Model Checking I
alias
Reactive Systems Verification

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Topics

- Safety and Liveness.
- Recall of Propositional Logic.
- Invariant properties. Examples.
- Invariant properties checking via Depth-First Search.

Material

Reading:

Chapter 3 of the book, pages 106–111.

More:

The slides in the following pages are taken from the material of the course “Introduction to Model Checking” held by Prof. Dr. Ir. Joost-Pieter Katoen at Aachen University.

Overview

Classification of LT-properties

safety properties *"nothing bad will happen"*

liveness properties *"something good will happen"*

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examples:

- mutual exclusion
- deadlock freedom
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- "each waiting process will eventually enter its critical section"
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} special case: **invariants** *"no bad state will be reached"*

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Propositional logic

$$\Phi ::= \mathit{true} \;\Big|\; a \;\Big|\; \Phi_1 \wedge \Phi_2 \;\Big|\; \neg\Phi$$

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IS2.5-2

$$\Phi ::= \textit{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \Phi_1 \vee \Phi_2 \mid \Phi_1 \rightarrow \Phi_2 \mid \ldots$$

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semantics: interpretation over a subsets of AP

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↑ atomic proposition, i.e., $a \in AP$

semantics: Let $A \subseteq AP$

$$
\begin{array}{lll}
A \models \textit{true} & & \\
A \models a & \text{iff} & a \in A \\
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e.g., $\{a, b\} \not\models (a \rightarrow \neg b) \vee c \qquad \{a, b\} \models a \vee c$

Propositional logic

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$A \models a$	iff	$a \in A$
$A \models \Phi_1 \wedge \Phi_2$	iff	$A \models \Phi_1$ and $A \models \Phi_2$
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for state s of a TS over AP: $s \models \Phi$ iff $L(s) \models \Phi$

Invariant

Invariant

Let E be an LT property over AP.

E is called an invariant if there exists a propositional formula Φ over AP such that

$$E = \left\{ A_0 A_1 A_2 \ldots \in \left(2^{AP}\right)^{\omega} : \forall i \geq 0.\, A_i \models \Phi \right\}$$

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Φ is called the invariant condition of E.

Examples for invariants

mutual exclusion (safety):

$$MUTEX = \begin{array}{l} \text{set of all infinite words } A_0\,A_1\,A_2\ldots \text{ s.t.} \\ \forall i \in \mathbb{N}.\ \ \text{crit}_1 \notin A_i \ \text{ or } \ \text{crit}_2 \notin A_i \end{array}$$

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deadlock freedom for 5 dining philosophers:

$$DF = \text{set of all infinite words } A_0 A_1 A_2 \ldots \text{ s.t. } \forall i \in \mathbb{N}\ \exists j \in \{0,1,2,3,4\}.\ \text{wait}_j \notin A_i$$

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Satisfaction of invariants

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iff $s \models \Phi$ for all states $s \in Reach(\mathcal{T})$

↑ set of reachable states in $\mathcal{T}$

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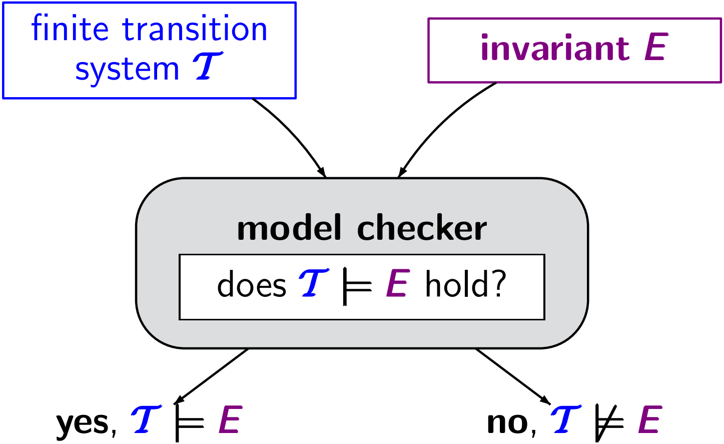
i.e., Φ holds in all initial states and is invariant under all transitions

