Model Checking I
alias
Reactive Systems Verification

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MSc in Computer Science, University of Camerino

Topics

- Parallelism
- Interleaving operator for Program Graphs
- Mutual Exclusion
- Peterson Algorithm for Mutual Exclusion

Material

Reading:

Chapter 2 of the book, pages 39–47.

More:

The slides in the following pages are taken from the material of the course "Introduction to Model Checking" held by Prof. Dr. Ir. Joost-Pieter Katoen at Aachen University.

Interleaving for program graphs

Interleaving for program graphs

. . . for modeling parallel systems with
subprocesses communicating via shared variables

Interleaving for program graphs

program graph $\mathcal{P}_1$
$(Loc_1, \ldots, \hookrightarrow_1, \ldots)$

program graph $\mathcal{P}_2$
$(Loc_2, \ldots, \hookrightarrow_2, \ldots)$

interleaving operator

$$\mathcal{P}_1 \,|||\, \mathcal{P}_2 \;=\; (Loc_1 \times Loc_2, \ldots, \hookrightarrow, \ldots)$$

Interleaving for program graphs

program graph $\mathcal{P}_1$
$(Loc_1, \ldots, \hookrightarrow_1, \ldots)$

program graph $\mathcal{P}_2$
$(Loc_2, \ldots, \hookrightarrow_2, \ldots)$

interleaving operator

$$\mathcal{P}_1 \,|||\, \mathcal{P}_2 \;=\; (Loc_1 \times Loc_2, \ldots, \hookrightarrow, \ldots)$$

$$\frac{\ell_1 \overset{g\,:\,\alpha}{\hookrightarrow_1} \ell_1'}{\langle \ell_1, \ell_2 \rangle \overset{g\,:\,\alpha}{\hookrightarrow} \langle \ell_1', \ell_2 \rangle} \qquad \frac{\ell_2 \overset{g\,:\,\alpha}{\hookrightarrow_2} \ell_2'}{\langle \ell_1, \ell_2 \rangle \overset{g\,:\,\alpha}{\hookrightarrow} \langle \ell_1, \ell_2' \rangle}$$

Example: interleaving for PG

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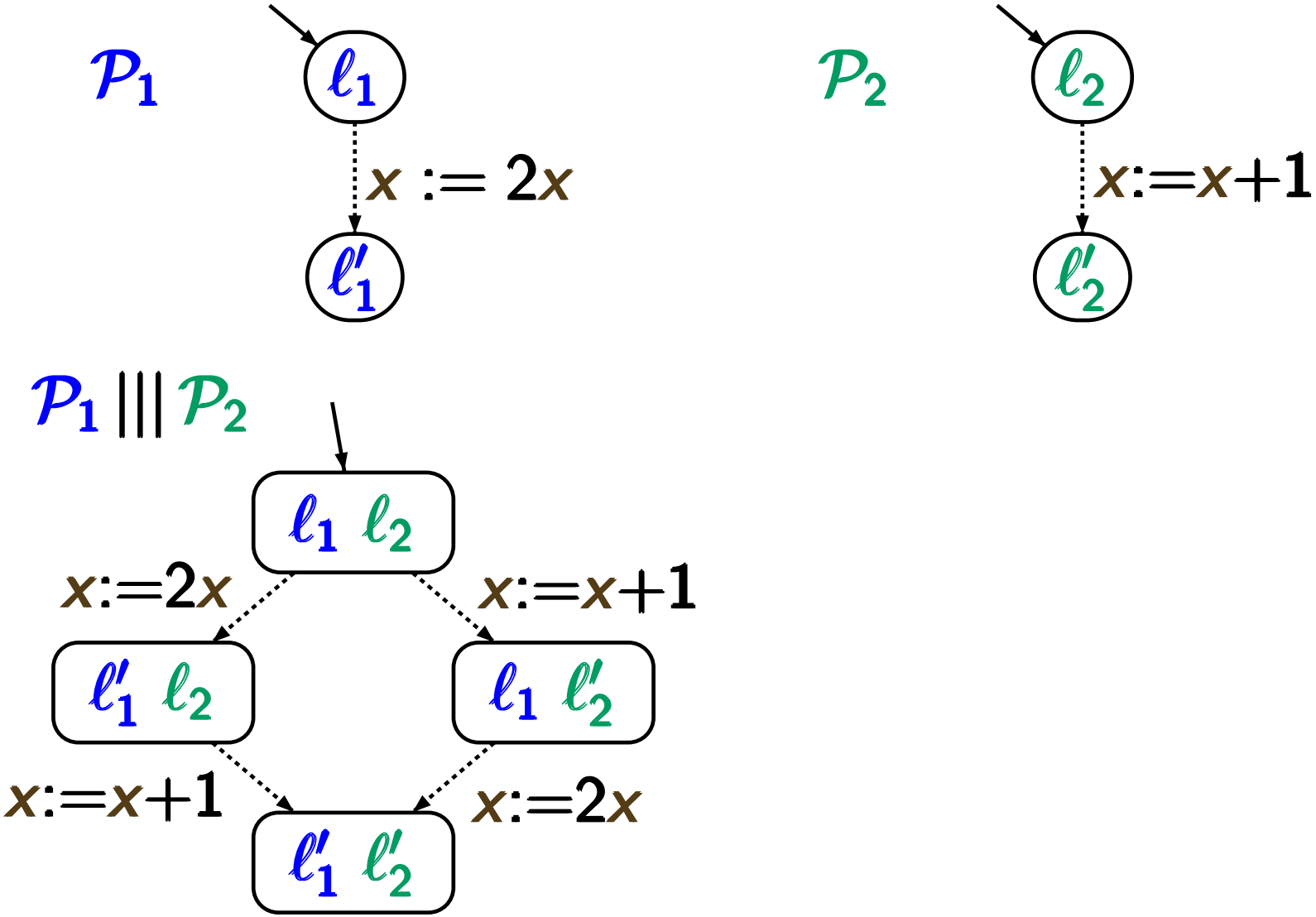

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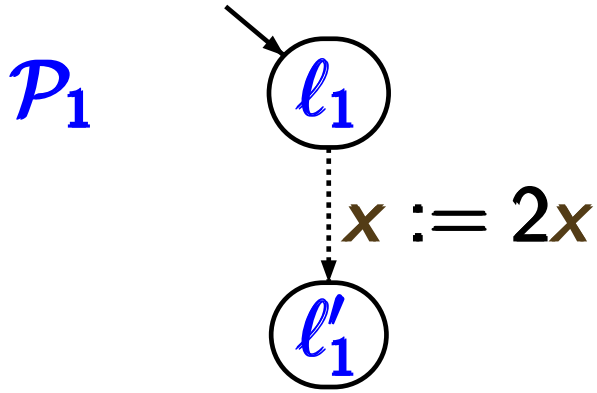

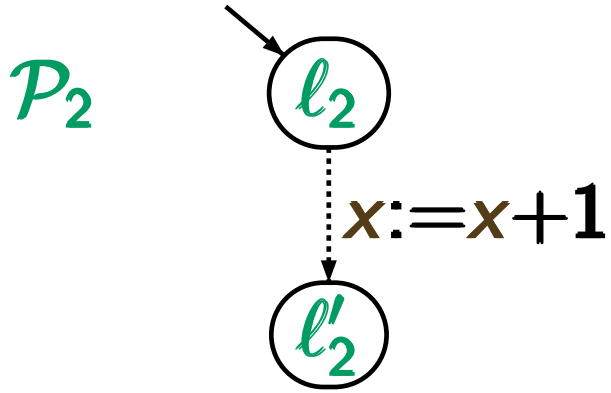

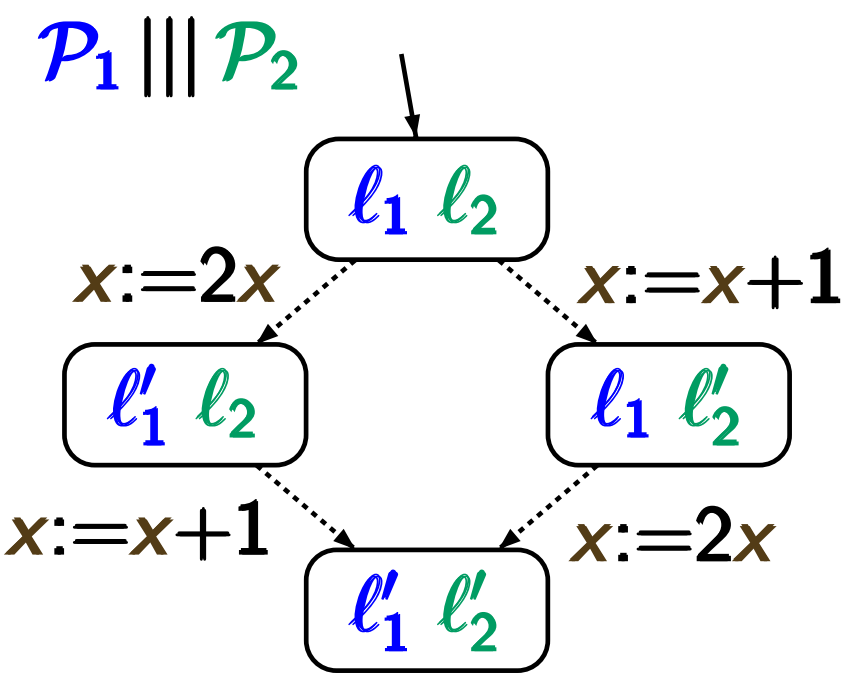

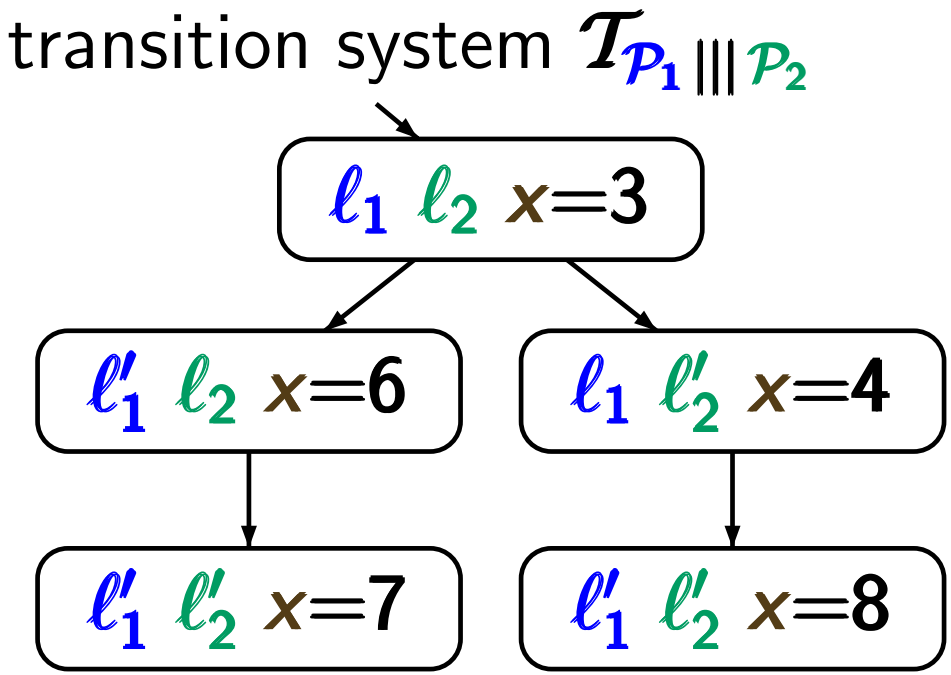

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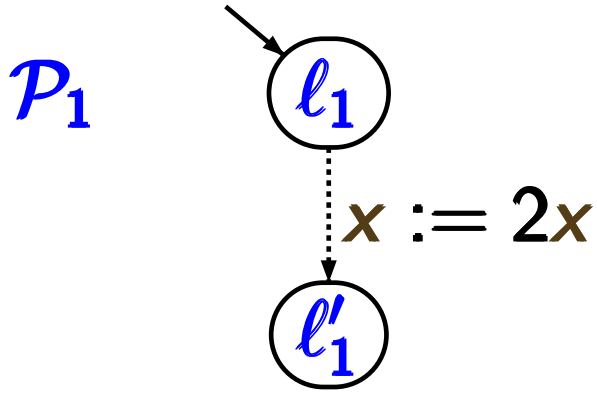
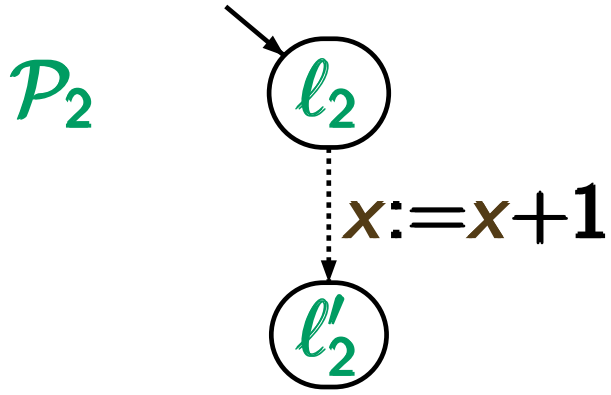
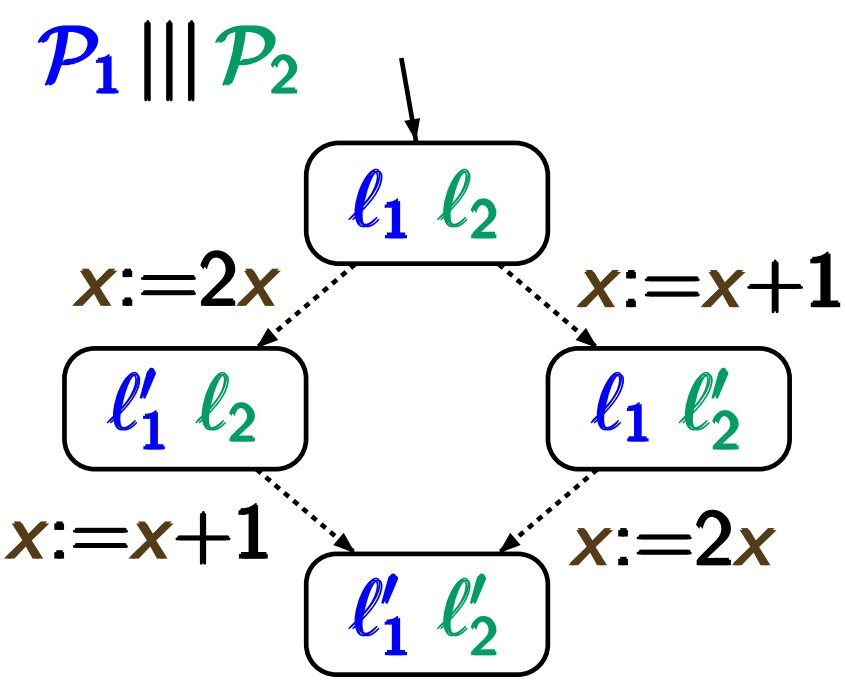
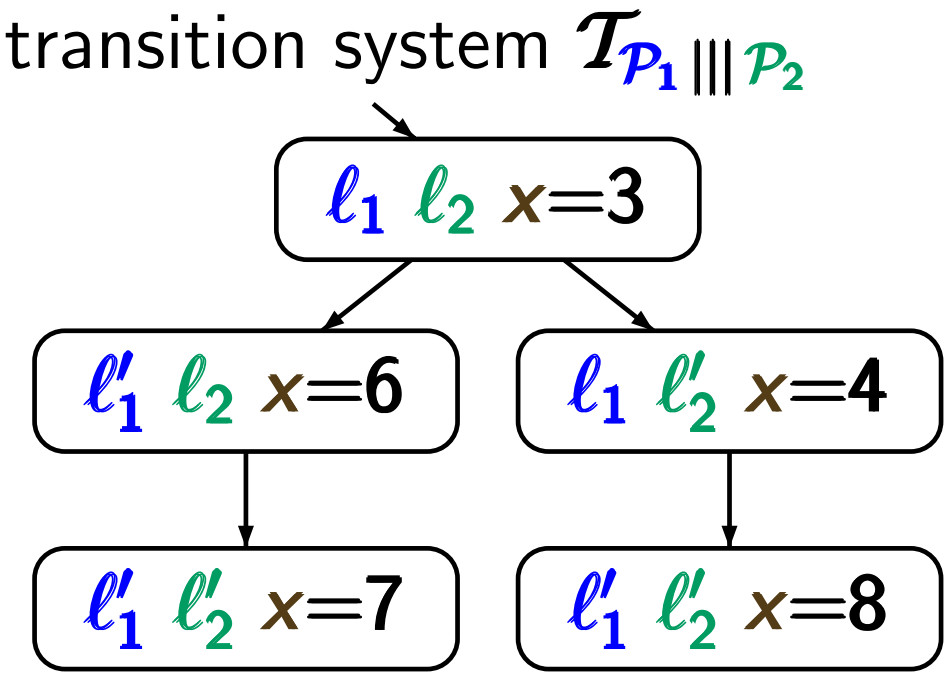


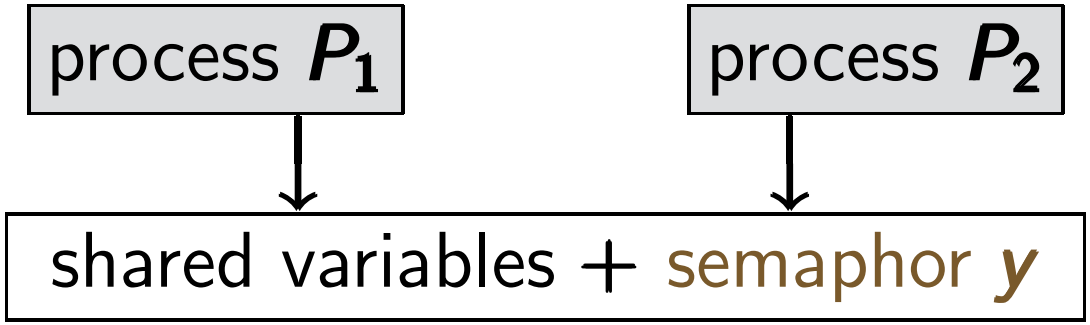






note: $\mathcal{T}_{\mathcal{P}_1 ||| \mathcal{P}_2} \neq \mathcal{T}_{\mathcal{P}_1} ||| \mathcal{T}_{\mathcal{P}_2}$

Mutual exclusion with semaphore



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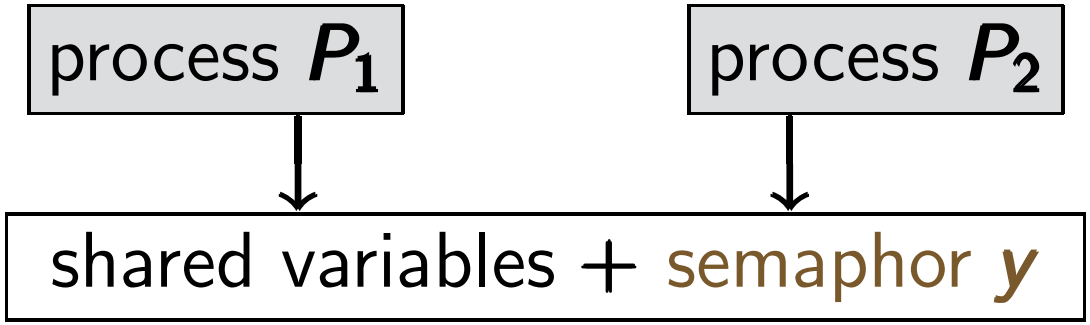

protocol for process P_i

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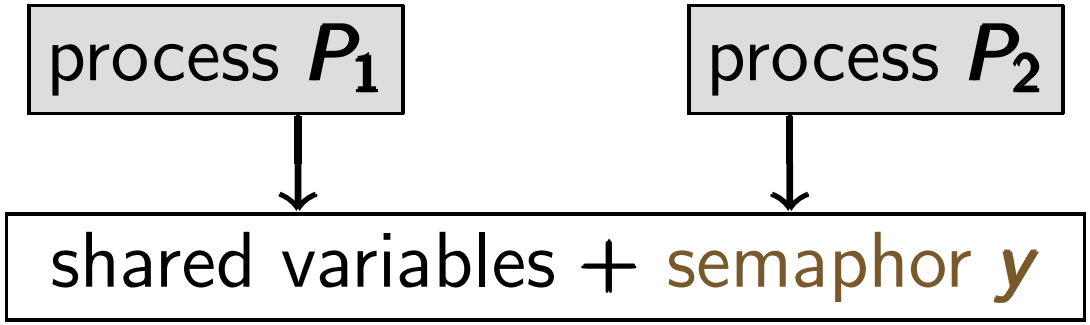

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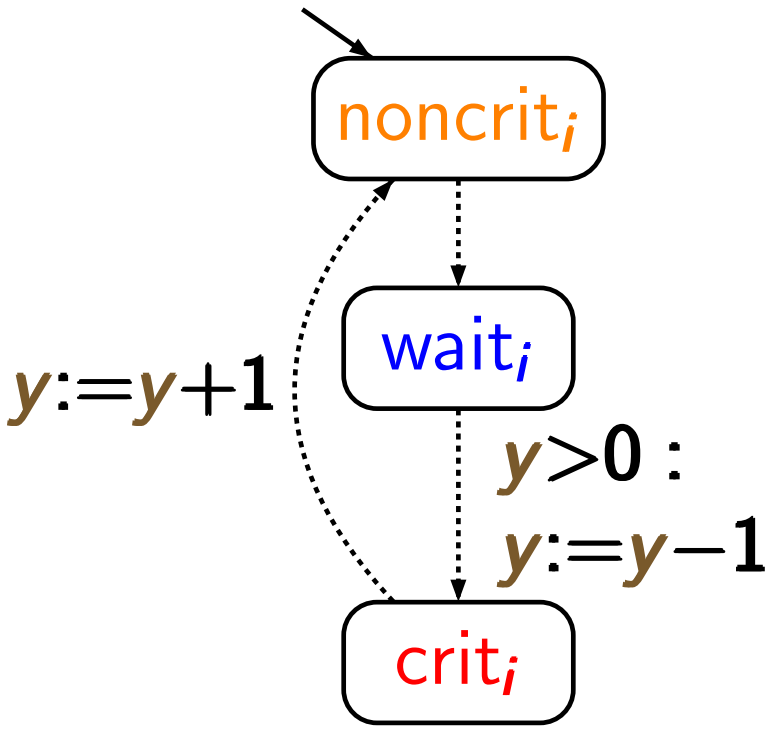

