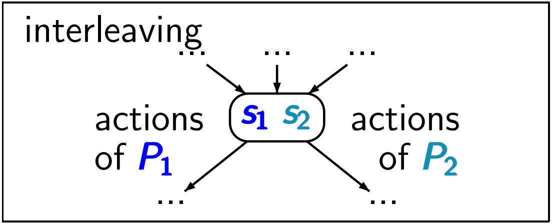


possible interleavings:

P_1 P_2 P_2 P_1 P_1 P_1 P_2 P_1 P_2 P_2 P_2 P_1 P_1 ...
P_1 P_1 P_2 P_1 P_1 P_2 P_1 P_1 P_2 P_1 P_1 P_2 P_1 ...
P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 ...

Process fairness

two independent non-communicating processes $P_1 \,|||\, P_2$



possible interleavings:

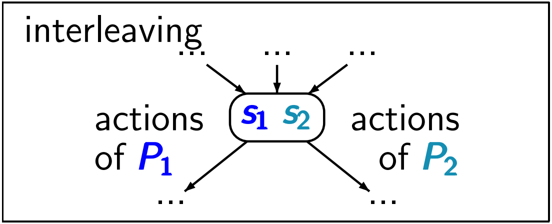
$P_1\ P_2\ P_2\ P_1\ P_1\ P_1\ P_2\ P_1\ P_2\ P_2\ P_2\ P_1\ P_1\ \ldots$ fair

$P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ \ldots$ fair

$P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ \ldots$ unfair

Process fairness

two independent non-communicating processes $P_1 \mid\mid\mid P_2$



possible interleavings:

P_1 P_2 P_2 P_1 P_1 P_1 P_2 P_1 P_2 P_2 P_2 P_1 P_1 ... fair

P_1 P_1 P_2 P_1 P_1 P_2 P_1 P_1 P_2 P_1 P_1 P_2 P_1 ... fair

P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 ... unfair

process fairness assumes an appropriate resolution of the nondeterminism resulting from interleaving and competitions

Nuances of fairness

- unconditional fairness

- strong fairness

- weak fairness

Nuances of fairness

- unconditional fairness, e.g.,

  every process enters gets its turn infinitely often.

- strong fairness

- weak fairness

Nuances of fairness

- unconditional fairness, e.g.,

  every process enters gets its turn infinitely often.

- strong fairness, e.g.,

  every process that is enabled infinitely often
  gets its turn infinitely often.

- weak fairness

Nuances of fairness

- unconditional fairness, e.g.,

  every process enters gets its turn infinitely often.

- strong fairness, e.g.,

  every process that is enabled infinitely often
  gets its turn infinitely often.

- weak fairness, e.g.,

  every process that is continuously enabled
  from a certain time instance on,
  gets its turn infinitely often.

Fairness for action-set

LF2.6-7

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set ***Act***, $A \subseteq$ ***Act*** and

$\rho = s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} s_2 \xrightarrow{\alpha_2} \ldots$ infinite execution fragment

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and

$\rho = s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} s_2 \xrightarrow{\alpha_2} \ldots$ infinite execution fragment

we will provide conditions for

- unconditional A-fairness of ρ
- strong A-fairness of ρ
- weak A-fairness of ρ

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using the following notations:

$$Act(s_i) = \{\beta \in Act : \exists s' \text{ s.t. } s_i \xrightarrow{\beta} s'\}$$

Fairness for action-set

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$$\overset{\infty}{\exists} \;\widehat{=}\; \text{“there exists infinitely many ...”}$$

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and

$\rho = s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} s_2 \xrightarrow{\alpha_2} \dots$ infinite execution fragment

we will provide conditions for

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using the following notations:

$$Act(s_i) = \{\beta \in Act : \exists s' \text{ s.t. } s_i \xrightarrow{\beta} s'\}$$

$\overset{\infty}{\exists}$ $\widehat{=}$ "there exists infinitely many ..."

$\overset{\infty}{\forall}$ $\widehat{=}$ "for all, but finitely many ..."

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and

$\rho = s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} s_2 \xrightarrow{\alpha_2} \ldots$ infinite execution fragment

- ρ is unconditionally A-fair, if

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and

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- ρ is unconditionally A-fair, if $\overset{\infty}{\exists}\, i \geq 0.\, \alpha_i \in A$

"actions in A will be taken infinitely many times"

Fairness for action-set

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$$\overset{\infty}{\exists}\, i \geq 0.\, A \cap Act(s_i) \neq \varnothing \quad \Longrightarrow \quad \overset{\infty}{\exists}\, i \geq 0.\, \alpha_i \in A$$

"If infinitely many times some action in A is enabled, then actions in A will be taken infinitely many times."

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and

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- ρ is weakly A-fair, if

Fairness for action-set

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$$\overset{\infty}{\forall}\, i \geq 0.\, A \cap Act(s_i) \neq \varnothing \implies \overset{\infty}{\exists}\, i \geq 0.\, \alpha_i \in A$$

"If from some moment, actions in A are enabled, then actions in A will be taken infinitely many times."