Invariant checking



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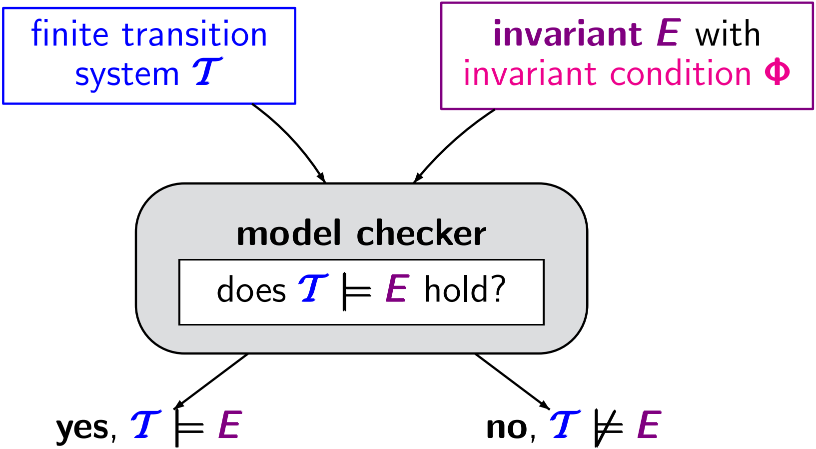

perform a graph analysis (**DFS** or **BFS**) to check whether $s \models \Phi$ for all $s \in Reach(\mathcal{T})$

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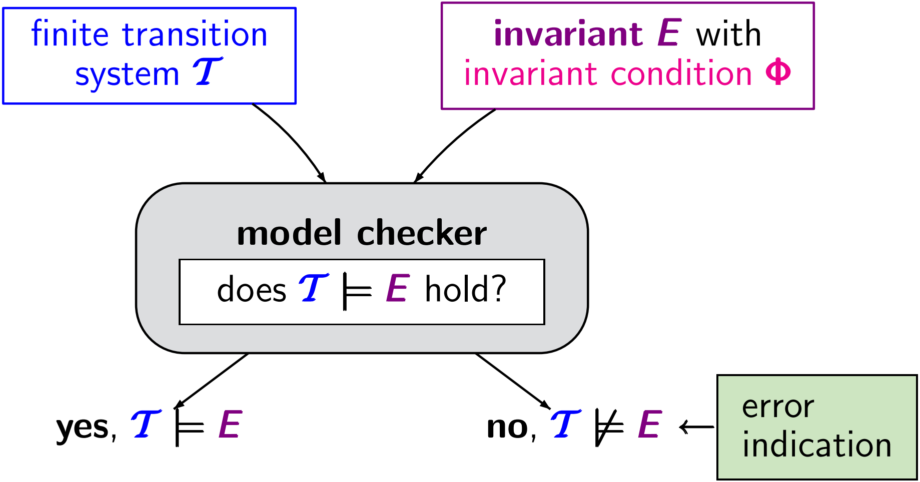

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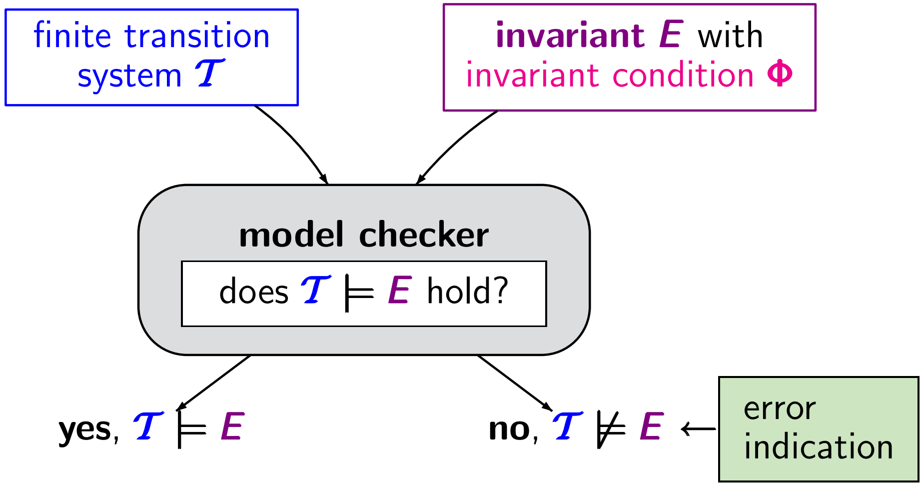

error indication: initial path fragment $s_0\, s_1 \ldots s_{n-1} s_n$
such that $s_i \models \Phi$ for $0 \leq i < n$ and $s_n \not\models \Phi$

DFS-based invariant checking

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FOR ALL s_0 ∈ S_0 DO
      IF DFS(s_0, Φ) THEN
         return "no"
      FI
OD
return "yes"
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$DFS(s_0, \Phi)$ returns "true" iff depth-first search from state s_0 leads to some state t with $t \not\models \Phi$

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```
π := ∅  ⟵  stack for error indication
FOR ALL s0 ∈ S0 DO
      IF DFS(s0, Φ) THEN
            return "no" and reverse(π)
      FI
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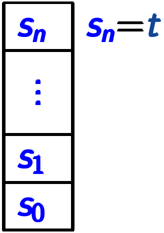

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$U := \varnothing$ ⟵ stores the "processed" states