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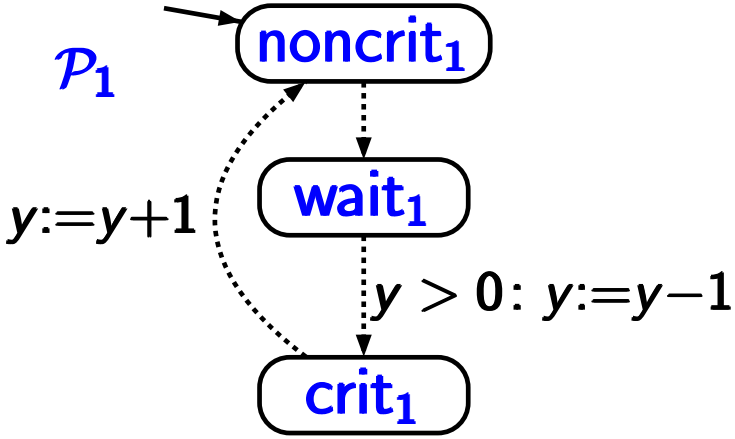

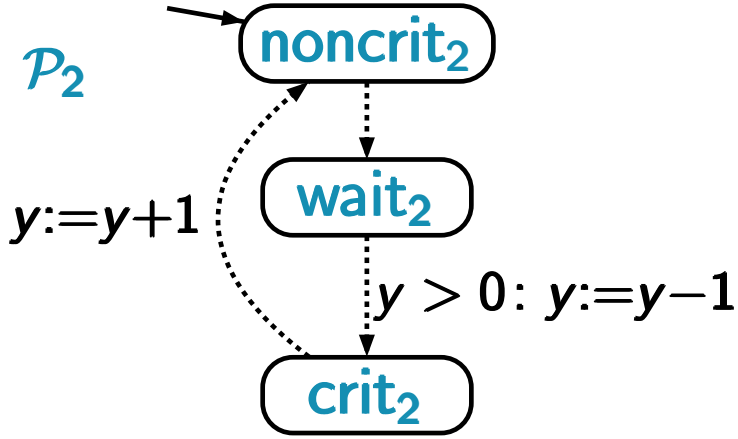

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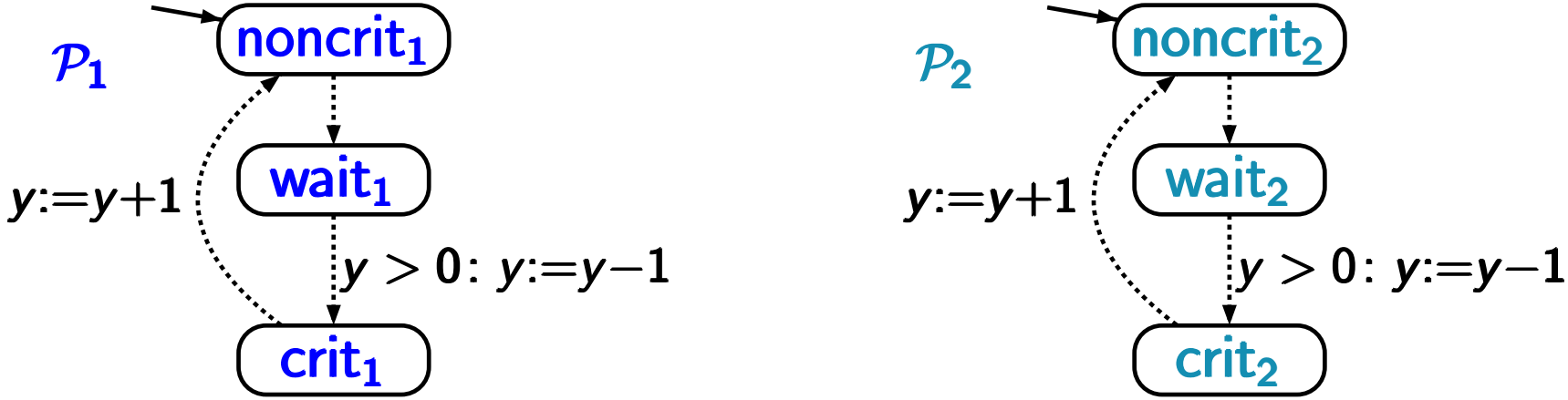
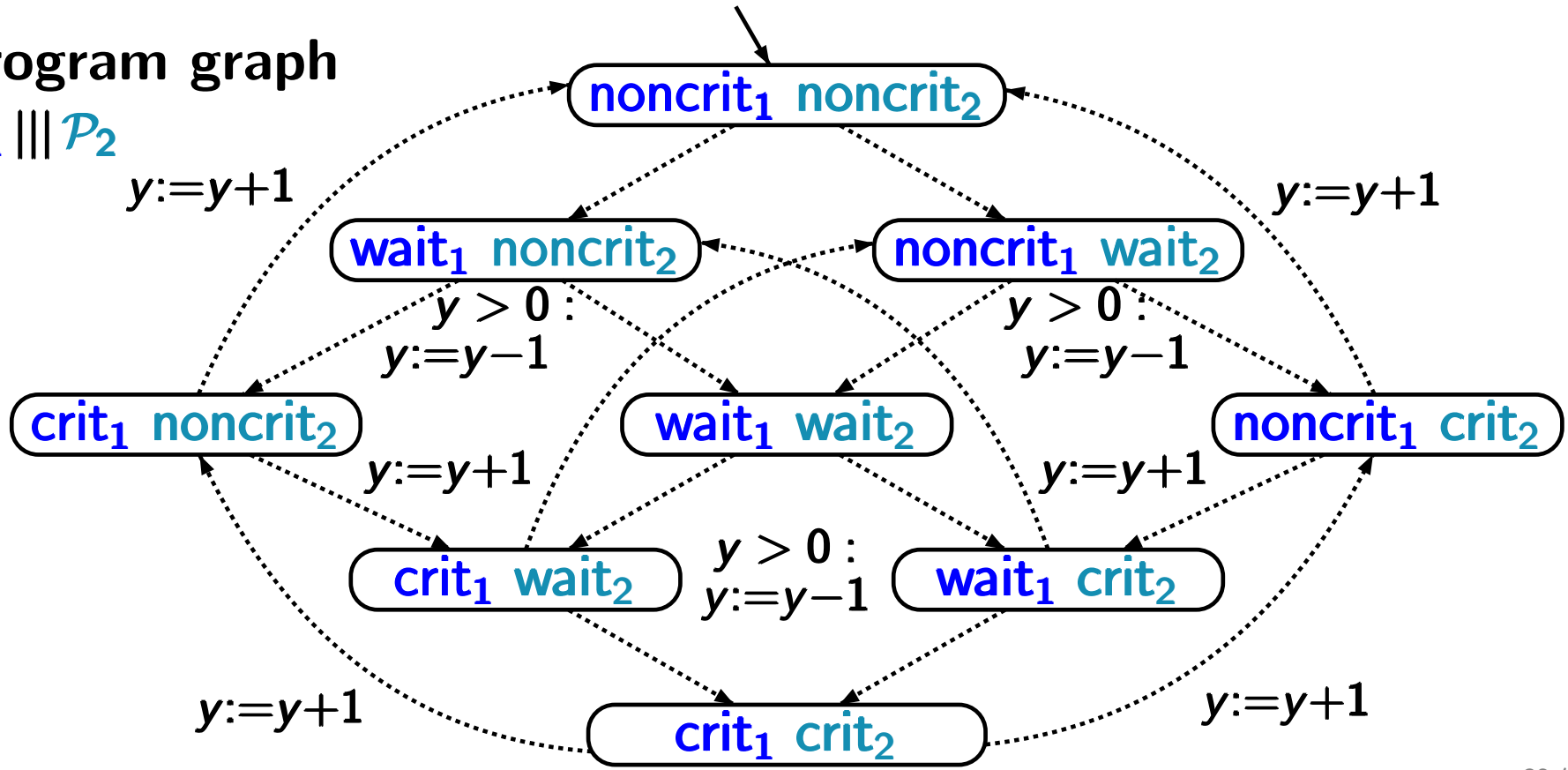


program graph

$\mathcal{P}_1 \,|||\, \mathcal{P}_2$



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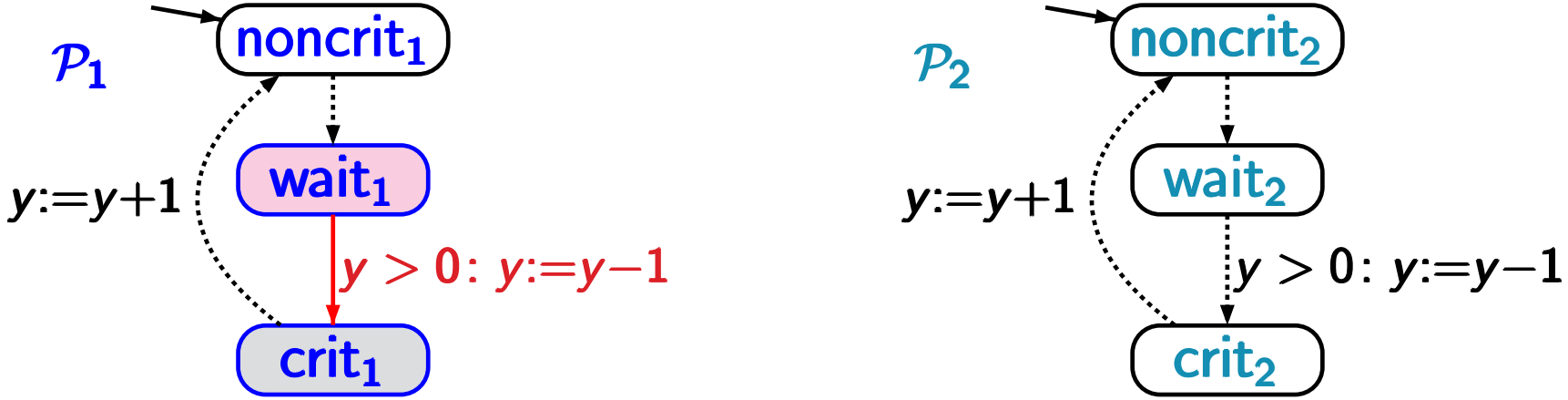

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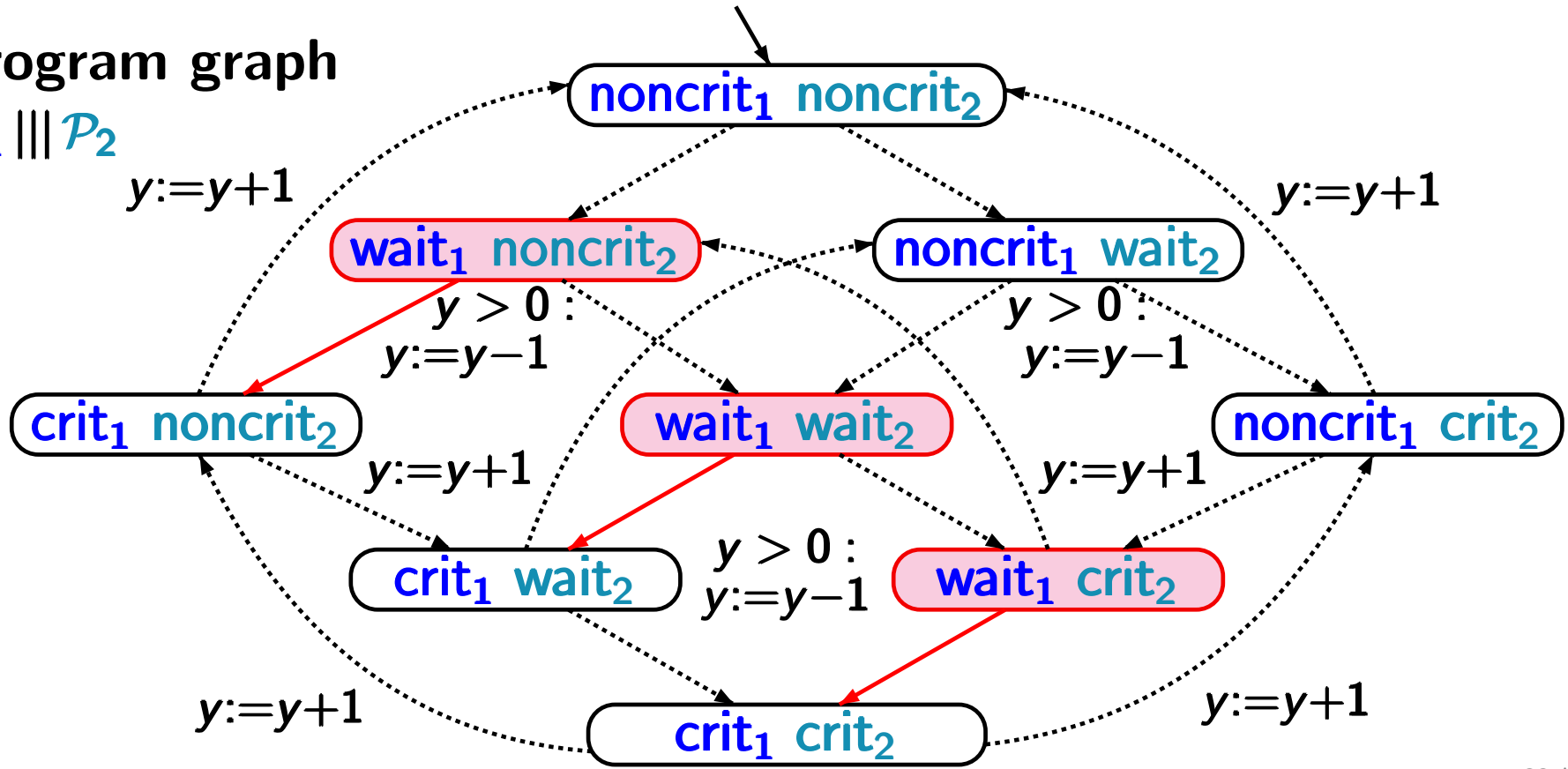
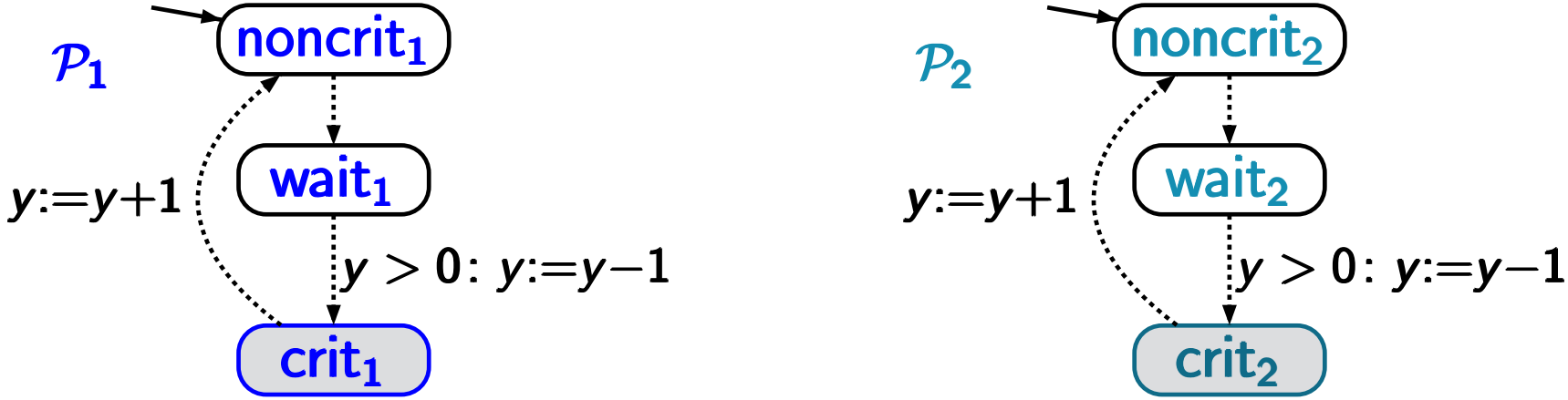


Mutual exclusion with semaphore



program graph

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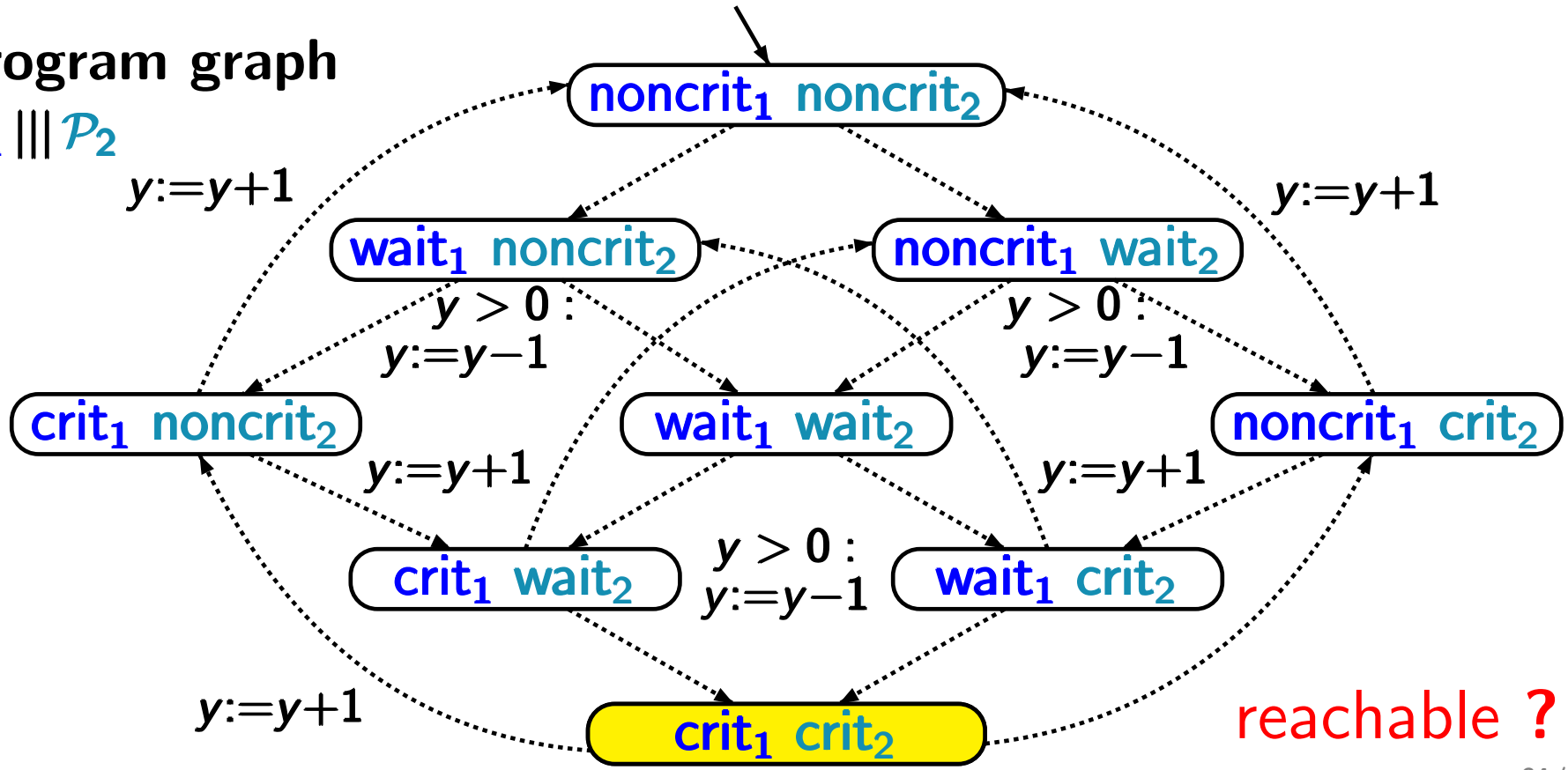
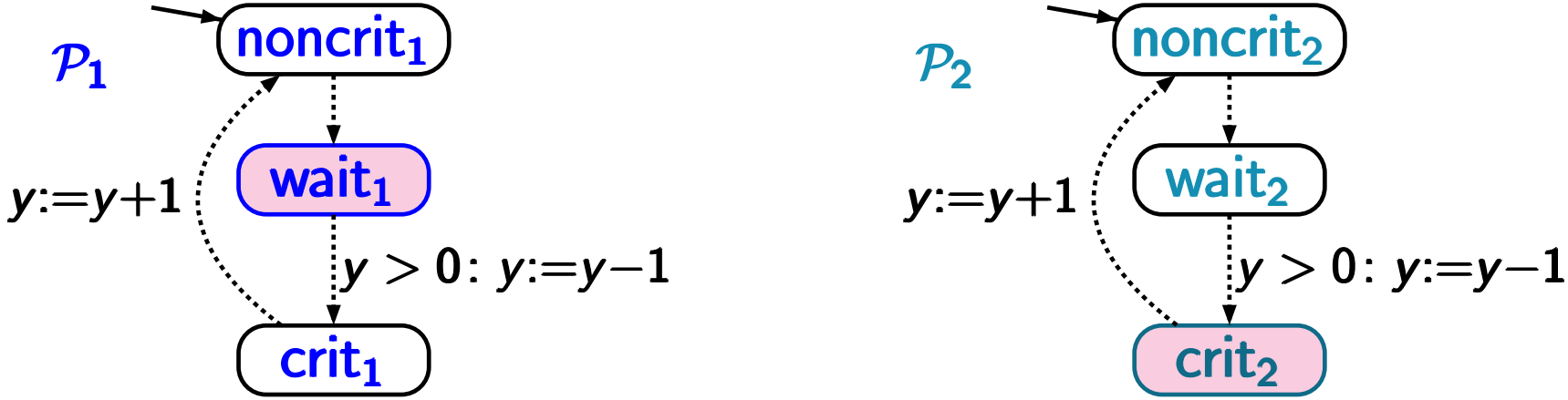


reachable ?