Fairness for action-set

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unconditionally A-fair	$\Longrightarrow$	strongly A-fair
	$\Longrightarrow$	weakly A-fair

Fairness for action-set

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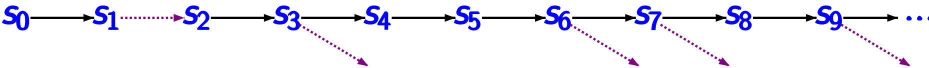
unconditionally A-fair	$\Longrightarrow$	strongly A-fair
	$\Longrightarrow$	weakly A-fair

Strong and weak action fairness

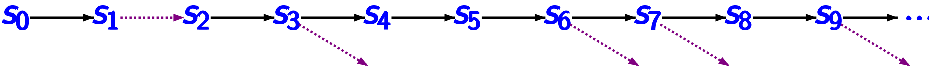
strong ***A***-fairness is *violated* if



- no ***A***-actions are executed from a certain moment
- ***A***-actions are enabled infinitely many times

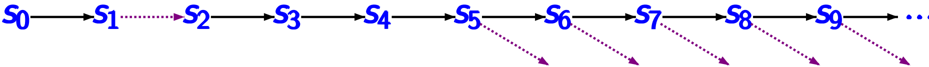
Strong and weak action fairness

strong ***A***-fairness is *violated* if



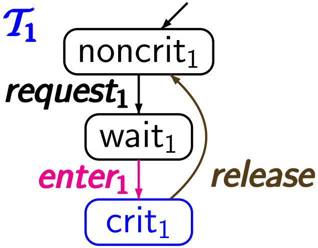
- no ***A***-actions are executed from a certain moment
- ***A***-actions are enabled infinitely many times

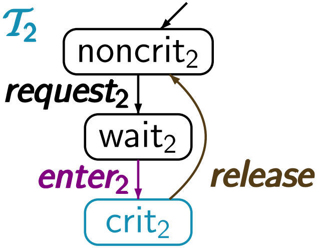
weak ***A***-fairness is *violated* if



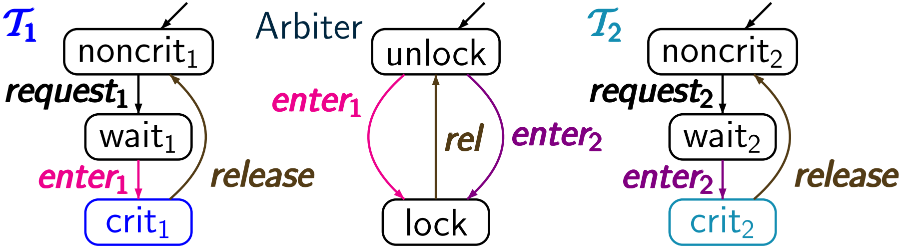
- no ***A***-actions are executed from a certain moment
- ***A***-actions are continuously enabled from some moment on

Mutual exclusion with arbiter





Mutual exclusion with arbiter



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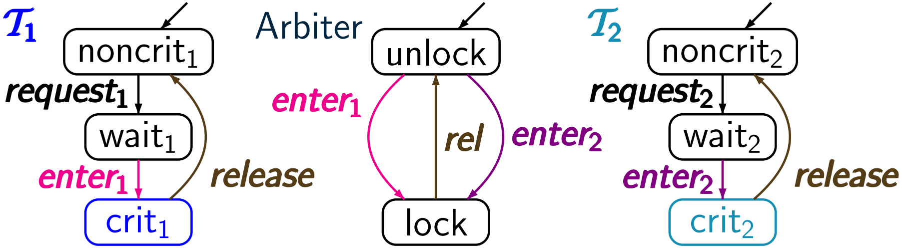
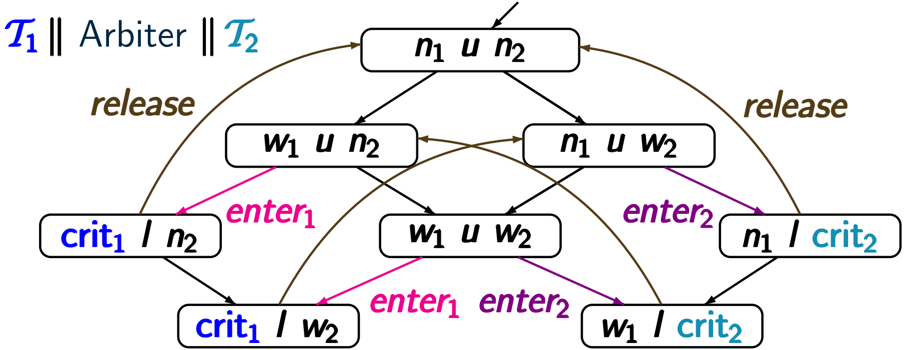
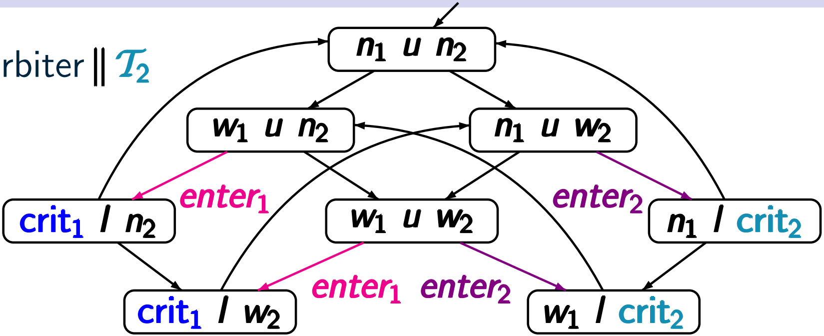




Unconditional, strongly or weakly fair?