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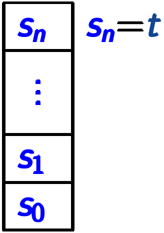

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Recursive algorithm $DFS(s, \Phi)$

IS2.5-8

"searches" for a path fragment $s \ldots t$ with $t \not\models \Phi$

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     IF s ⊨ Φ THEN

          ⋮

     FI
FI
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```

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IF s ∉ U THEN
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IF s ∉ U THEN
    IF s ⊭ Φ THEN return "true" FI
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        insert s in U;
        FOR ALL s' ∈ Post(s) DO
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Push(π, s);
IF s ∉ U THEN
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Pop(π); return "false"
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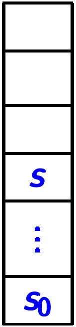

initial state

Recursive algorithm $DFS(s, \Phi)$

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IF $s \notin U$ THEN
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    IF $s \models \Phi$ THEN
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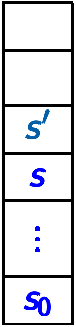

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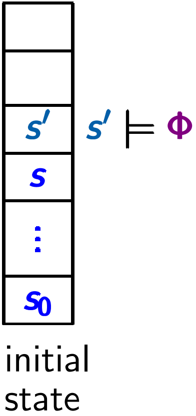

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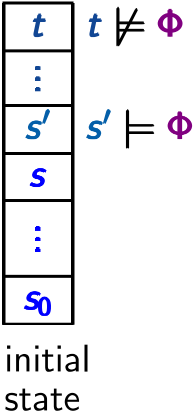
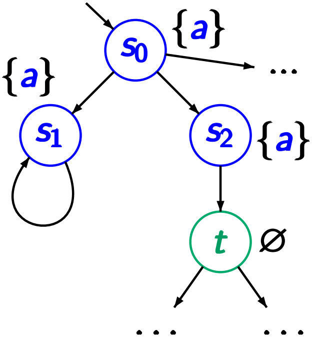


Example: invariant checking



invariant
condition a

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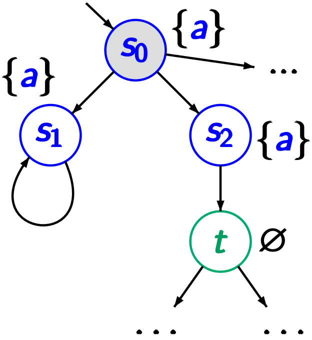

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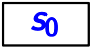
$DFS(s_0, a)$

stack π



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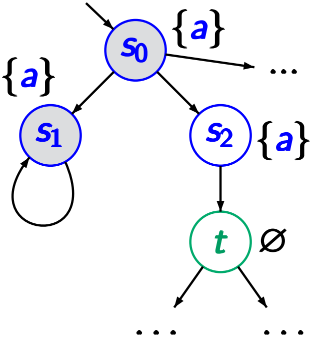

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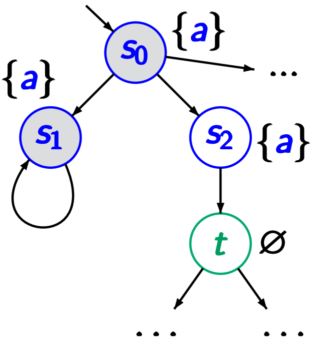

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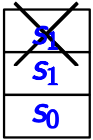

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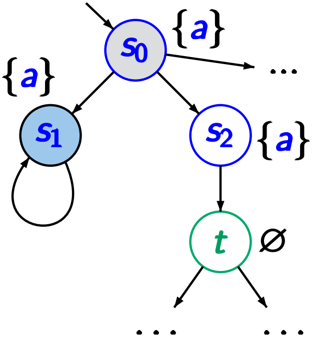

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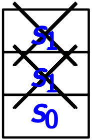

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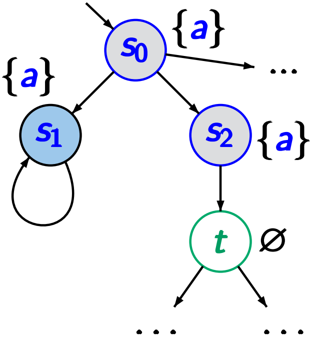

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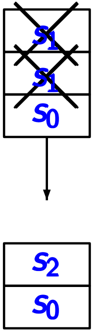
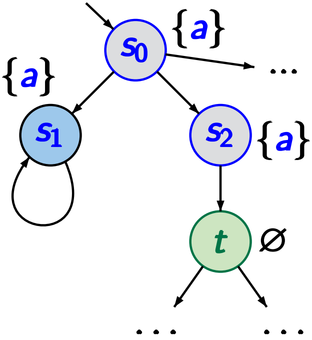