Mutual exclusion with semaphore



program graph

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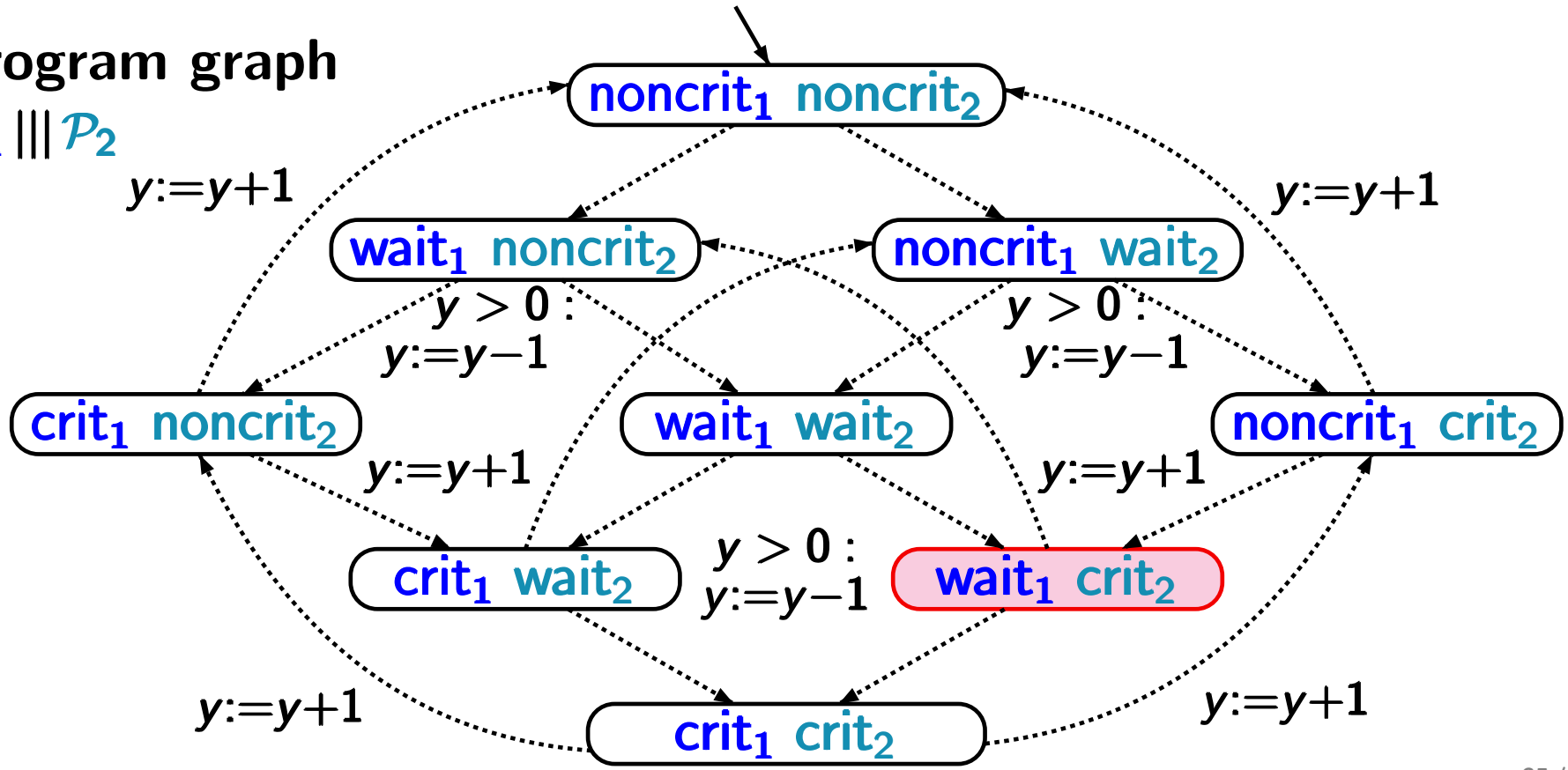
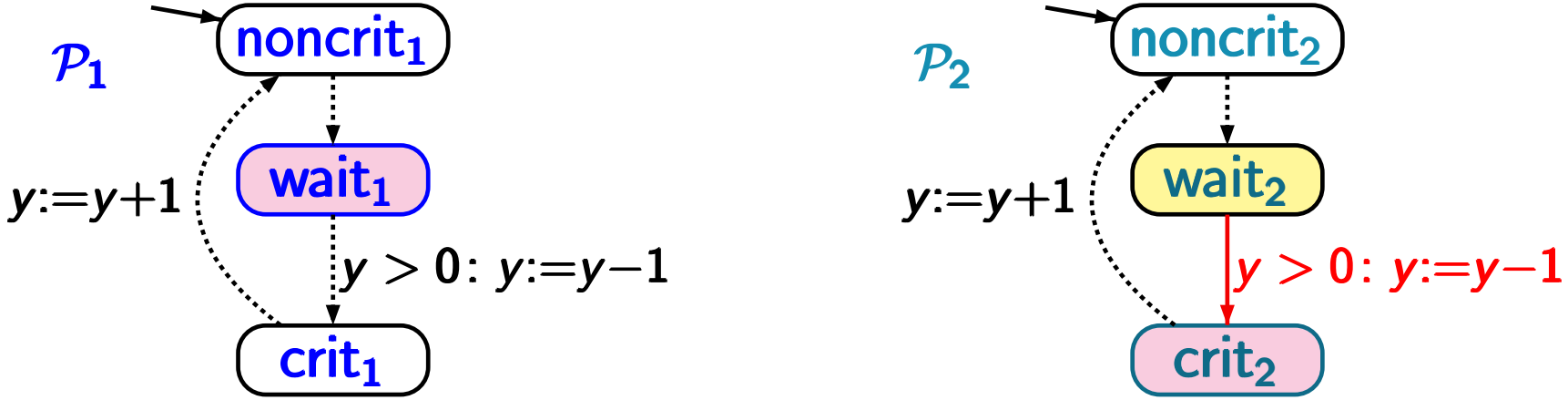
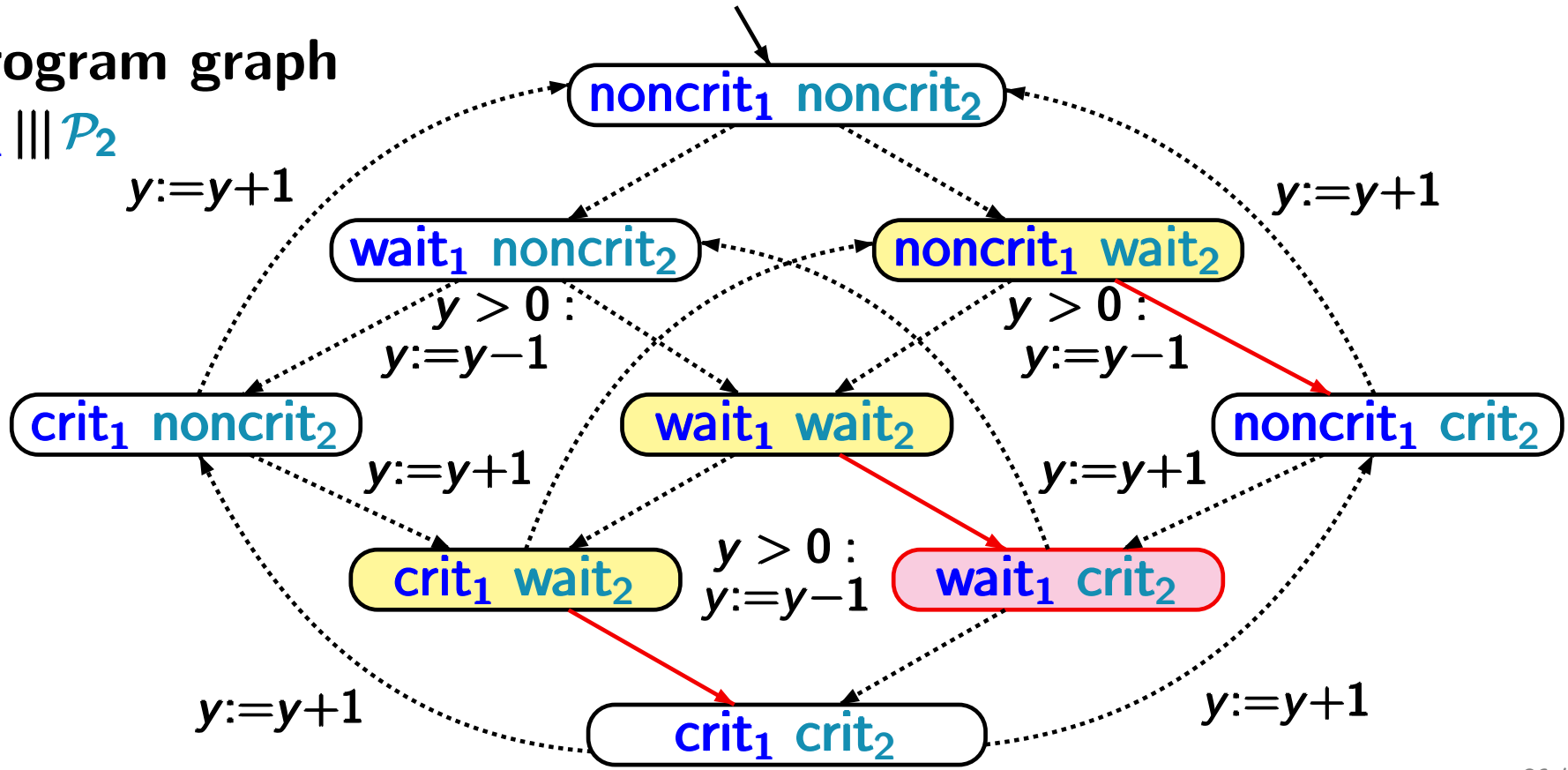


Mutual exclusion with semaphore

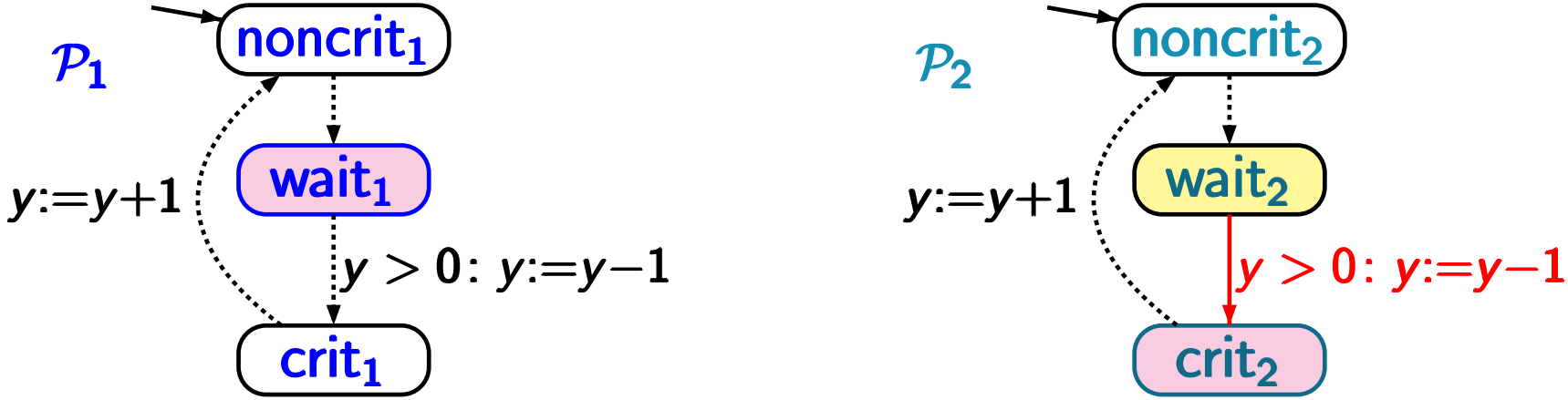


program graph

$\mathcal{P}_1 \,|||\, \mathcal{P}_2$



Mutual exclusion with semaphore



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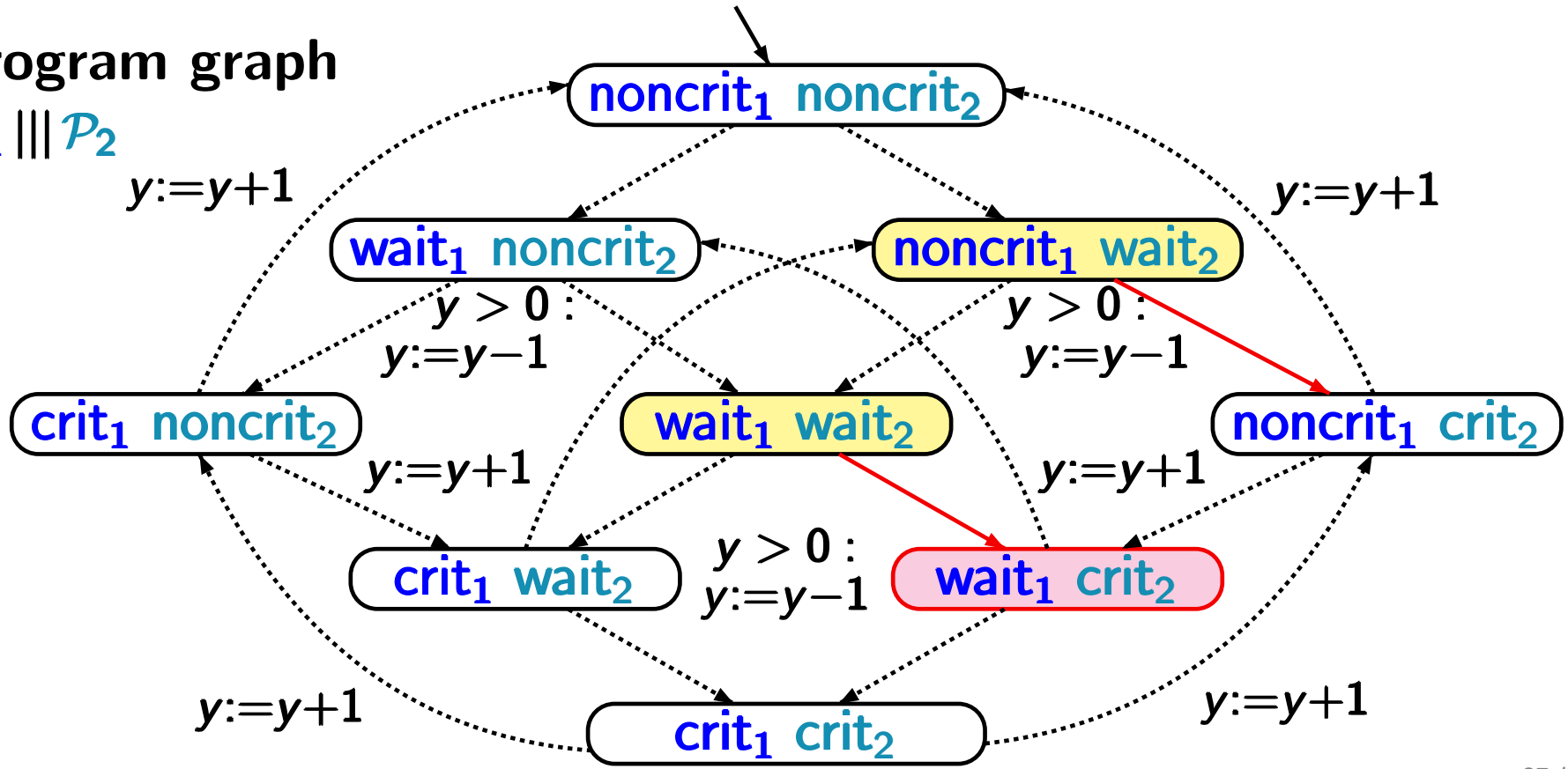

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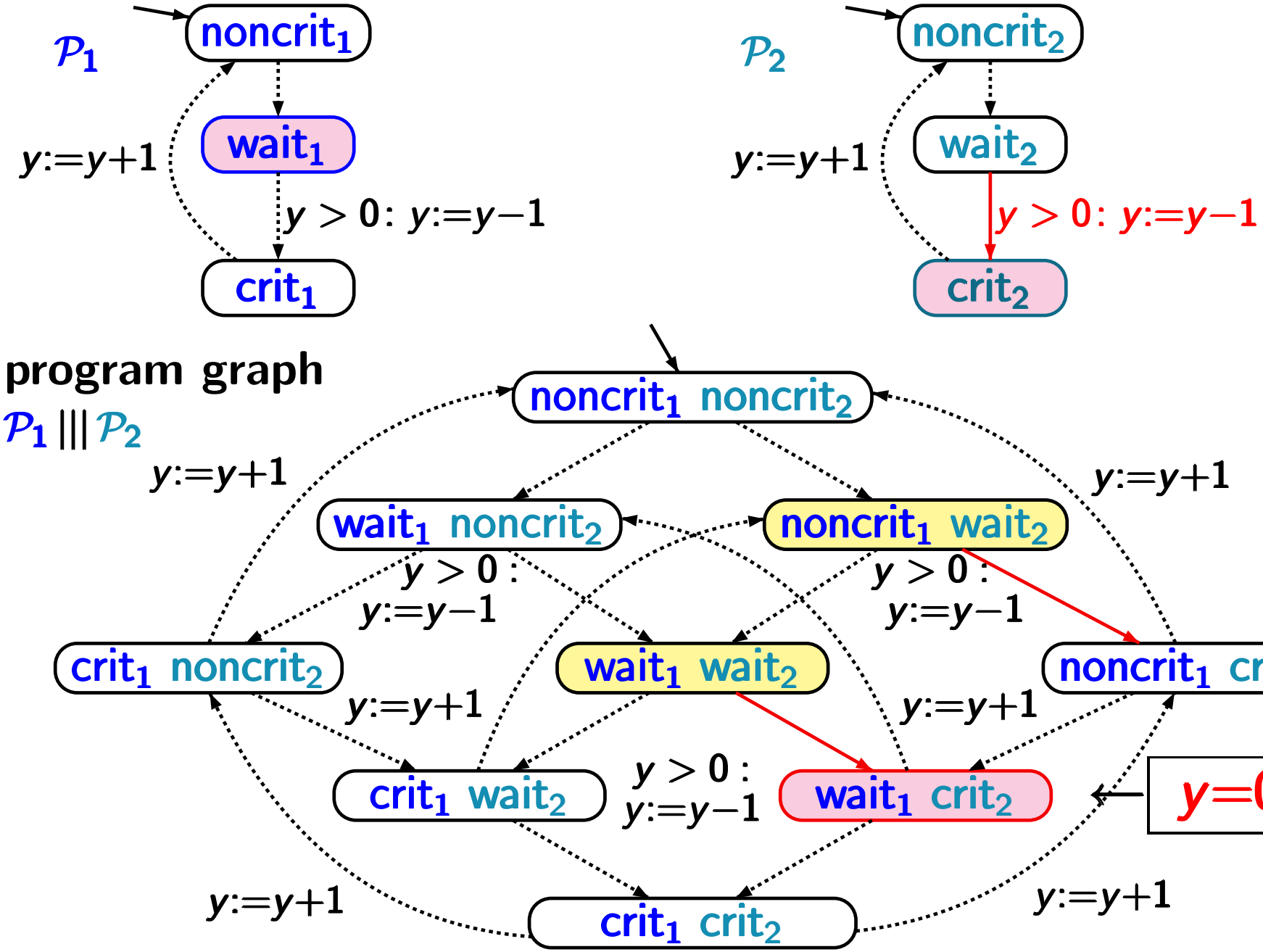