$\mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$



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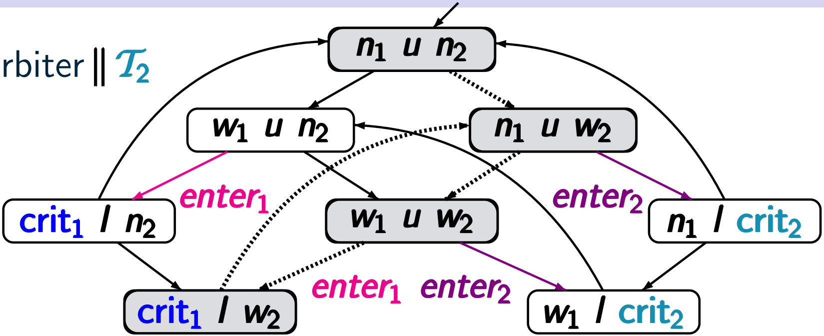

fairness for action set $A = \{enter_1\}$:

$$\langle n_1, u, n_2 \rangle \longrightarrow \Big(\langle n_1, u, w_2 \rangle \longrightarrow \langle w_1, u, w_2 \rangle \longrightarrow \langle crit_1, l, w_2 \rangle \Big)^\omega$$

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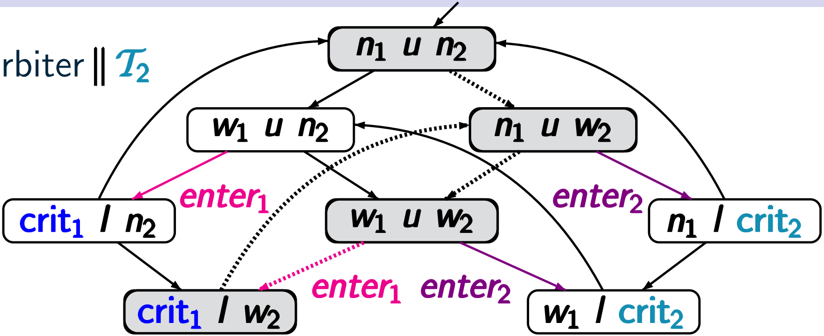

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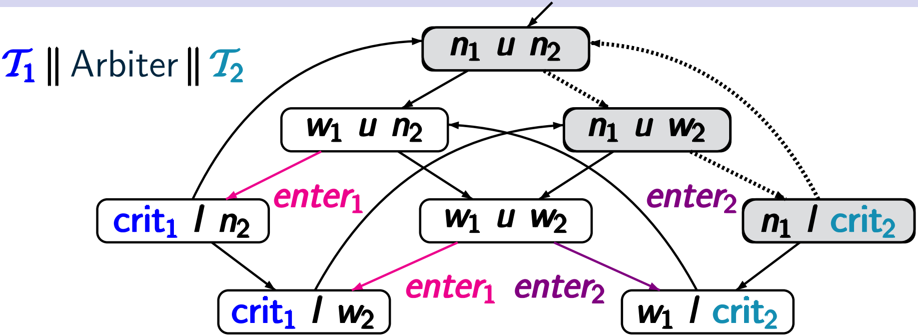

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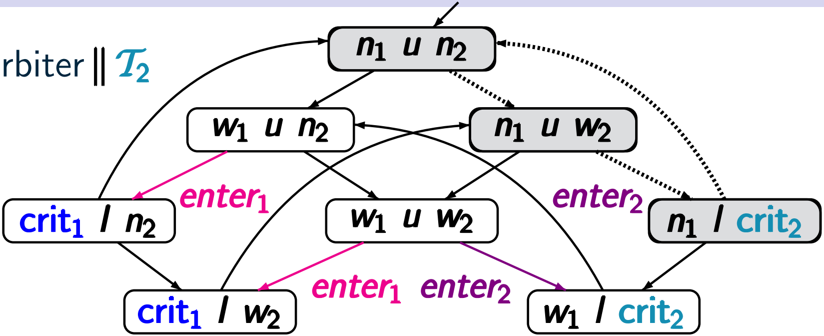

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- unconditional A-fairness: **no**
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- weak A-fairness: **yes** $\leftarrow$ strongly A-fair

Unconditional, strongly or weakly fair?

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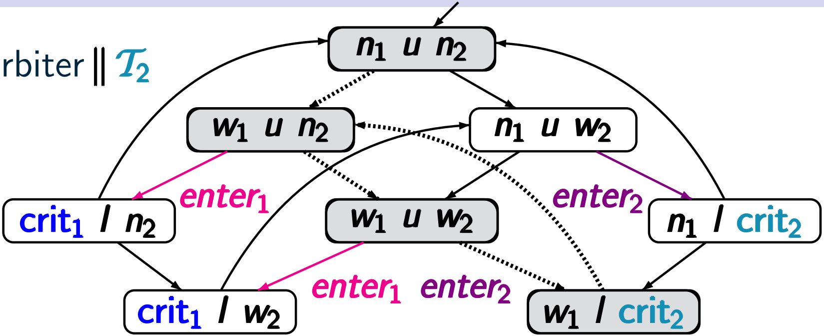