Example: invariant checking



invariant condition a

$s_0, s_1, s_2 \models a$

$t \not\models a$

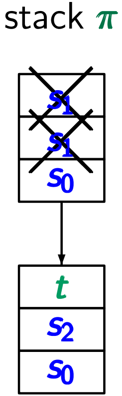
$DFS(s_0, a)$

$DFS(s_1, a)$

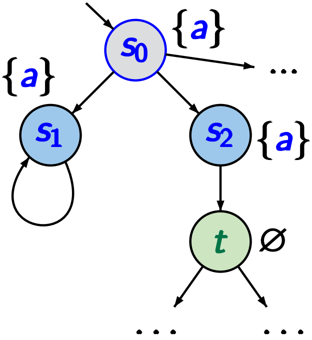
$DFS(s_1, a)$

$DFS(s_2, a)$

$DFS(t, a)$



Example: invariant checking



invariant
condition a

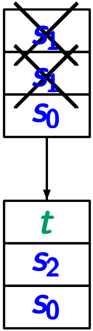
$$s_0, s_1, s_2 \models a$$
$$t \not\models a$$

$DFS(s_0, a)$

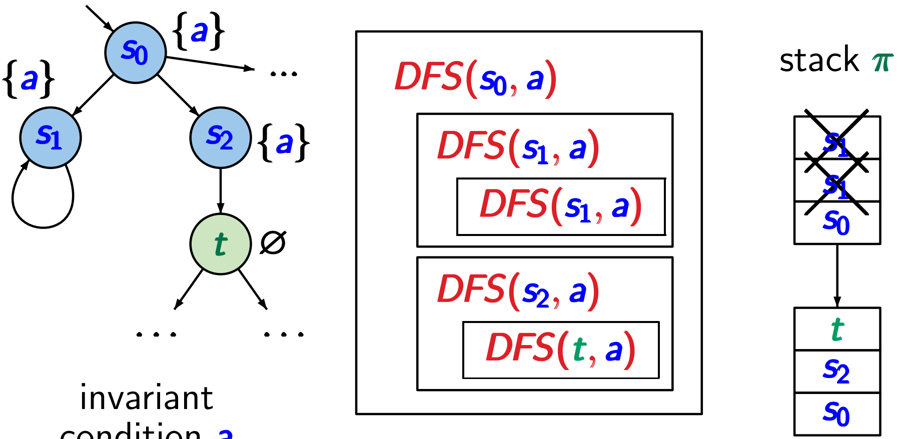
$DFS(s_1, a)$
$DFS(s_1, a)$

$DFS(s_2, a)$
$DFS(t, a)$

stack π



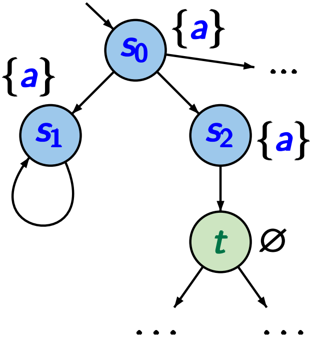
Example: invariant checking

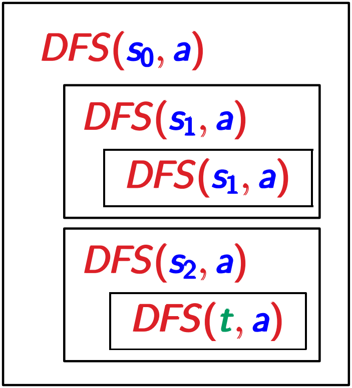


invariant condition a

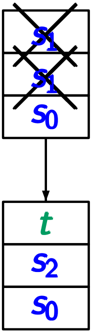
$$s_0, s_1, s_2 \models a$$
$$t \not\models a$$

Example: invariant checking





stack π



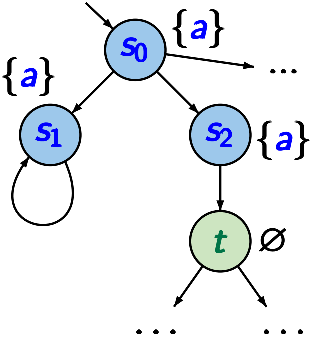
invariant condition a

$s_0, s_1, s_2 \models a$

$t \not\models a$

$s_0 \not\models$ "always a"

Example: invariant checking



invariant condition a

$$s_0, s_1, s_2 \models a$$
$$t \not\models a$$

$DFS(s_0, a)$

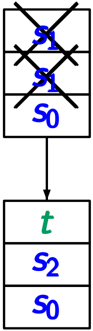
$DFS(s_1, a)$

$DFS(s_1, a)$

$DFS(s_2, a)$

$DFS(t, a)$

stack π



$s_0 \not\models$ "always a" $\longleftarrow$ error indication: $s_0\, s_2\, t$