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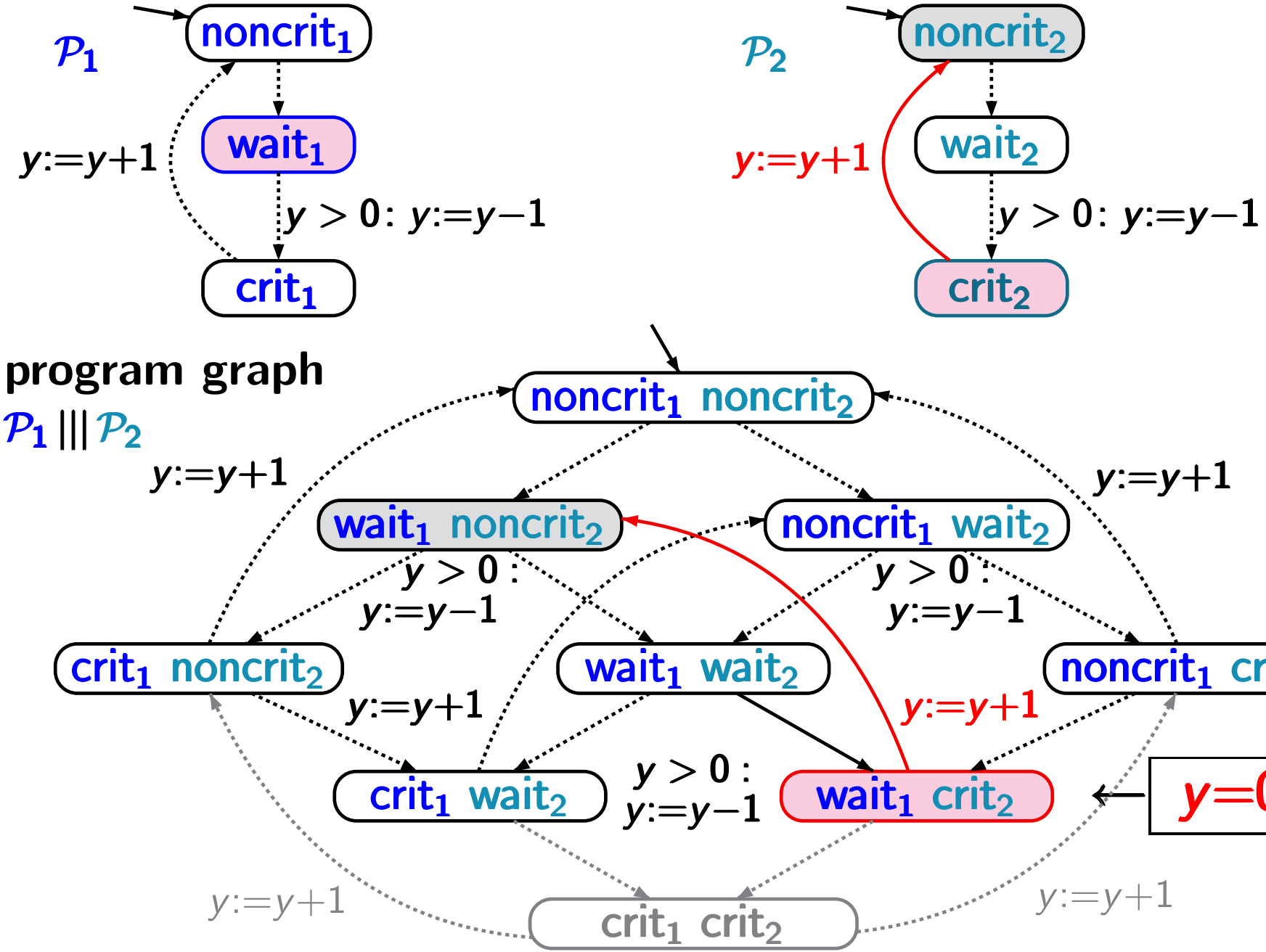
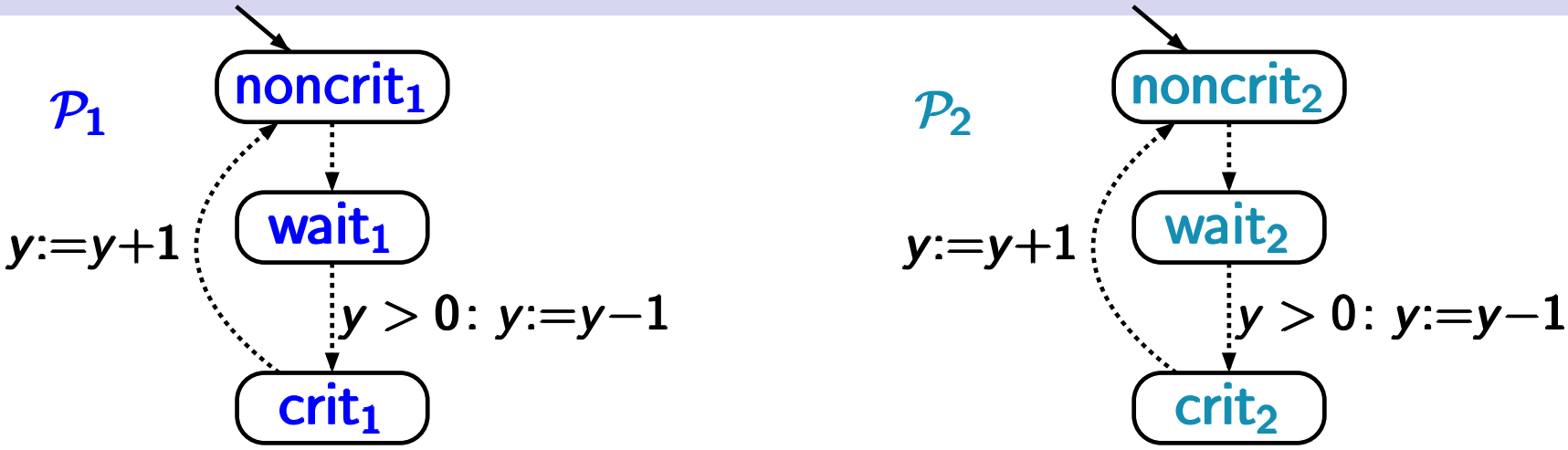
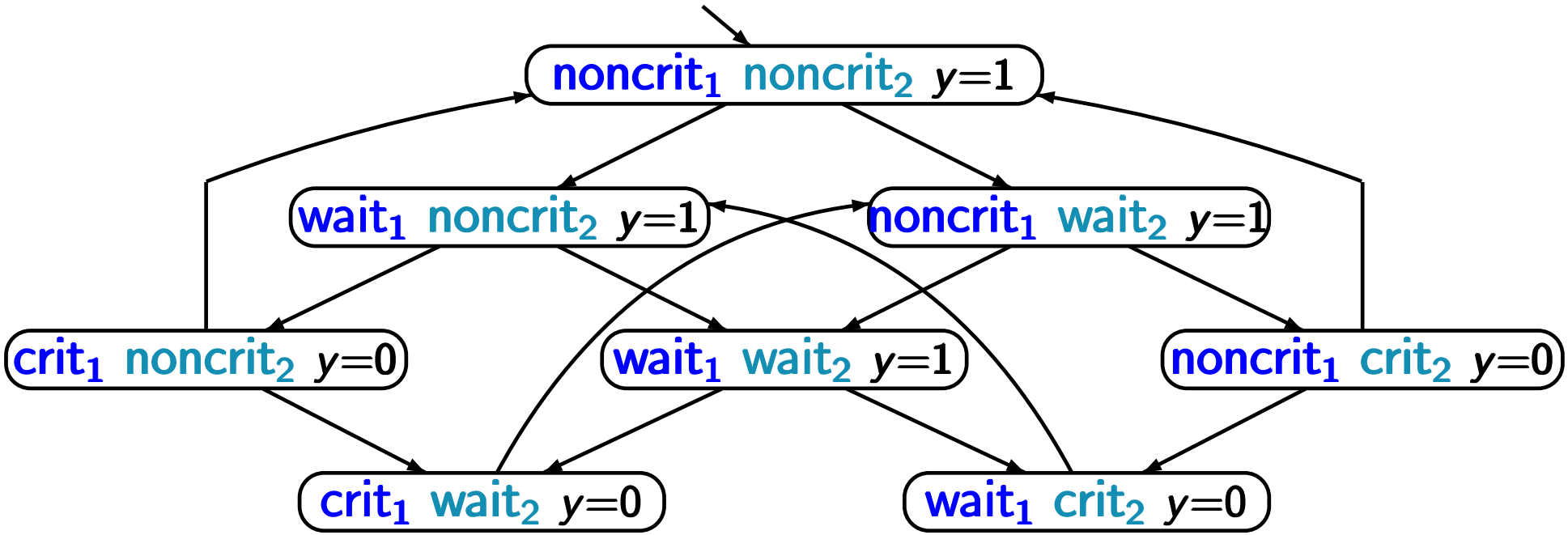


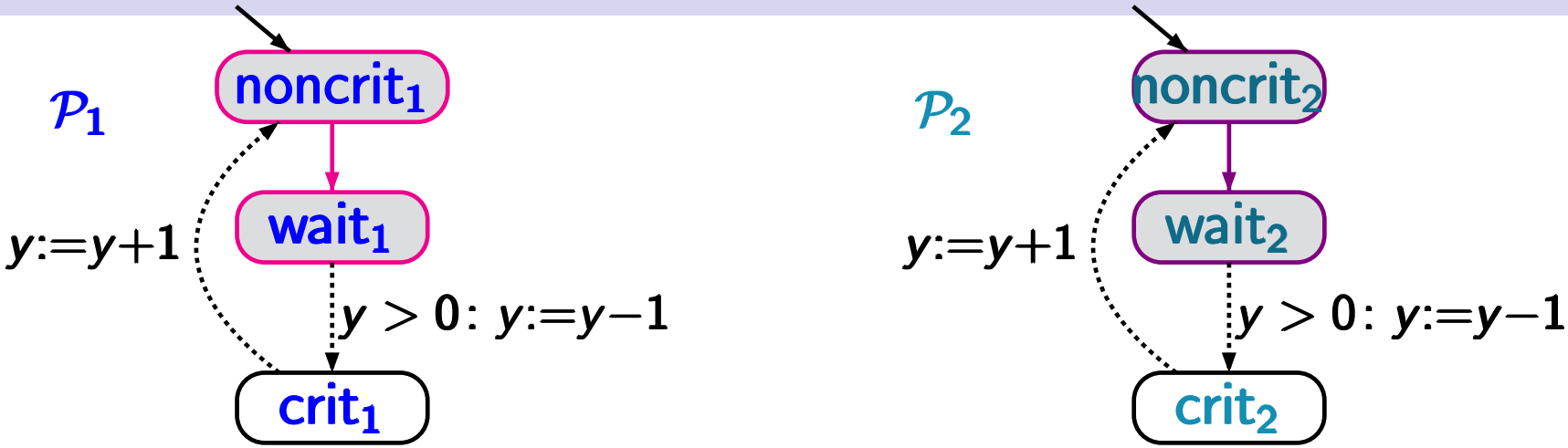
TS for mutual exclusion with semaphore



reachable fragment of the transition system $\mathcal{T}_{\mathcal{P}_1 \,|||\, \mathcal{P}_2}$



Concurrency of the request actions



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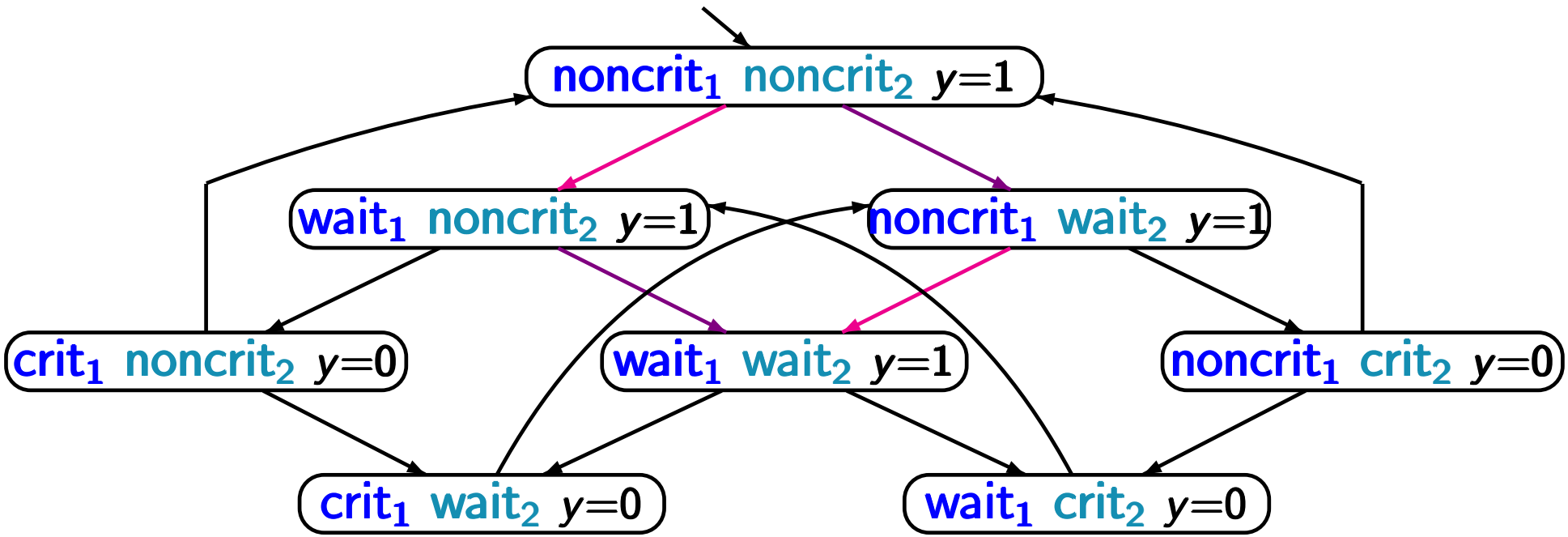

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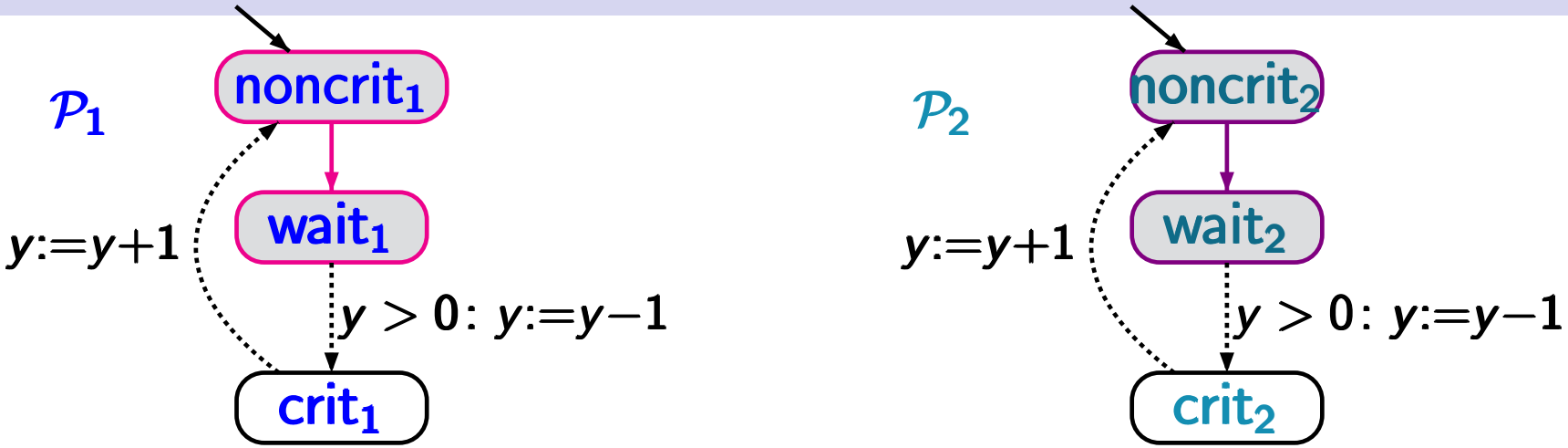
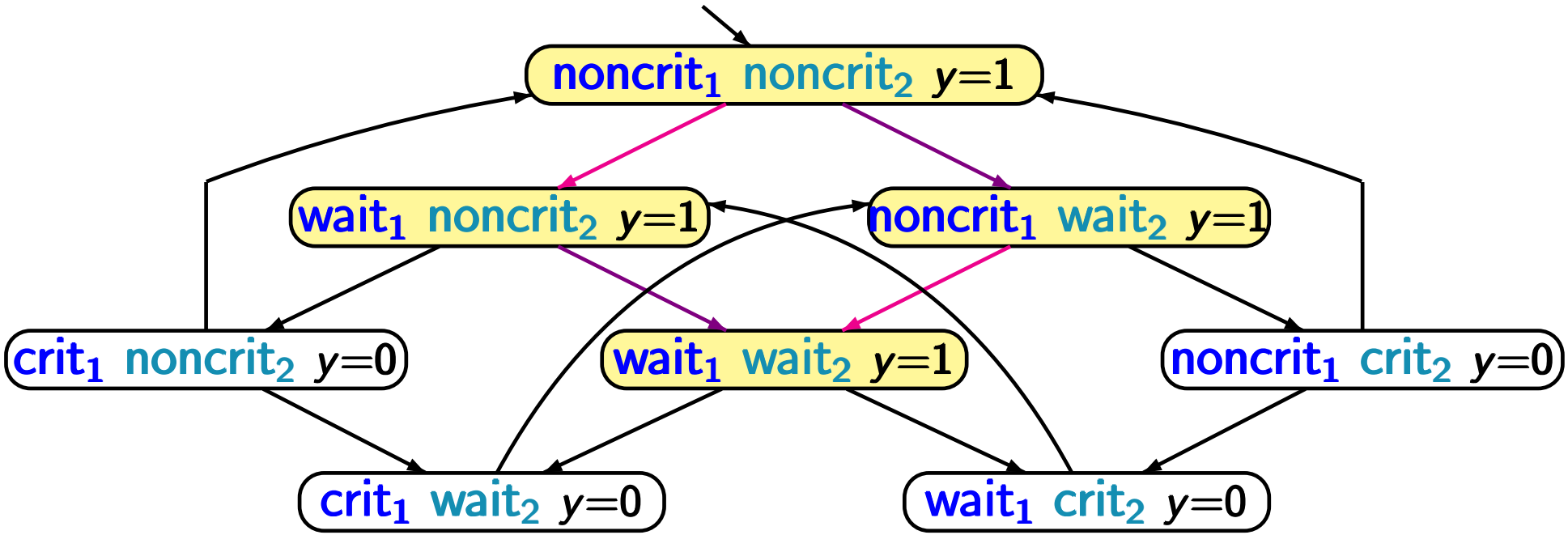
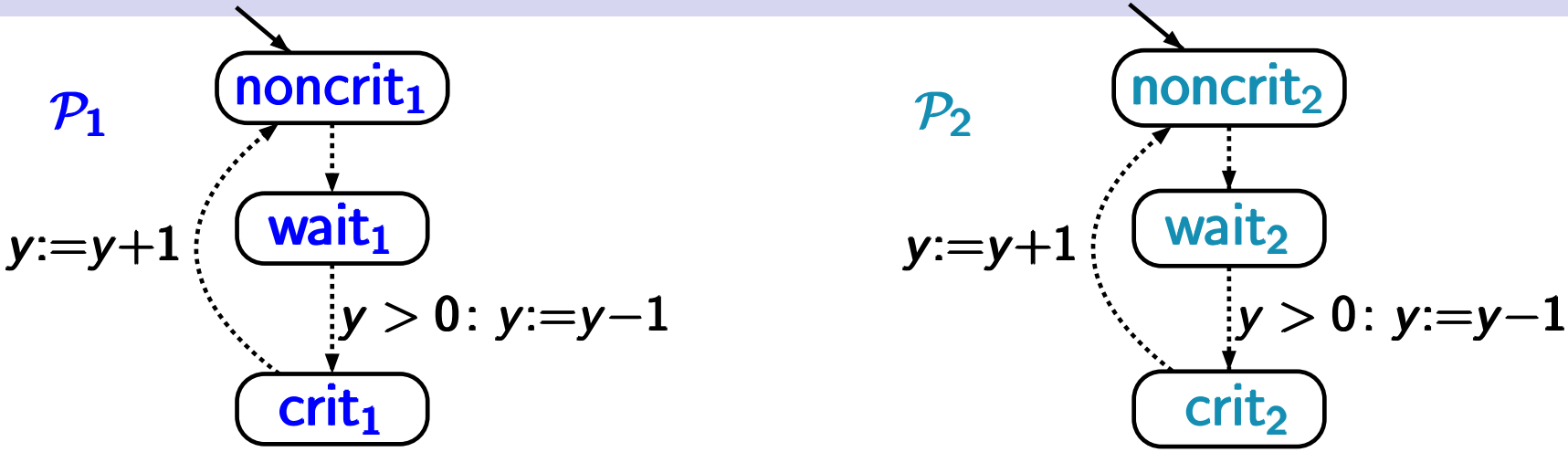


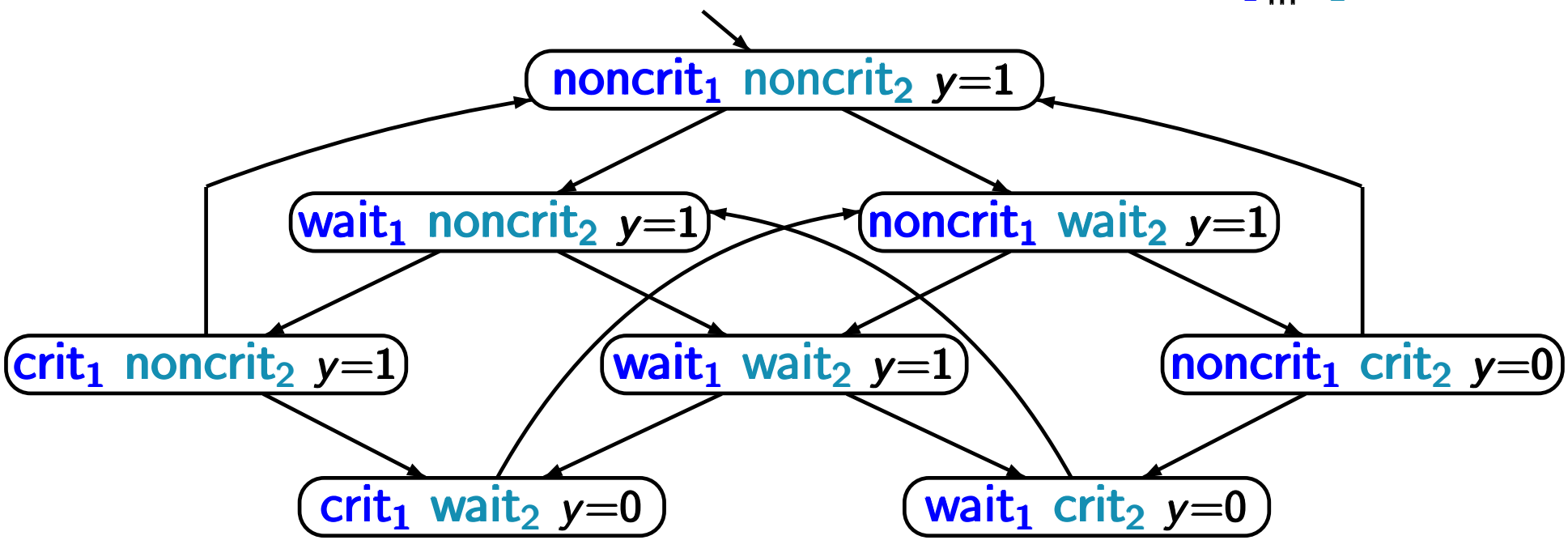
interleaving of the independent request actions



Competition



reachable fragment of the transition system $\mathcal{T}_{\mathcal{P}_1 \,|||\, \mathcal{P}_2}$



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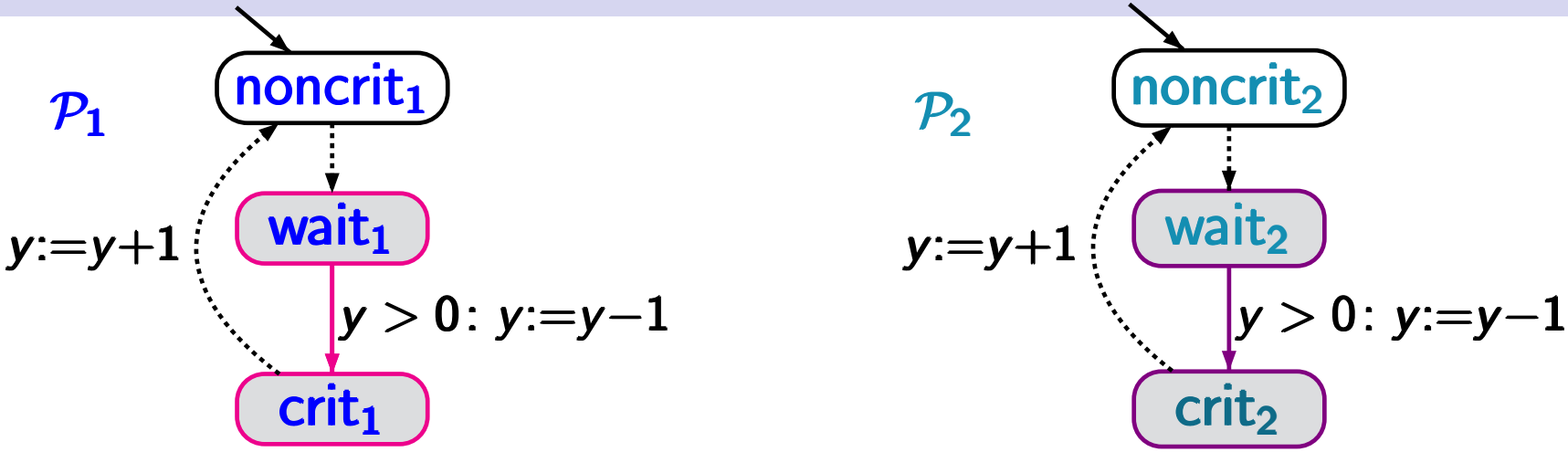