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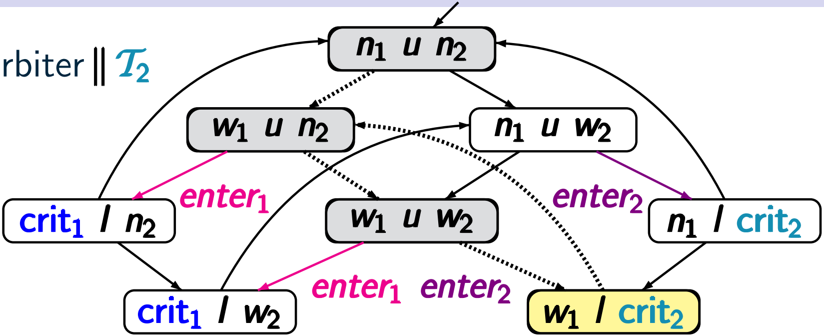

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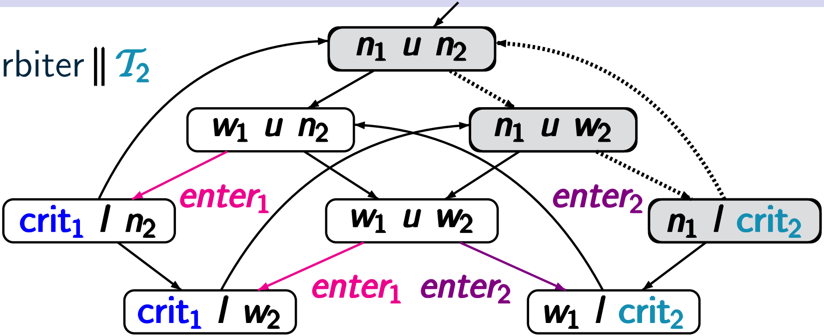

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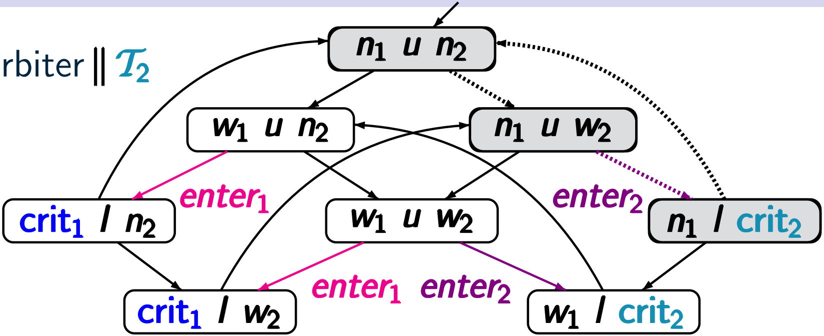

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Action-based fairness assumptions

Action-based fairness assumptions

Let $\mathcal{T}$ be a transition system with action-set Act.
A fairness assumption for $\mathcal{T}$ is a triple

$$\mathcal{F} = (\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak})$$

where $\mathcal{F}_{ucond}$, $\mathcal{F}_{strong}$, $\mathcal{F}_{weak} \subseteq 2^{Act}$.

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An execution ρ is called $\mathcal{F}$-fair iff

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$$FairTraces_{\mathcal{F}}(\mathcal{T}) \stackrel{\text{def}}{=} \{trace(\rho) : \rho \text{ is a } \mathcal{F}\text{-fair execution of } \mathcal{T}\}$$

Fair satisfaction relation

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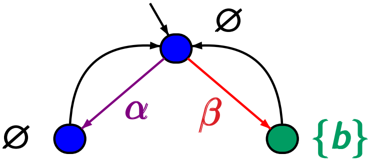
If $\mathcal{T}$ is a TS and E a LT property over AP then:

$$\mathcal{T} \models_{\mathcal{F}} E \quad \overset{\text{def}}{\Longleftrightarrow} \quad FairTraces_{\mathcal{F}}(\mathcal{T}) \subseteq E$$

Example: fair satisfaction relation



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- no unconditional fairness condition
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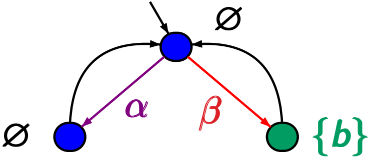

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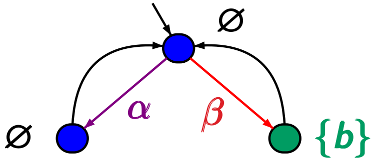

$\mathcal{T} \models_{\mathcal{F}}$ "infinitely often b" ?

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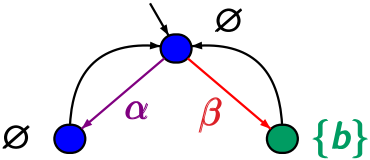

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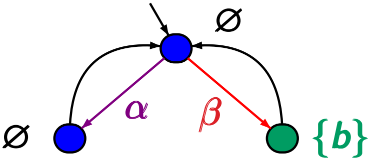

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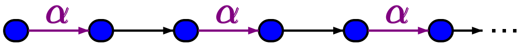
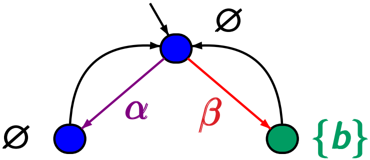


$\mathcal{F}$-fair

actions in $\{\alpha, \beta\}$ are executed infinitely many times

Example: fair satisfaction relation



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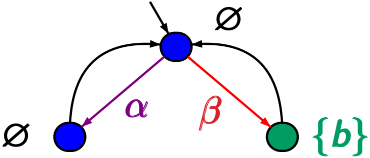

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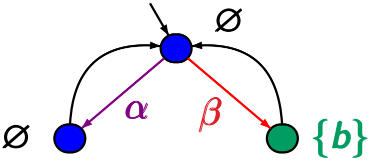

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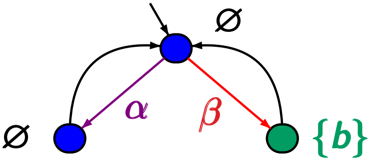

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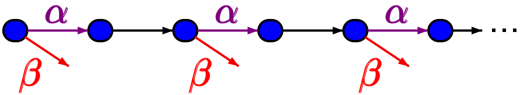

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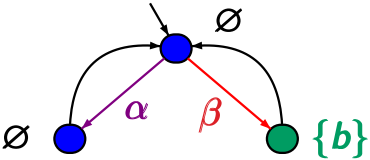

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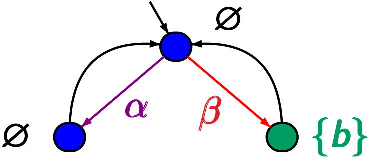

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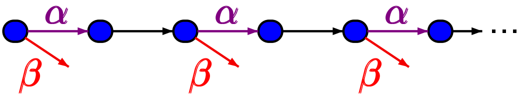


is not $\mathcal{F}$-fair

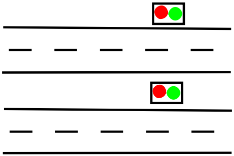
Which type of fairness?

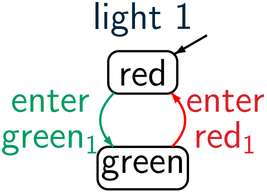
LF2.6-13A

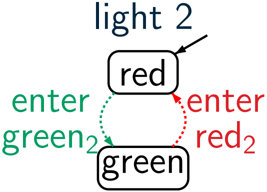
Which type of fairness?

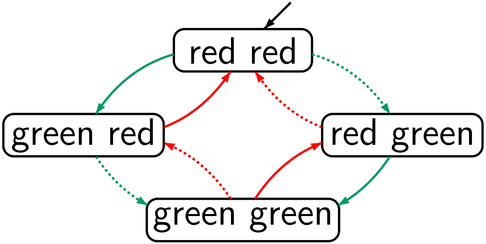
fairness assumptions should be
as weak as possible

Two independent traffic lights









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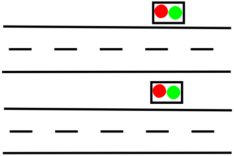

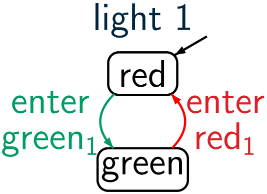

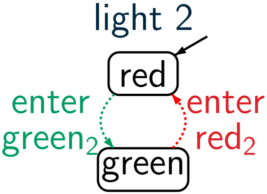

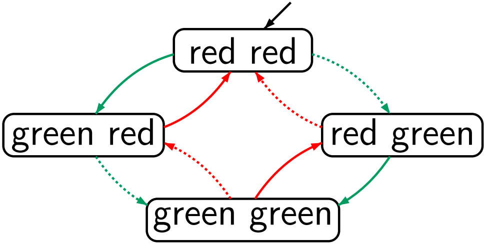

fairness assumption $\mathcal{F}$:

$\mathcal{F}_{ucond} = ?$

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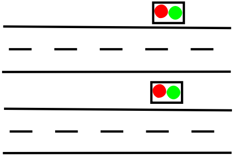
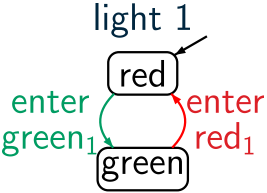
$\mathcal{F}_{weak} = ?$

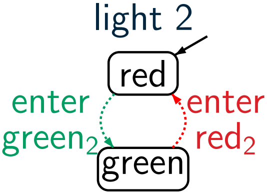
light 1 $|||$ light 2 $\models_{\mathcal{F}} E$

$E \mathrel{\widehat{=}}$ "both lights are infinitely often green"

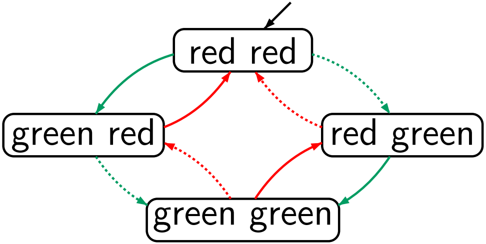
Two independent traffic lights







A_1 = actions of light 1
A_2 = actions of light 2



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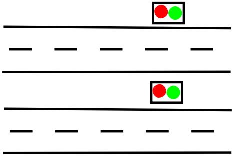