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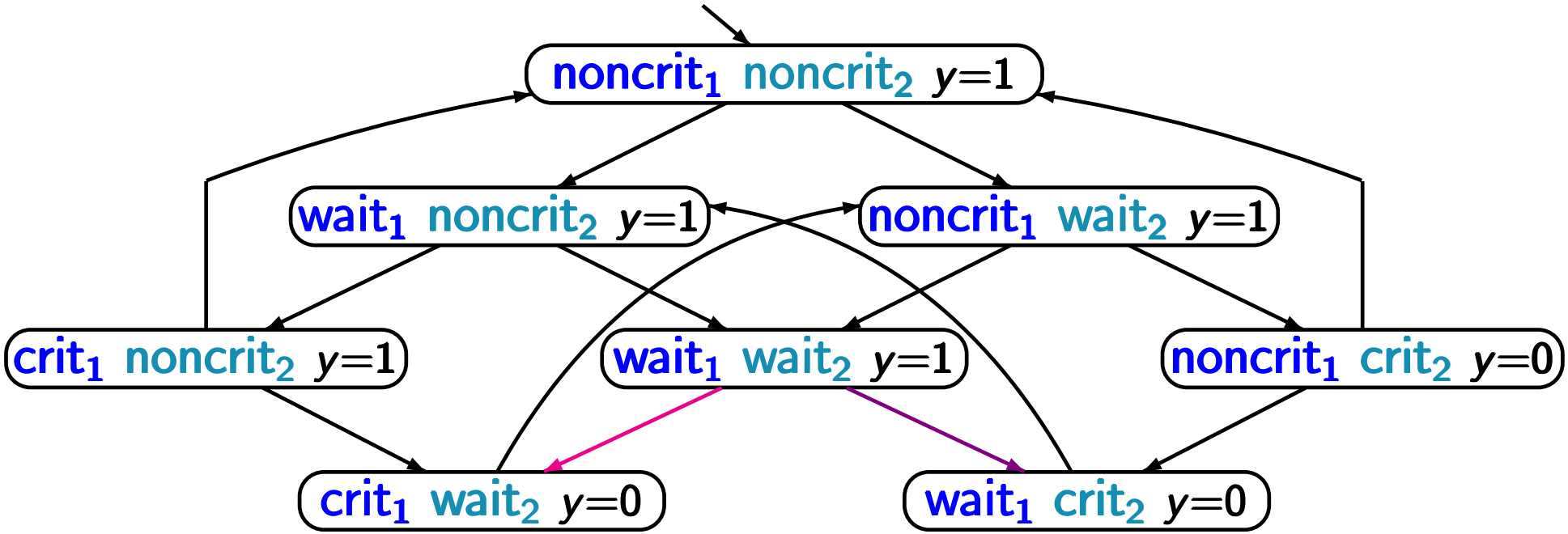

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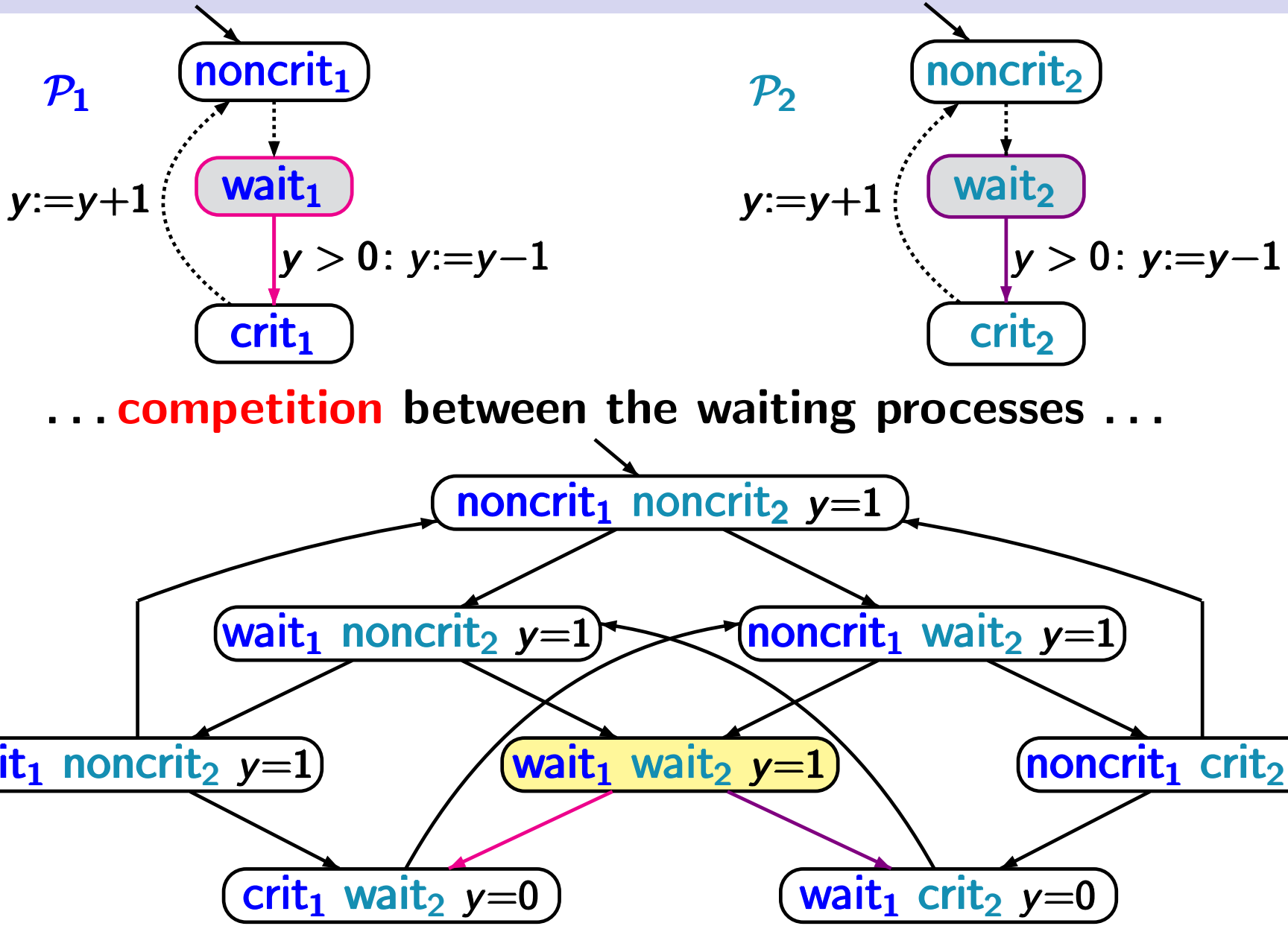

Peterson algorithm for mutual exclusion

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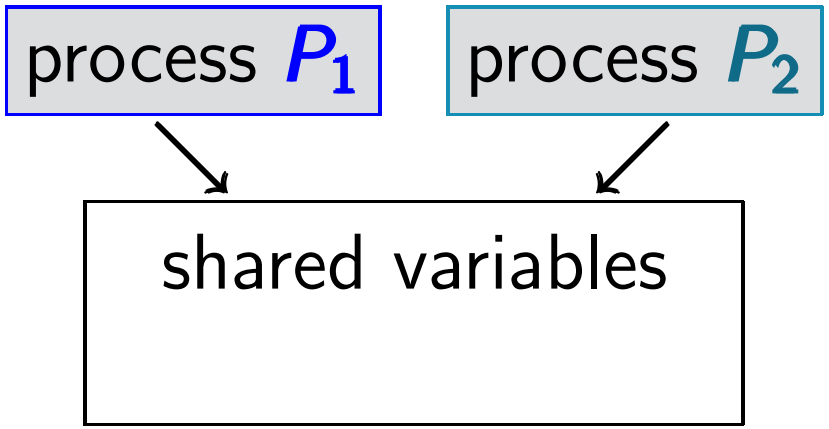

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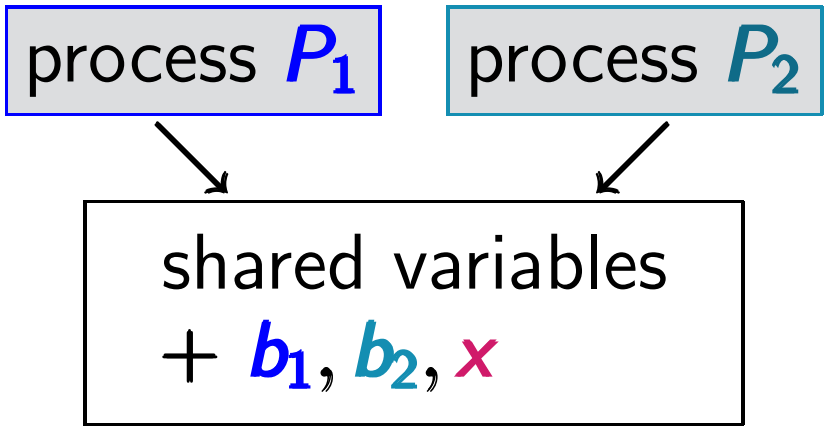

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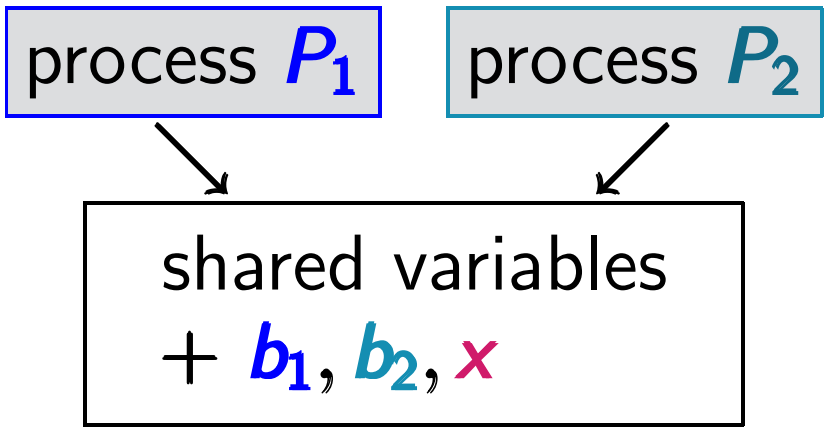

b_1, b_2 Boolean variables, $x \in \{1, 2\}$

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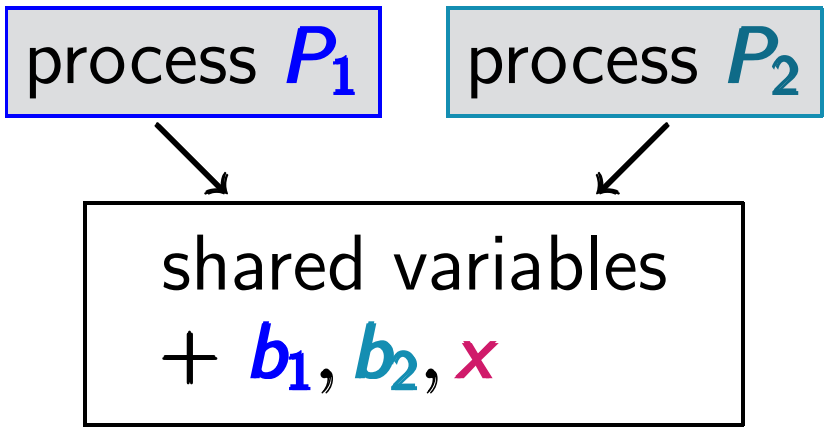

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```
LOOP FOREVER                                   (* protocol for P1 *)
    noncritical actions;
    b1:=1 ; x:=2;
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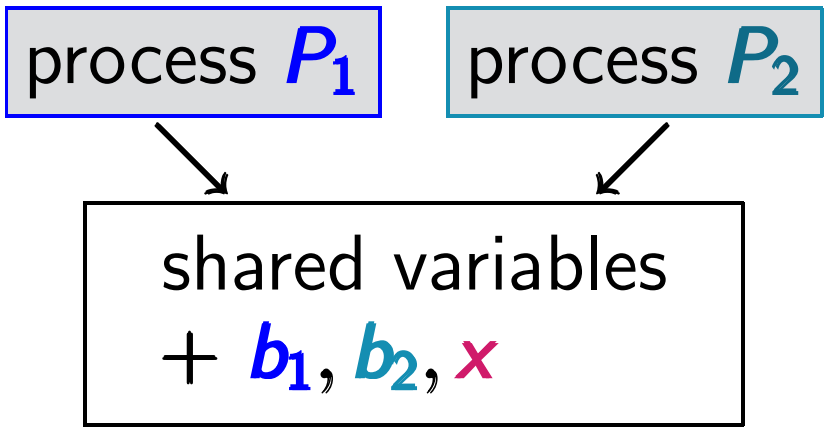

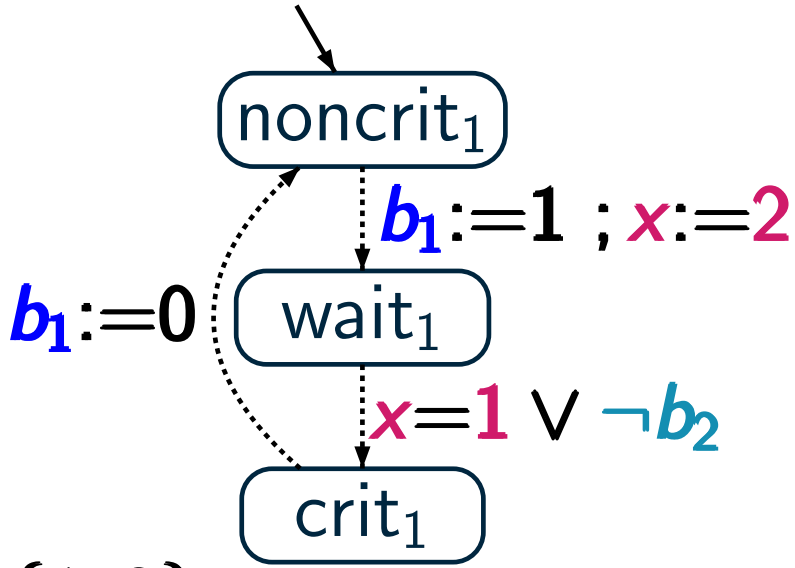

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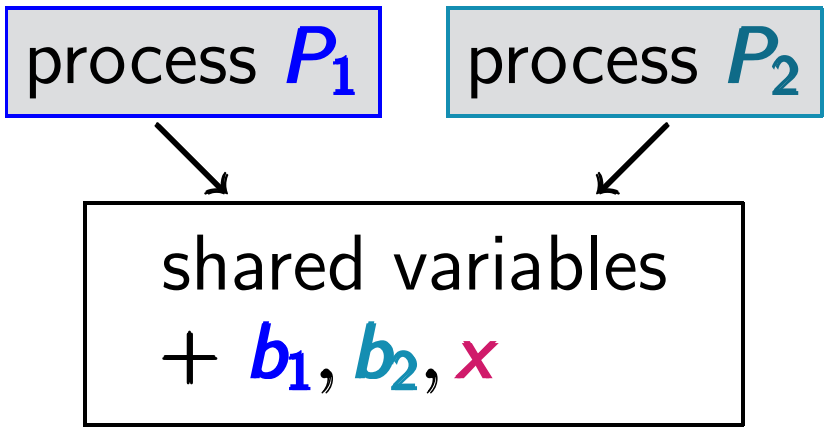

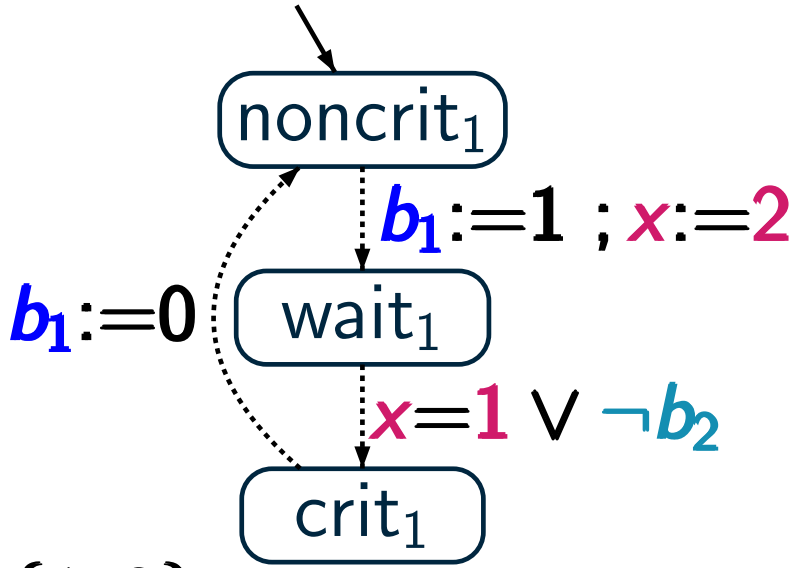

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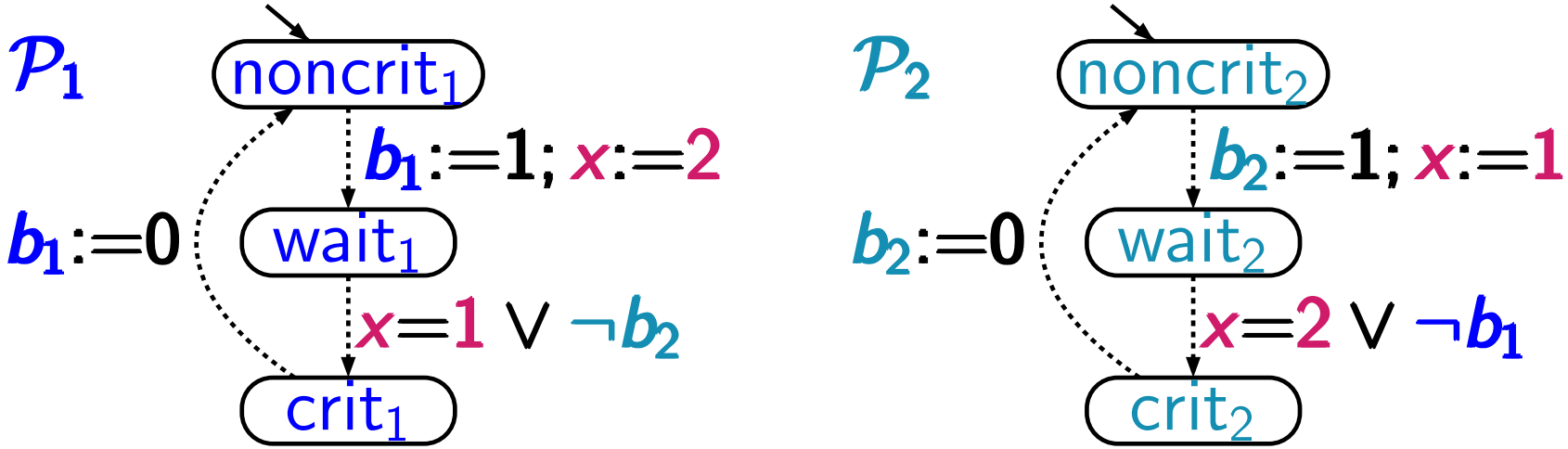
Program graphs for Peterson algorithm