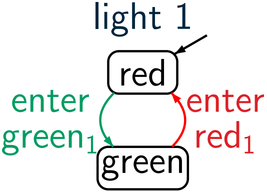

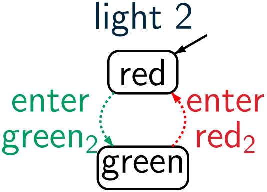

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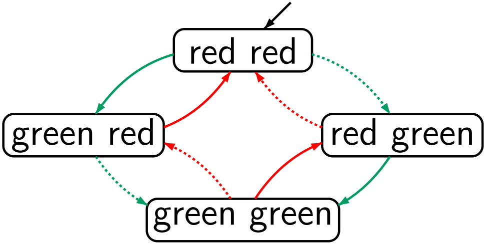

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light 1 $|||$ light 2 $\models_{\mathcal{F}}$ E

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Example: MUTEX with fair arbiter

$$\mathcal{T} = \mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$$

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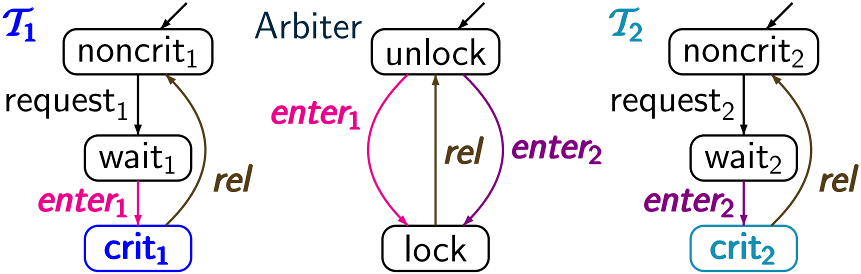

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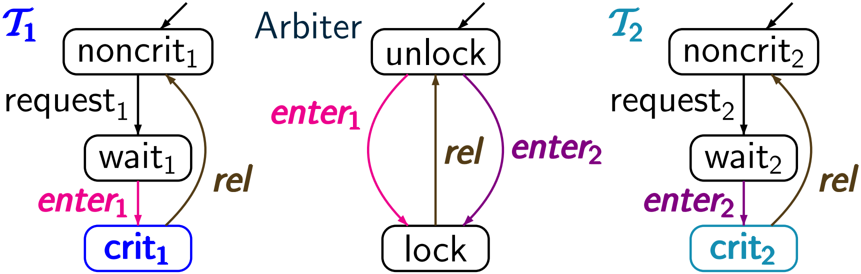

$\mathcal{T}_1$ and $\mathcal{T}_2$ compete to communicate with the arbiter by means of the actions $enter_1$ and $enter_2$, respectively

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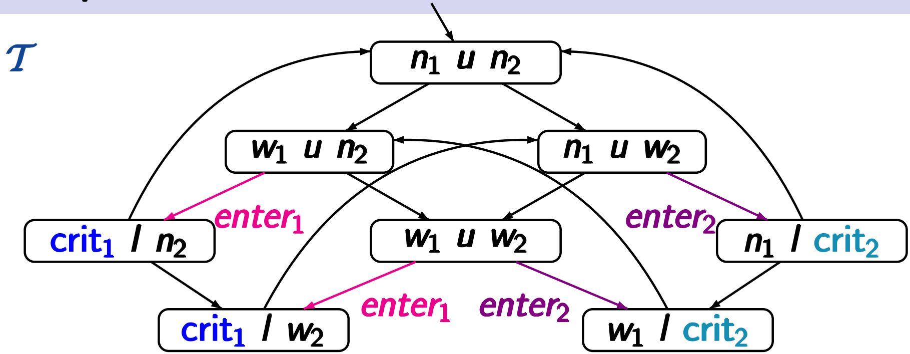
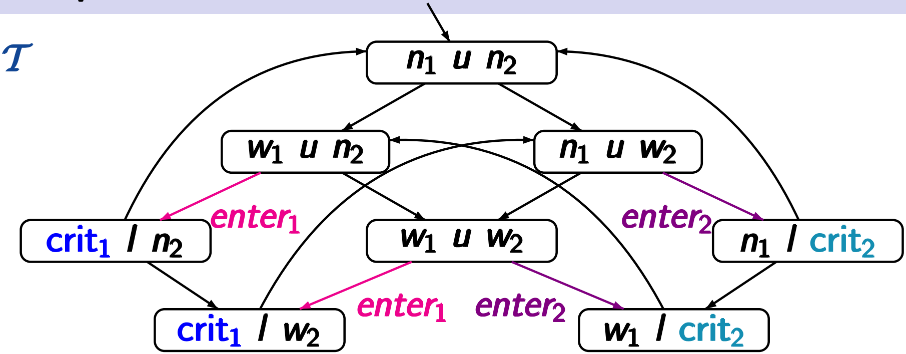