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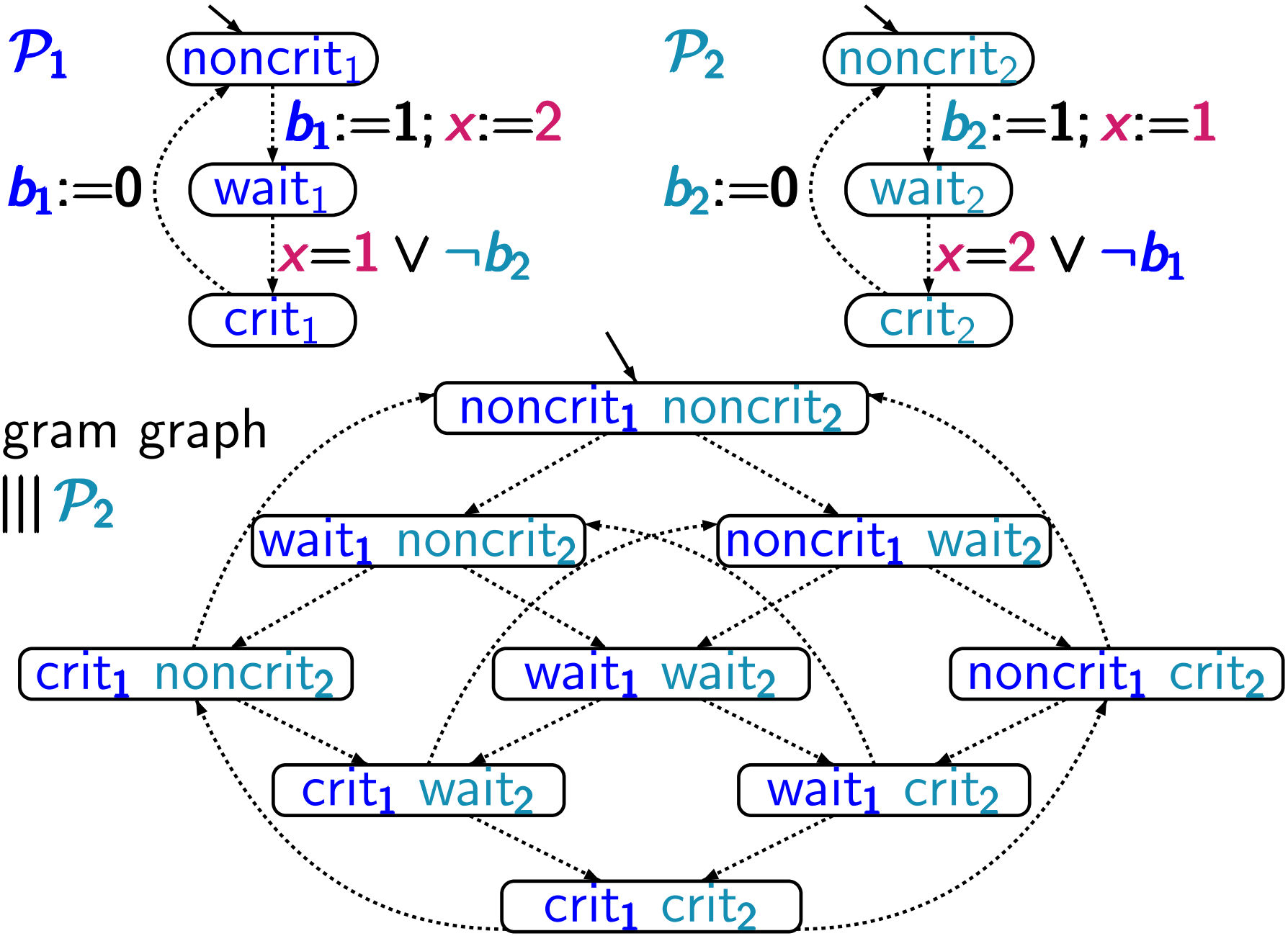
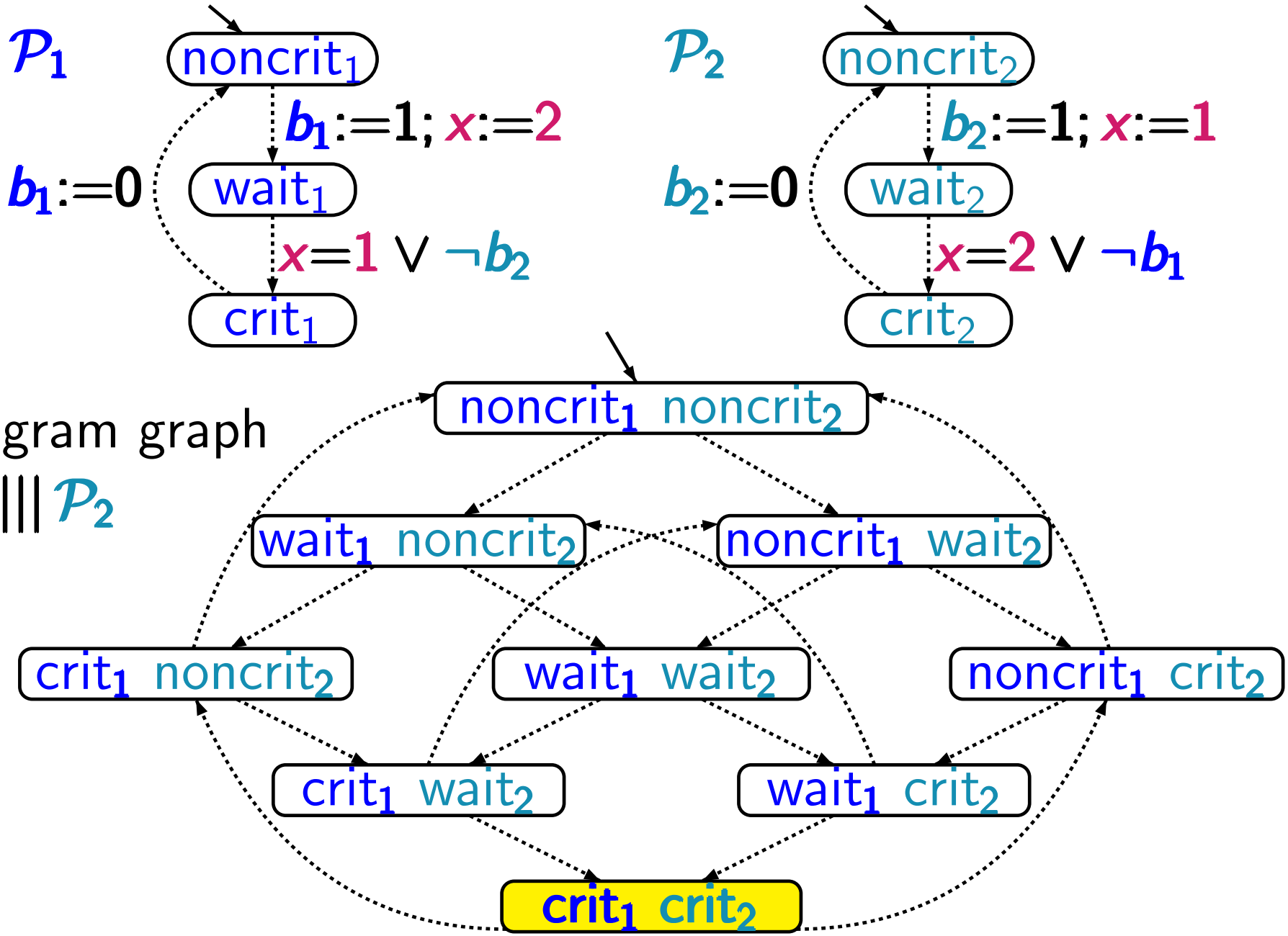
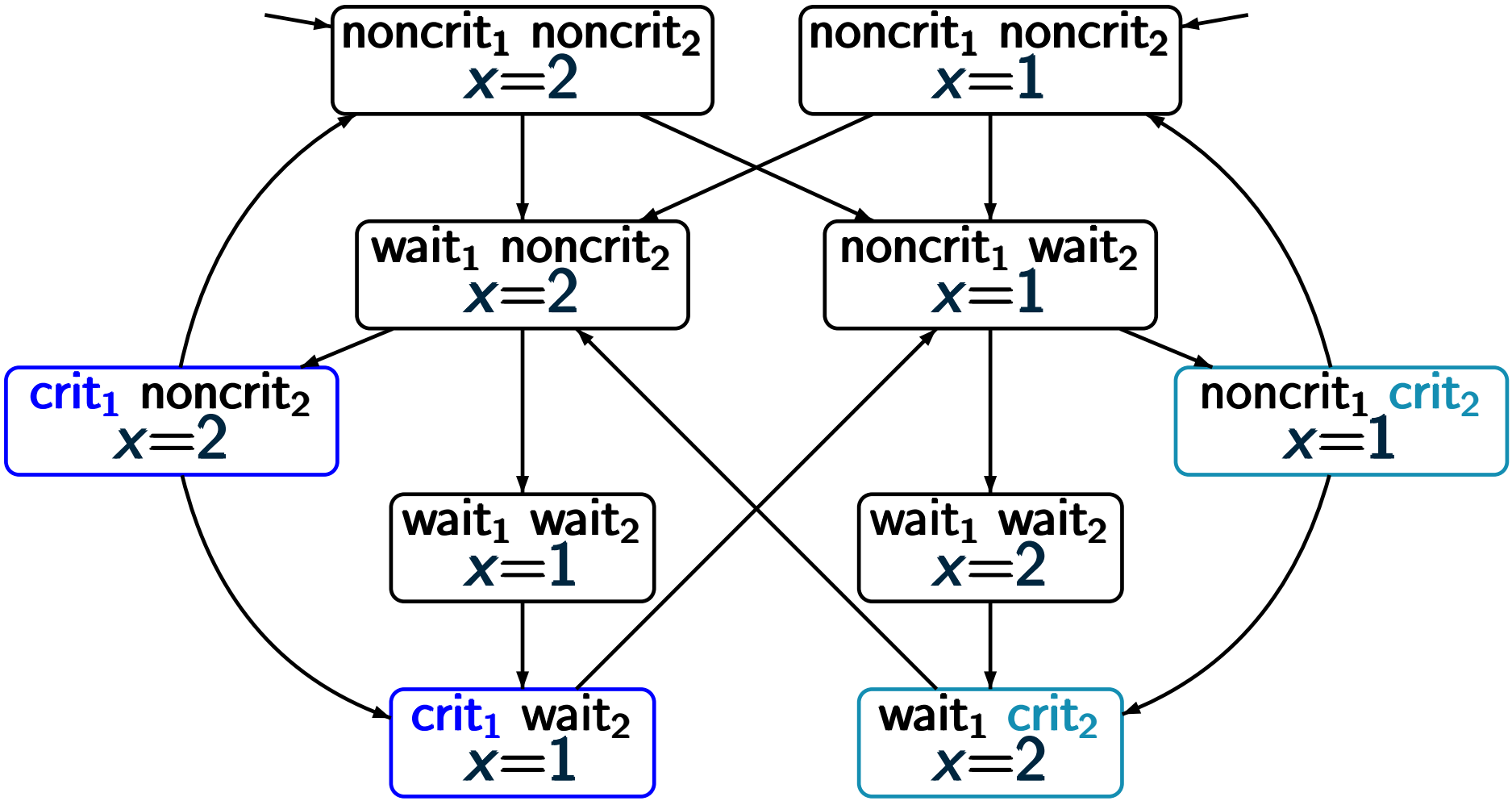


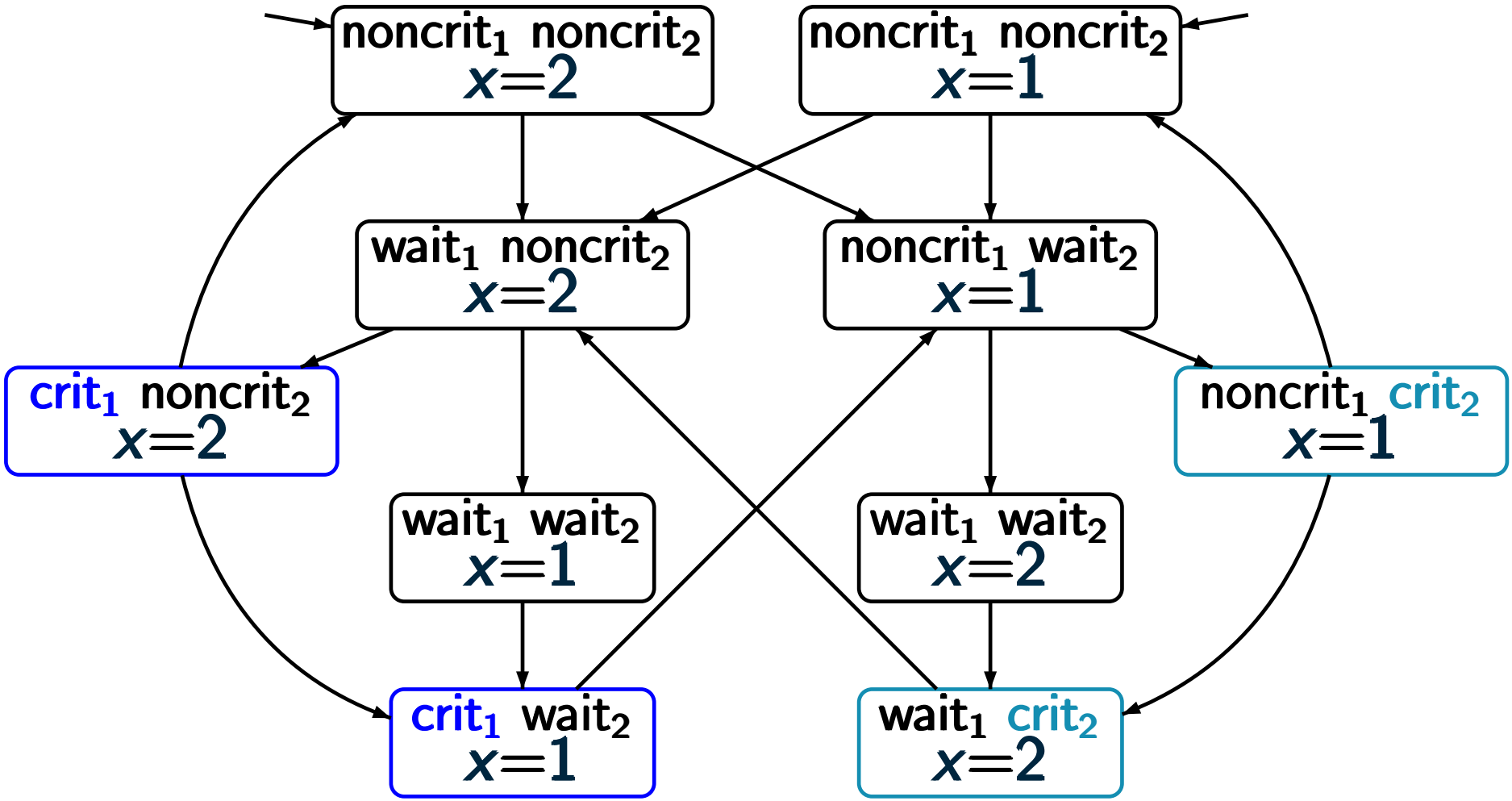
Program graphs for Peterson algorithm



TS for the Peterson algorithm



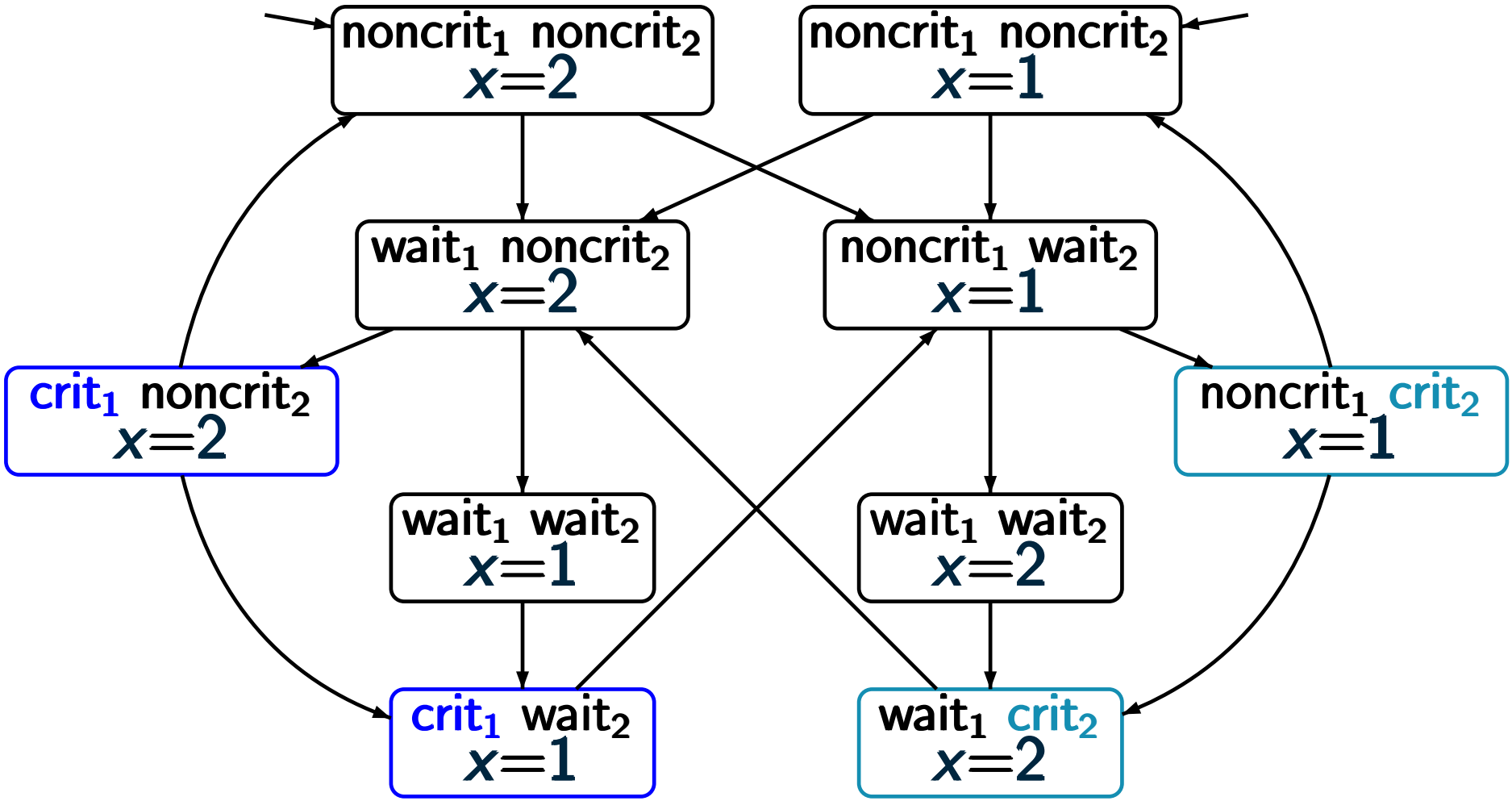
TS for the Peterson algorithm



value of b_1 is given by $\mathbf{wait_1} \vee \mathbf{crit_1}$

value of b_2 is given by $\mathbf{wait_2} \vee \mathbf{crit_2}$

TS for the Peterson algorithm



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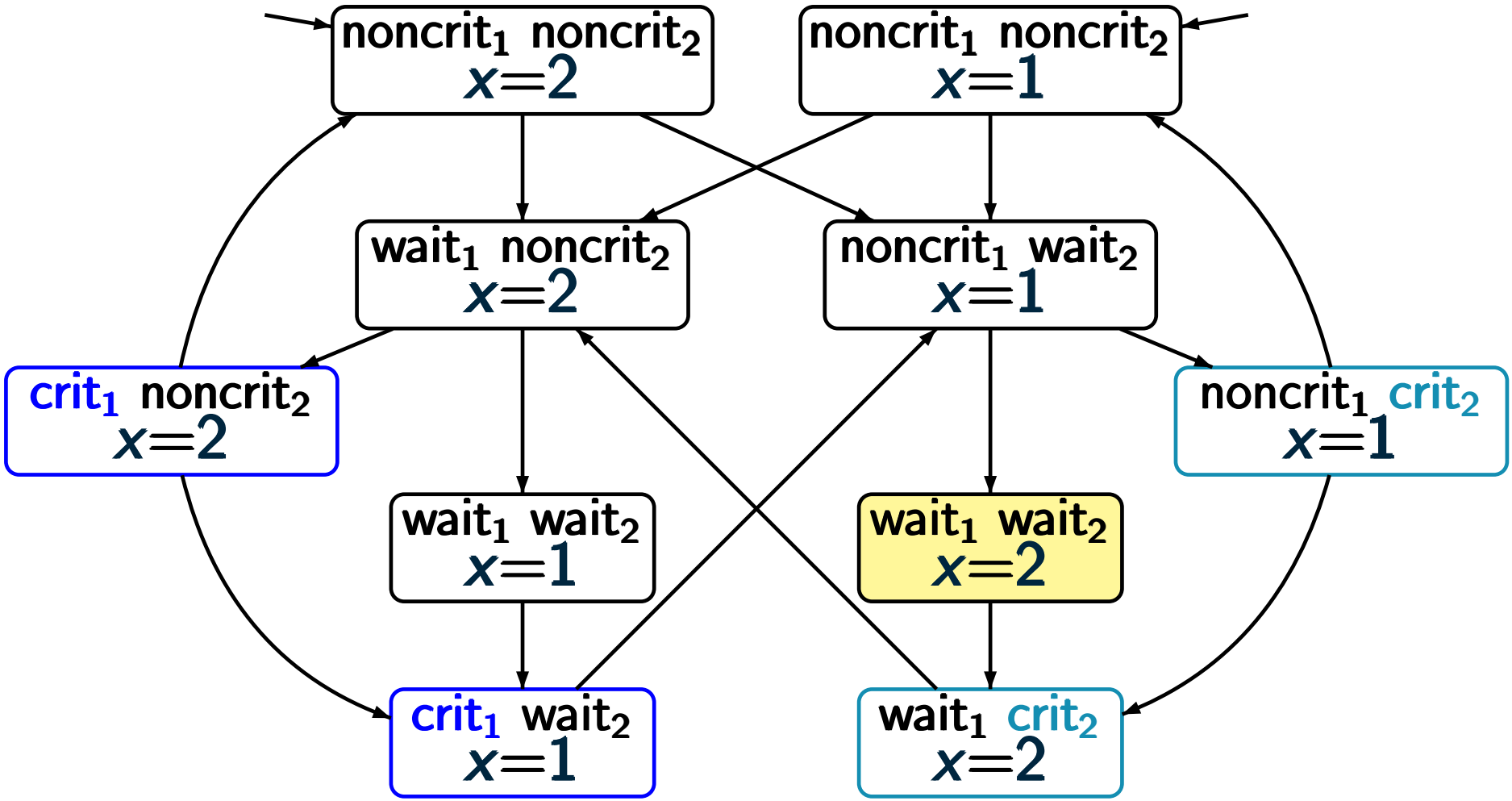
\+ unreachable states