LT property E: each waiting process eventually enters its critical section

$\mathcal{T} \not\models E$

Example: MUTEX with fair arbiter



LT property E: each waiting process eventually enters its critical section

fairness assumption $\mathcal{F}$

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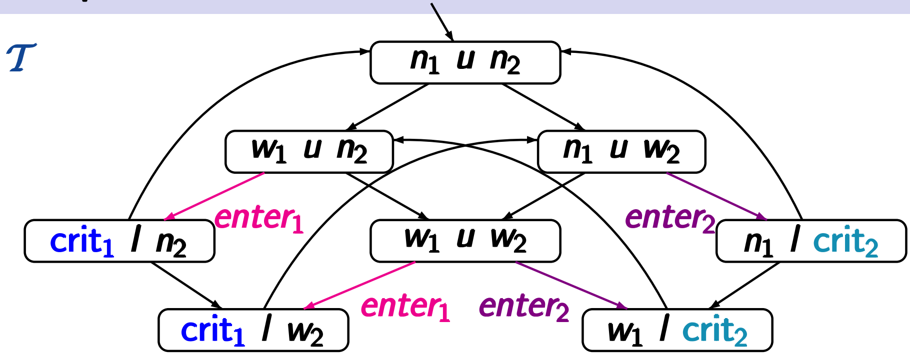
$\mathcal{F}_{weak} = \{\{enter_1\}, \{enter_2\}\}$

does $\mathcal{T} \models_{\mathcal{F}} E$ hold ?

Example: MUTEX with fair arbiter



LT property E: each waiting process eventually enters its critical section

fairness assumption $\mathcal{F}$

$\mathcal{F}_{ucond} = \mathcal{F}_{strong} = \varnothing$

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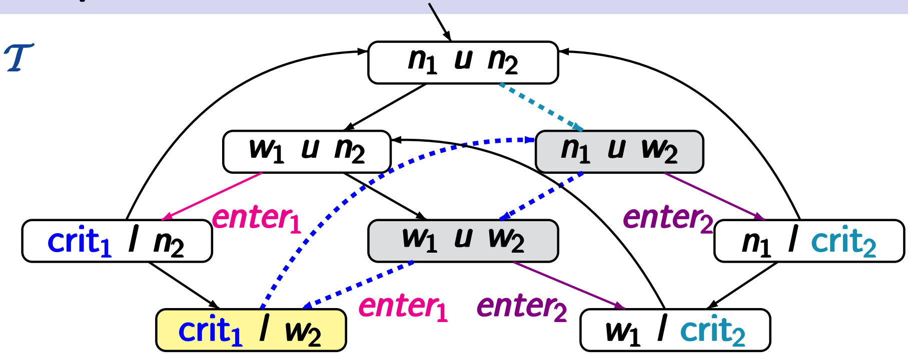

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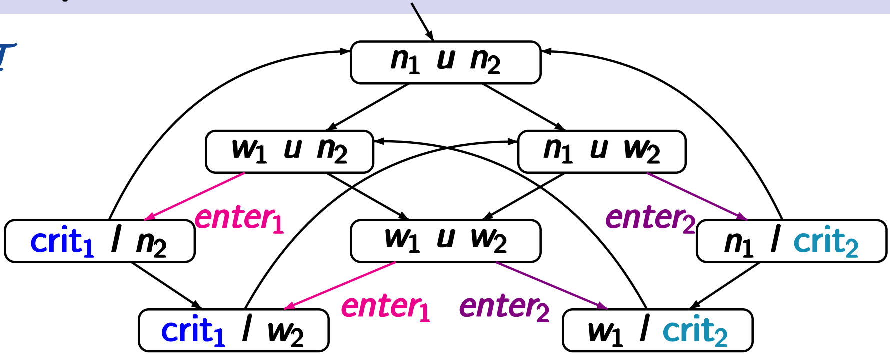
as $enter_2$ is not enabled in $\langle crit_1, l, w_2 \rangle$

Example: MUTEX with fair arbiter

$\mathcal{T}$



E: each waiting process eventually enters its crit. section

$\mathcal{F}_{ucond} = ?$
$\mathcal{F}_{strong} = ?$
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but $\mathcal{T} \models_{\mathcal{F}} E$

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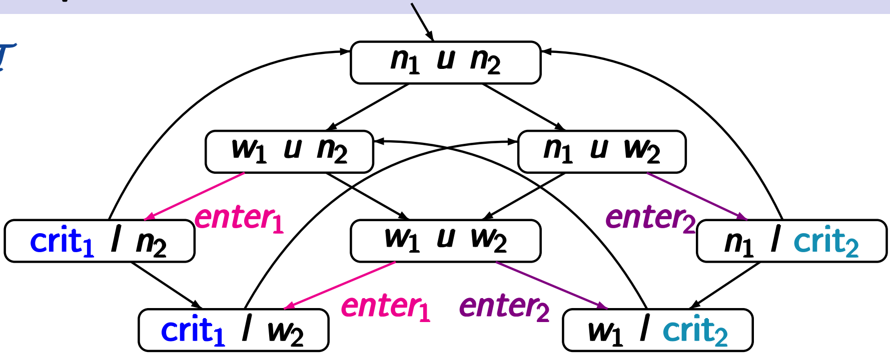

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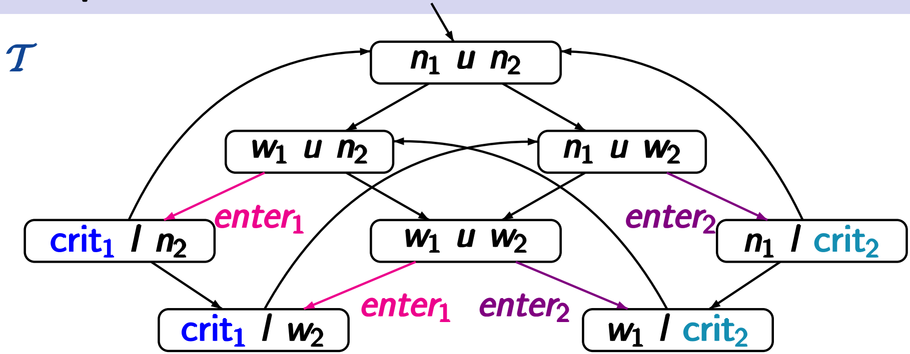

E: each waiting process eventually enters its crit. section

D: each process enters its critical section infinitely often

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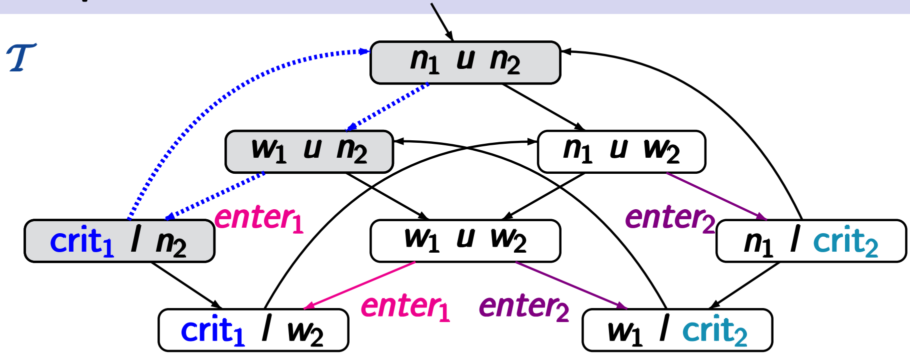

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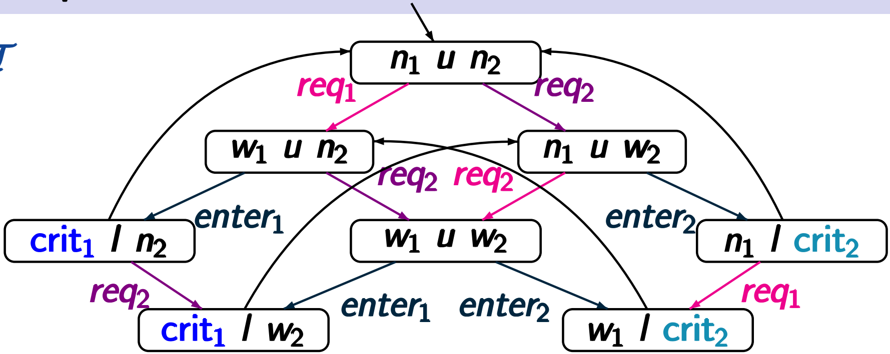

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Process fairness

LF2.6-19

Process fairness

For asynchronous systems:

parallelism = interleaving + fairness

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rule of thumb:

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  - ∗ choice between dependent actions
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- weak fairness for the nondetermism obtained from the interleaving of independent actions
- unconditional fairness: only of theoretical interest

Purpose of fairness conditions

parallelism = interleaving + fairness

Process fairness and other fairness conditions

- can compensate information loss due to interleaving
  or rule out other unrealistic pathological cases
- can be requirements for a scheduler
  or requirements for environment
- can be verifiable system properties

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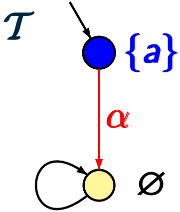
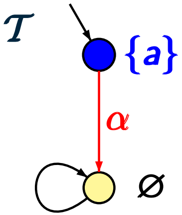
liveness properties:	fairness can be essential
safety properties:	fairness is irrelevant

Fairness



fairness assumption $\mathcal{F}$:
unconditional fairness for action set $\{\alpha\}$

does $\mathcal{T} \models_{\mathcal{F}}$ "infinitely often a" hold ?



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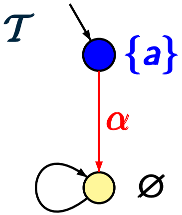

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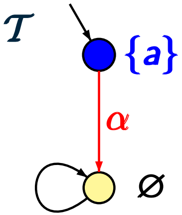
Realizability of fairness assumptions



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Realizability requires that each initial finite path fragment can be extended to a $\mathcal{F}$-fair path

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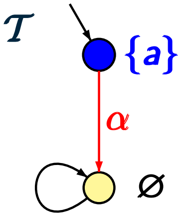

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Fairness assumption $\mathcal{F}$ is said to be realizable for a transition system $\mathcal{T}$ if for each reachable state s in $\mathcal{T}$ there exists a $\mathcal{F}$-fair path starting in s

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$\uparrow$

can always be guaranteed by a scheduler, i.e., an instance that resolves the nondeterminism in $\mathcal{T}$

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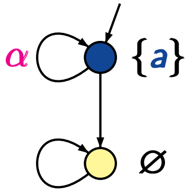

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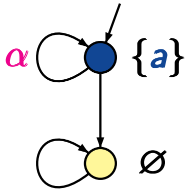

$\mathcal{F}$: unconditional fairness for $\{\alpha\}$

E = invariant "always a"

$\mathcal{T} \not\models E$, but $\mathcal{T} \models_{\mathcal{F}} E$