TS for the Peterson algorithm



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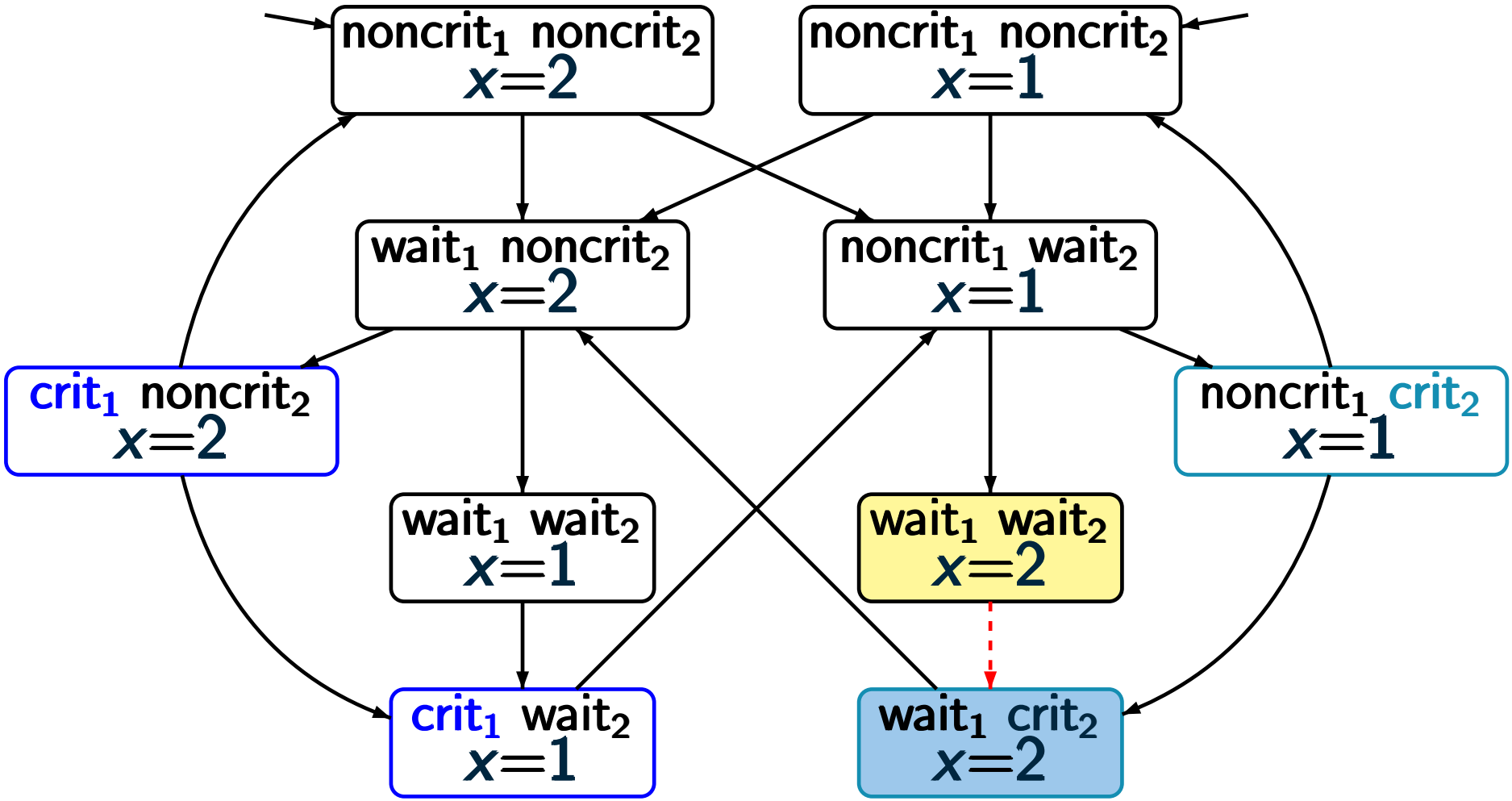

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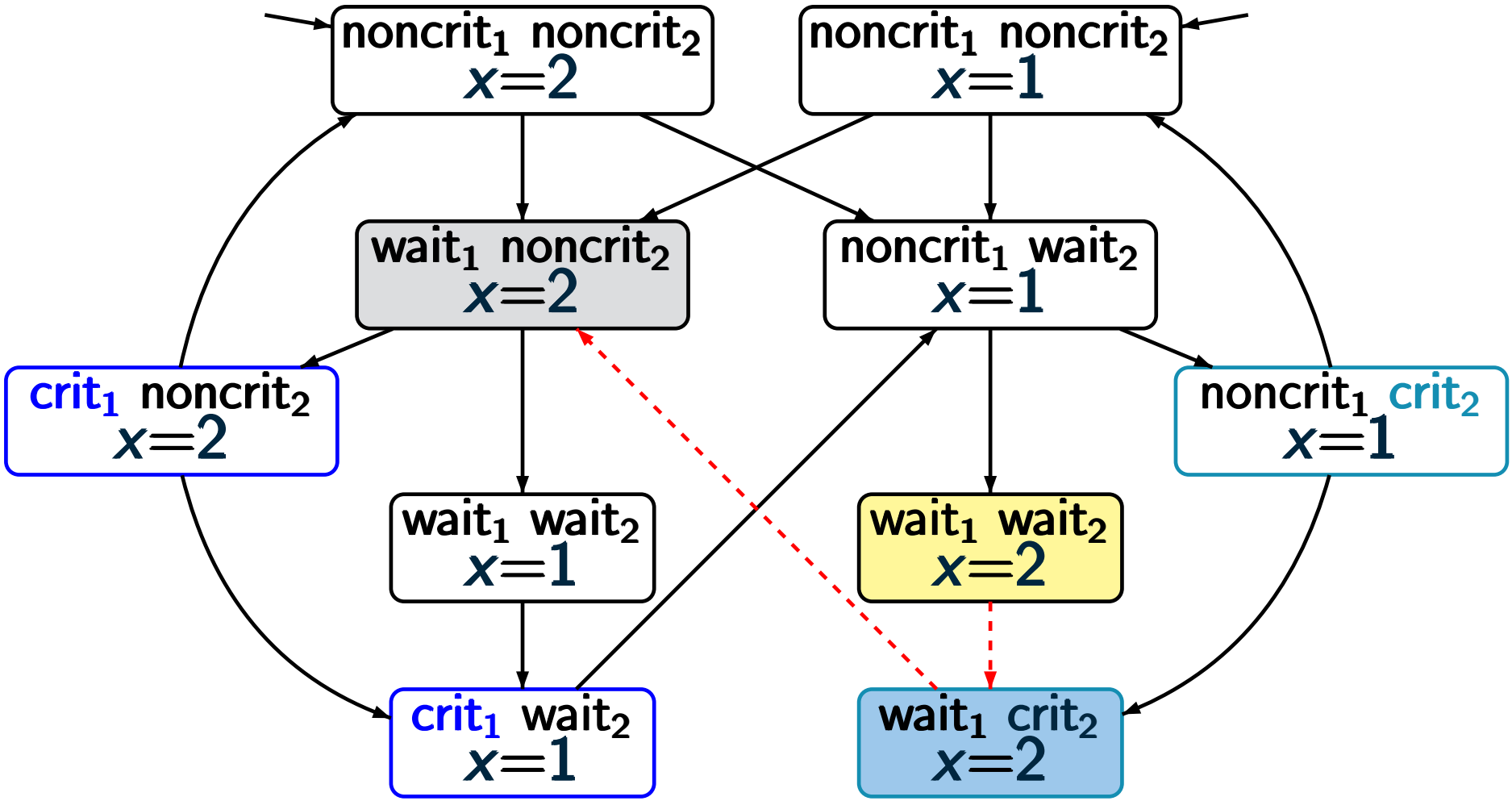

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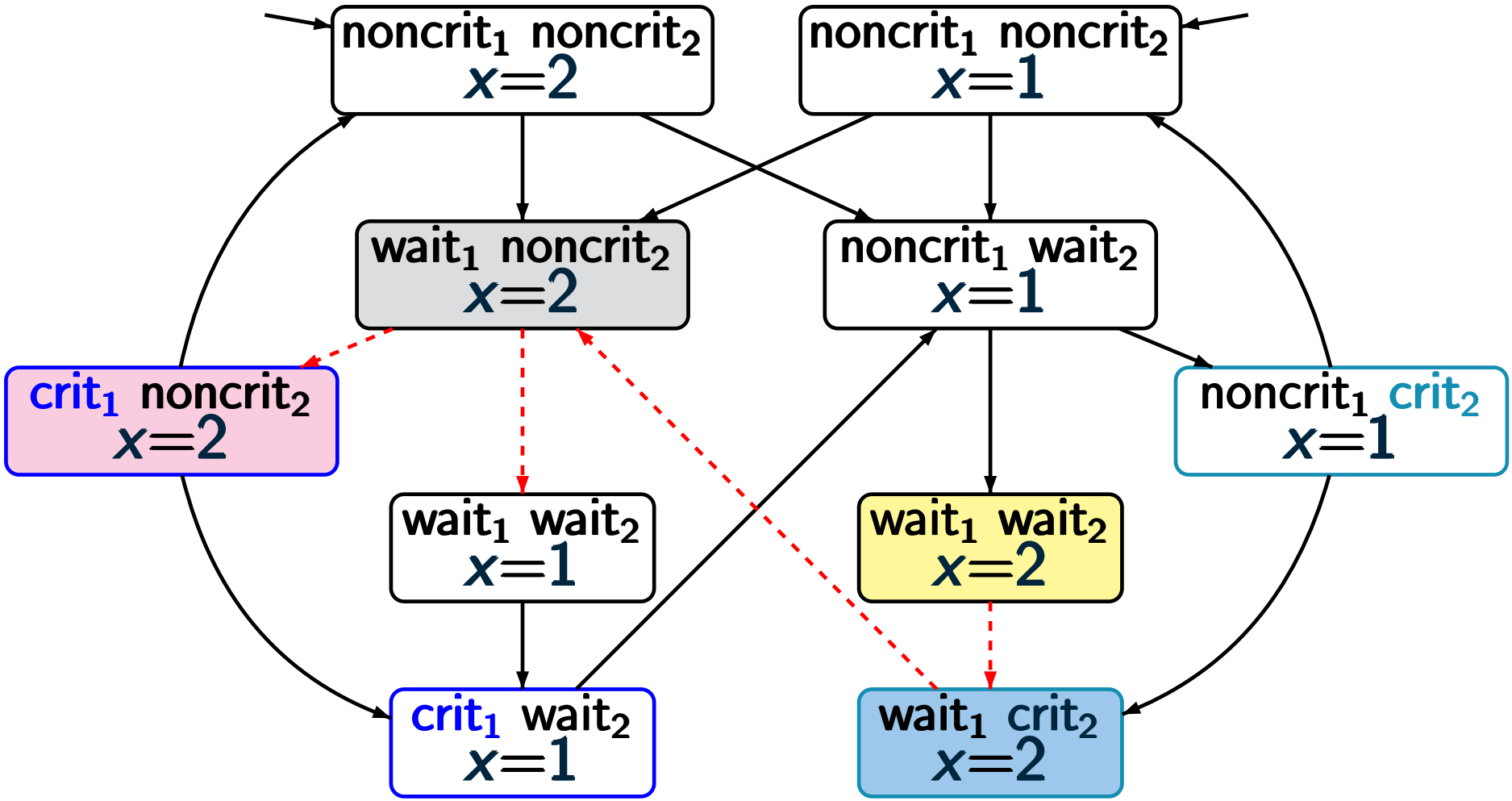

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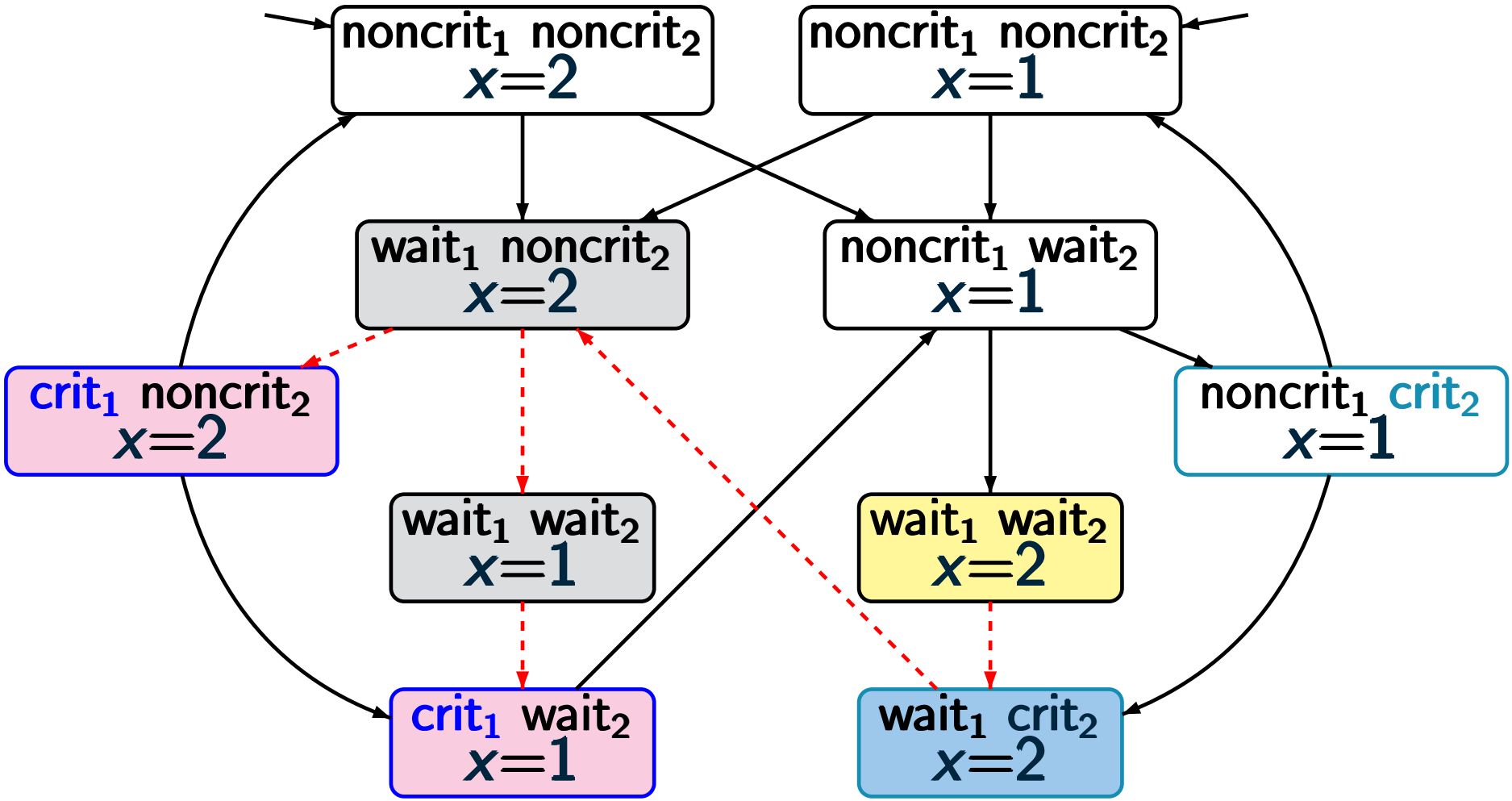
TS for the Peterson algorithm



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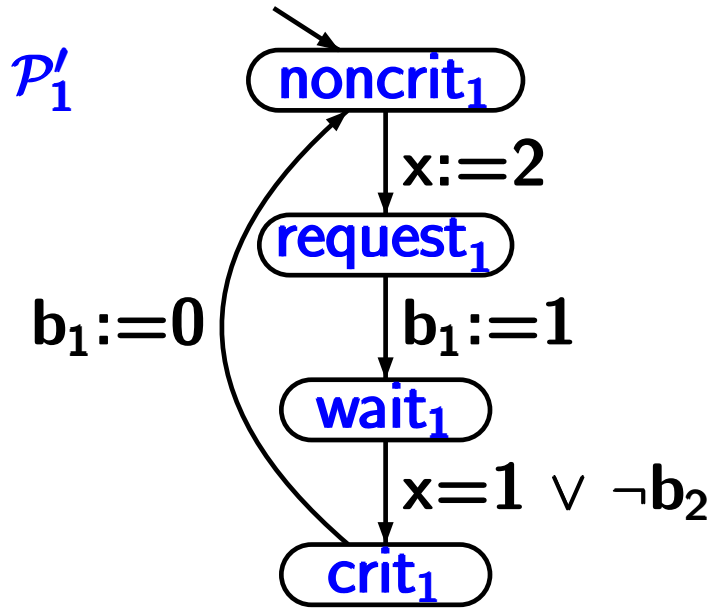
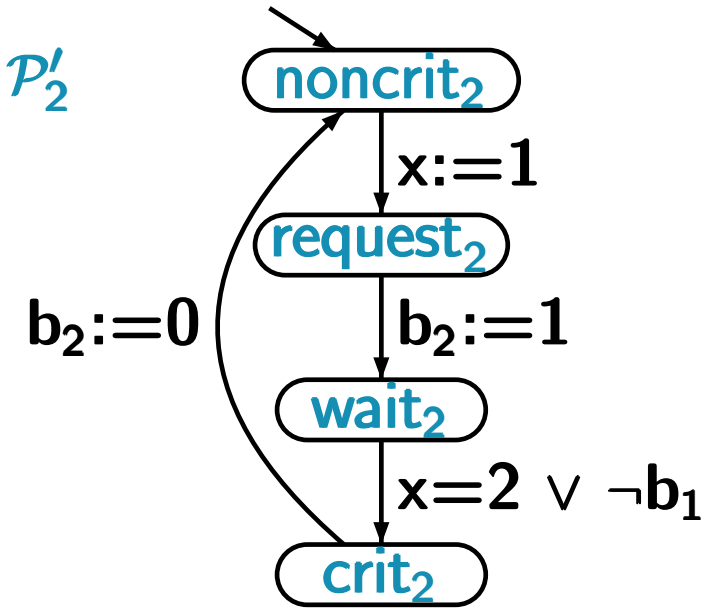
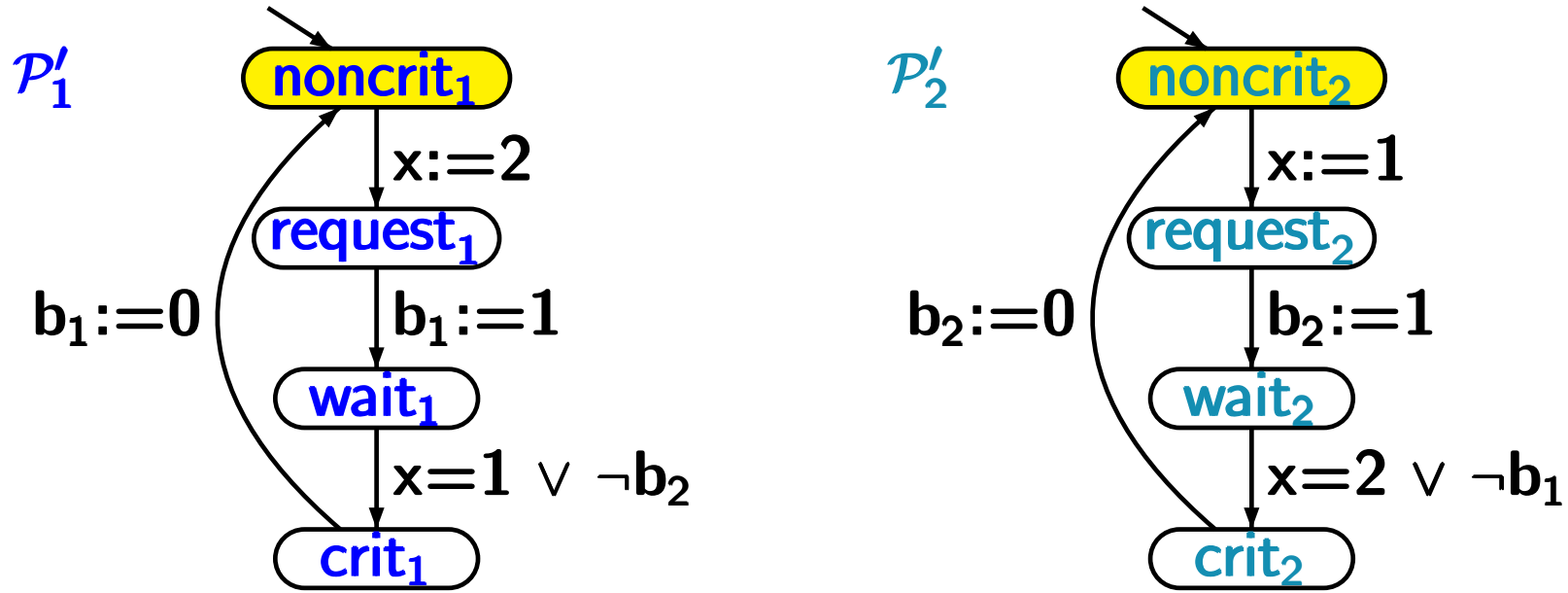
+ unreachable states

Variant of Peterson algorithm





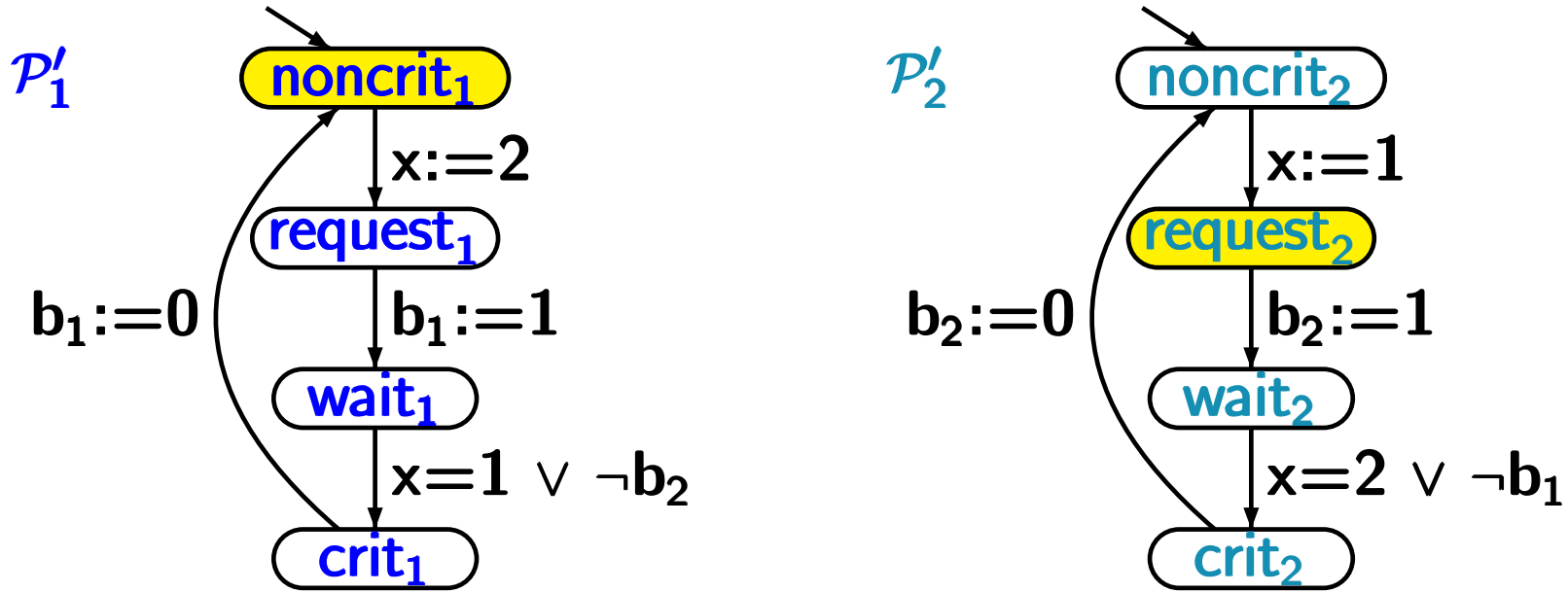
Variant of Peterson algorithm



possible executions

noncrit$_1$ noncrit$_2$ x=1 ¬b$_1$ ¬b$_2$

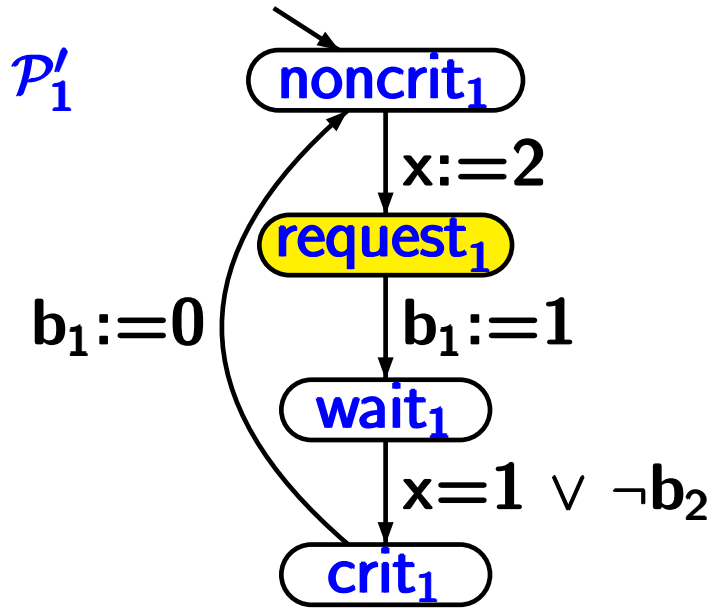
Variant of Peterson algorithm

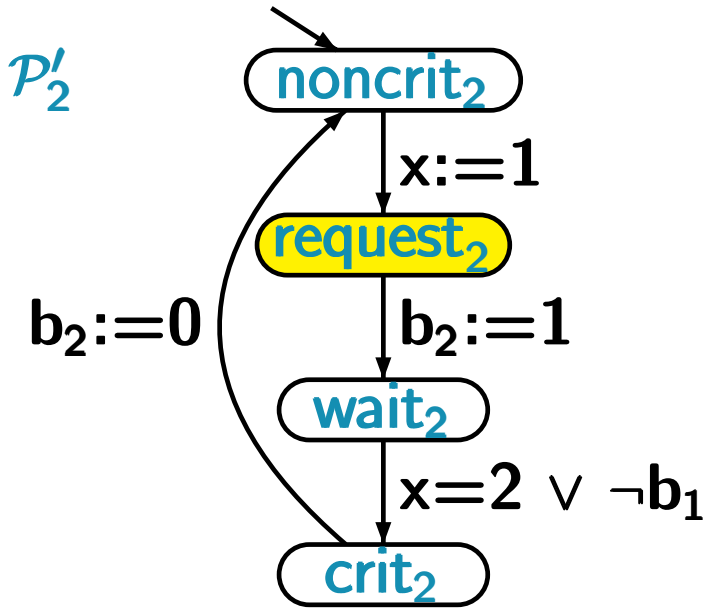


possible executions

noncrit$_1$	noncrit$_2$	x=1	¬b$_1$	¬b$_2$
noncrit$_1$	request$_2$	x=1	¬ b$_1$	¬b$_2$

Variant of Peterson algorithm

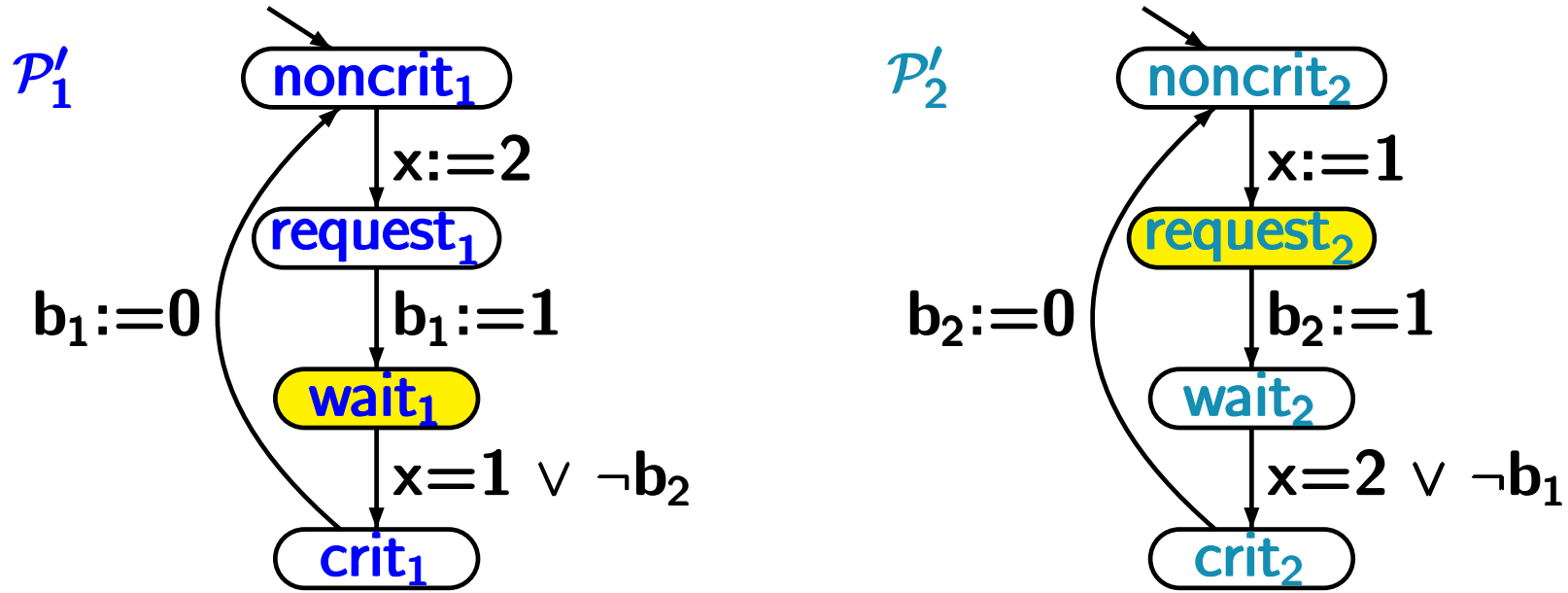




possible executions

noncrit$_1$	noncrit$_2$	x=1	¬b$_1$	¬b$_2$
noncrit$_1$	request$_2$	x=1	¬ b$_1$	¬b$_2$
request$_1$	request$_2$	x=2	¬b$_1$	¬b$_2$

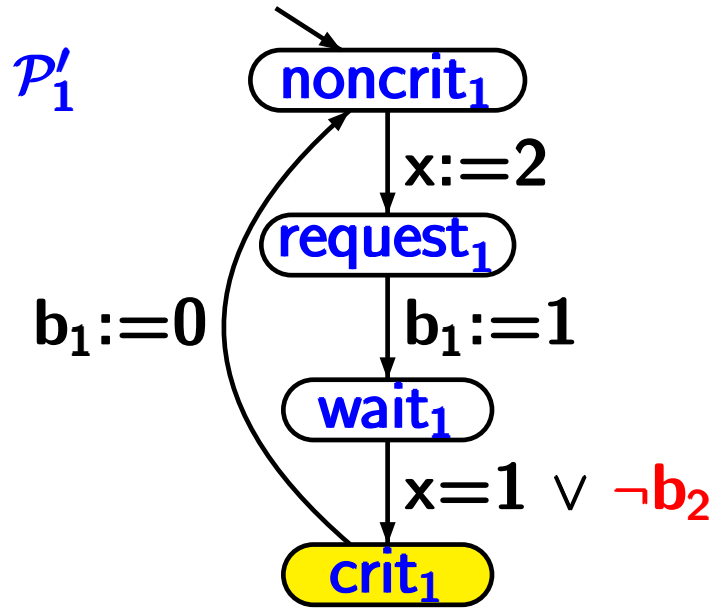
Variant of Peterson algorithm

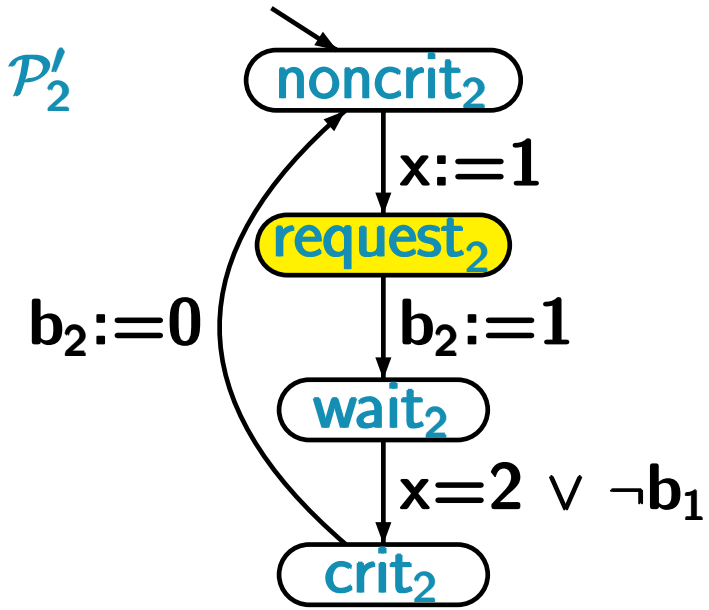


possible executions

noncrit$_1$	noncrit$_2$	x=1	¬b$_1$	¬b$_2$
noncrit$_1$	request$_2$	x=1	¬ b$_1$	¬b$_2$
request$_1$	request$_2$	x=2	¬b$_1$	¬b$_2$
wait$_1$	request$_2$	x=2	b$_1$	¬b$_2$

Variant of Peterson algorithm

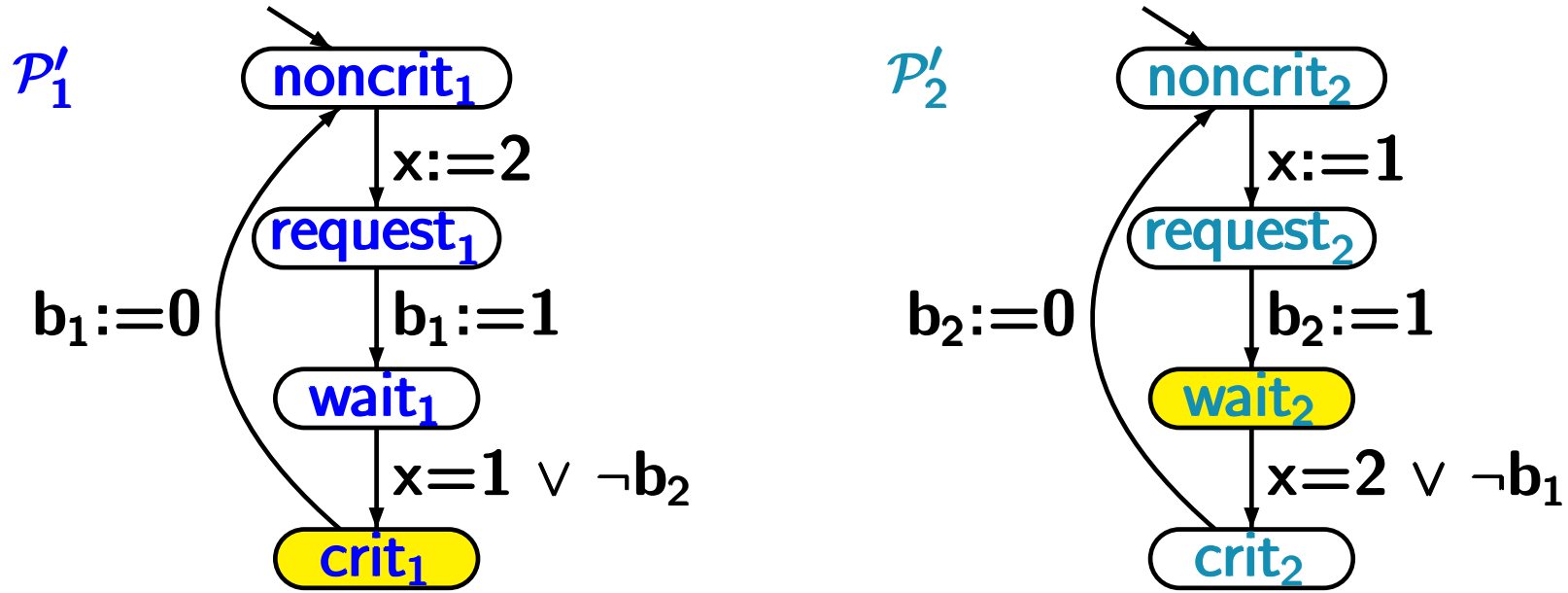




possible executions

noncrit$_1$	noncrit$_2$	x=1	¬b$_1$	¬b$_2$
noncrit$_1$	request$_2$	x=1	¬ b$_1$	¬b$_2$
request$_1$	request$_2$	x=2	¬b$_1$	¬b$_2$
wait$_1$	request$_2$	x=2	b$_1$	¬b$_2$
crit$_1$	request$_2$	x=2	b$_1$	¬b$_2$

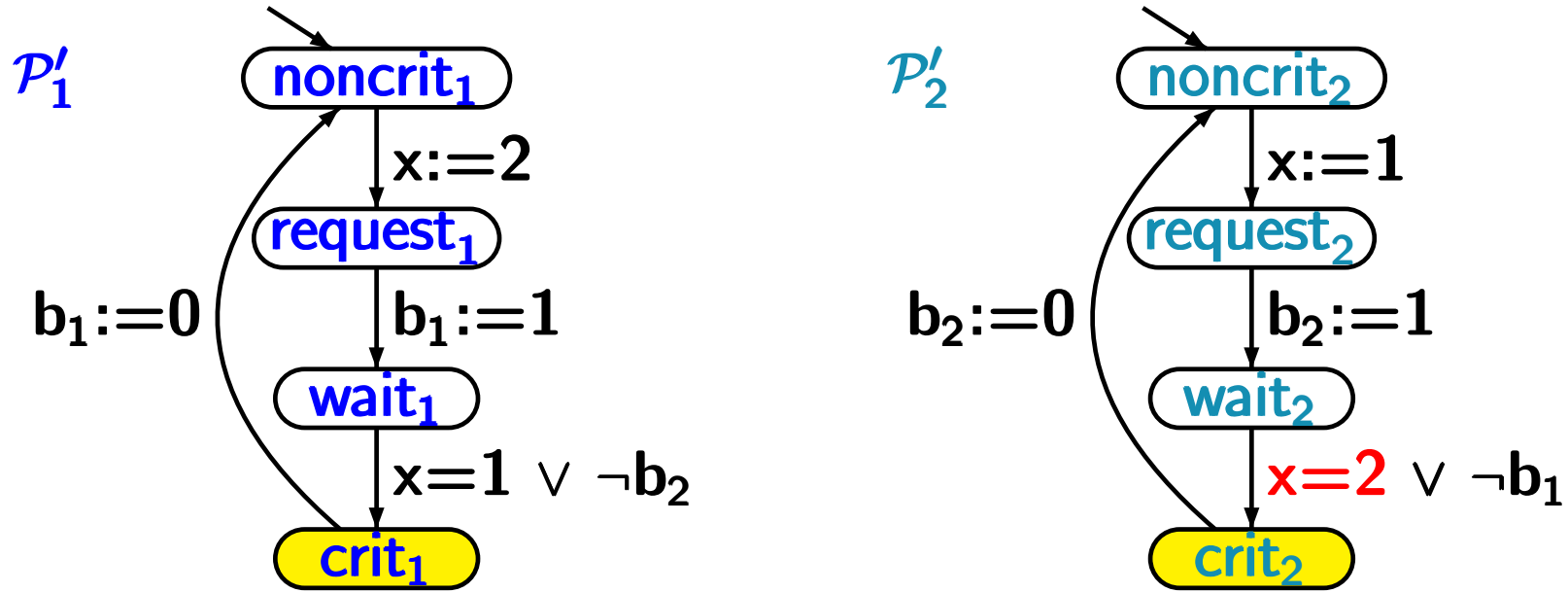
Variant of Peterson algorithm



possible executions

noncrit$_1$	noncrit$_2$	x=1	¬b$_1$	¬b$_2$
noncrit$_1$	request$_2$	x=1	¬ b$_1$	¬b$_2$
request$_1$	request$_2$	x=2	¬b$_1$	¬b$_2$
wait$_1$	request$_2$	x=2	b$_1$	¬b$_2$
crit$_1$	request$_2$	x=2	b$_1$	¬b$_2$
crit$_1$	wait$_2$	x=2	b$_1$	b$_2$

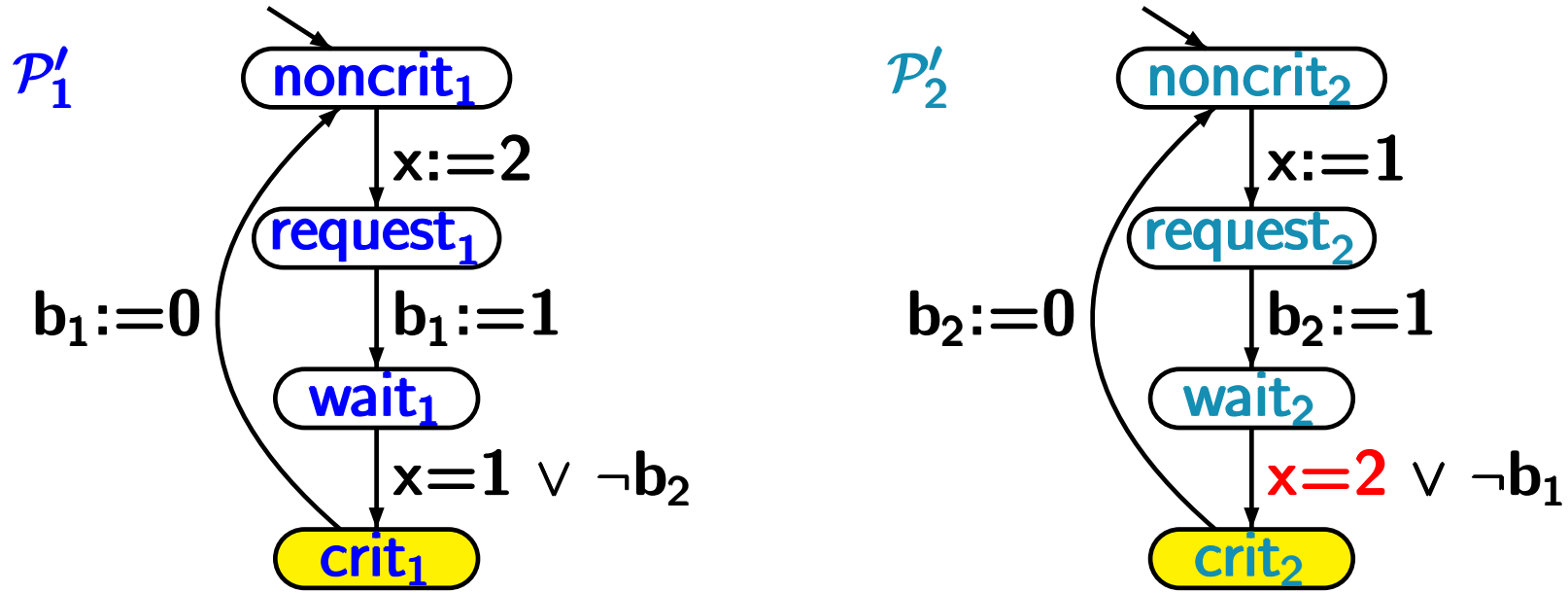
Variant of Peterson algorithm



possible executions

noncrit$_1$	noncrit$_2$	x=1	¬b$_1$	¬b$_2$
noncrit$_1$	request$_2$	x=1	¬ b$_1$	¬b$_2$
request$_1$	request$_2$	x=2	¬b$_1$	¬b$_2$
wait$_1$	request$_2$	x=2	b$_1$	¬b$_2$
crit$_1$	request$_2$	x=2	b$_1$	¬b$_2$
crit$_1$	wait$_2$	x=2	b$_1$	b$_2$
crit$_1$	crit$_2$	x=2	b$_1$	b$_2$

Variant of Peterson algorithm incorrect!



possible executions

noncrit$_1$	noncrit$_2$	x=1	¬b$_1$	¬b$_2$
noncrit$_1$	request$_2$	x=1	¬ b$_1$	¬b$_2$
request$_1$	request$_2$	x=2	¬b$_1$	¬b$_2$
wait$_1$	request$_2$	x=2	b$_1$	¬b$_2$
crit$_1$	request$_2$	x=2	b$_1$	¬b$_2$
crit$_1$	wait$_2$	x=2	b$_1$	b$_2$
crit$_1$	crit$_2$	x=2	b$_1$	b$_2$

How many states ...?

Given n processes by program graphs $\mathcal{P}_1, \ldots, \mathcal{P}_n$

How many states ...?

Given n processes by program graphs $\mathcal{P}_1, \ldots, \mathcal{P}_n$

- with 2 locations each
- over the set of variables $Var = \{x_1, \ldots, x_m\}$ with $Dom(x_i) = \{0, 1\}$

How many states has the transition system $\mathcal{T}_{\mathcal{P}_1 \,|||\, \ldots \,|||\, \mathcal{P}_n}$?

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answer: $2^n \cdot 2^m$

State explosion problem

Given n processes by program graphs $\mathcal{P}_1, \ldots, \mathcal{P}_n$

- with 2 locations each
- over the set of variables $Var = \{x_1, \ldots, x_m\}$ with $Dom(x_i) = \{0, 1\}$

How many states has the transition system $\mathcal{T}_{\mathcal{P}_1 \,|||\, \ldots \,|||\, \mathcal{P}_n}$?

answer: $2^n \cdot 2^m$

state explosion: size of transition systems grows

- exponentially in the number of parallel processes
- exponentially in the number of variables