Model Checking I
alias
Reactive Systems Verification

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Topics

- Parallelism and Communication
- Synchronous Message Passing
- Examples

Material

Reading:

Chapter 2 of the book, pages 47–53.

More:

The slides in the following pages are taken from the material of the course “Introduction to Model Checking” held by Prof. Dr. Ir. Joost-Pieter Katoen at Aachen University.

Operators for parallelism and communication

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- channel systems
  communication via shared variables + via channels
- synchronous product

Synchronous message passing

Synchronous message passing

$\mathcal{T}_1 = (S_1, Act_1, \rightarrow_1, \ldots)$, $\mathcal{T}_2 = (S_2, Act_2, \rightarrow_2, \ldots)$ TS

$Syn \subseteq Act_1 \cap Act_2$ set of synchronization actions

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composite transition system:

$$\mathcal{T}_1 \parallel_{Syn} \mathcal{T}_2 = (S_1 \times S_2, Act_1 \cup Act_2, \rightarrow, \ldots)$$

for modeling the concurrent execution of $\mathcal{T}_1$ and $\mathcal{T}_2$ with synchronization over all actions in Syn

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Mutual exclusion

by synchronous message passing

Mutual exclusion

by synchronous message passing using an arbiter

Mutual exclusion with an arbiter

protocol for process P_i

```
LOOP FOREVER DO
    noncritical actions
    request
    critical section
    release
    noncritical actions
OD
```

Mutual exclusion with an arbiter

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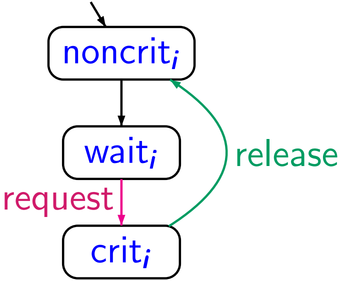
transition system $\mathcal{T}_i$



Mutual exclusion with an arbiter

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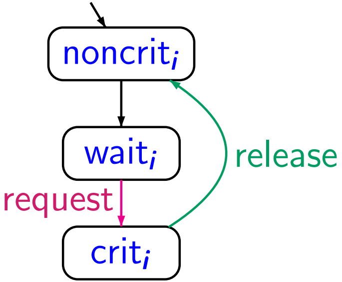
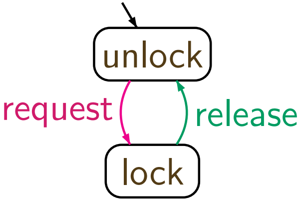


Arbiter:

selects nondeterministically a synchronization partner $\mathcal{T}_1$ or $\mathcal{T}_2$



Mutual exclusion with an arbiter

$(\mathcal{T}_1 \;|||\; \mathcal{T}_2) \;\|_{Syn}\; Arbiter$ where $Syn = \{\text{request}, \text{release}\}$

Mutual exclusion with an arbiter

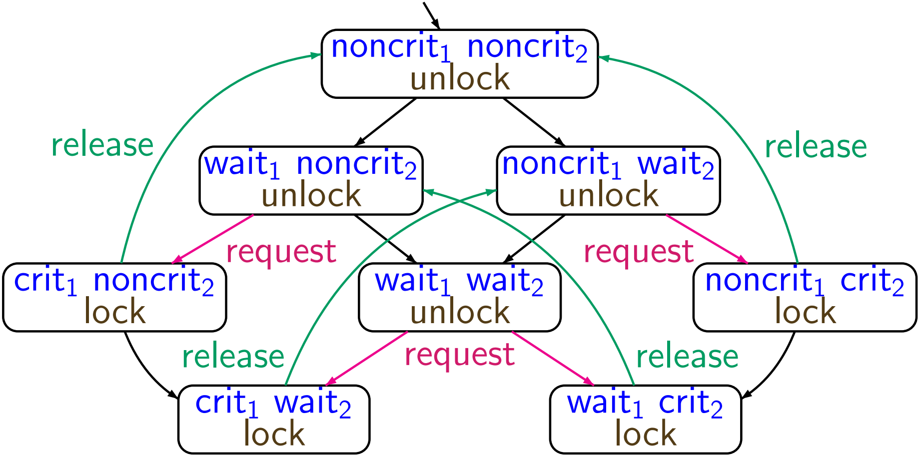
$$(\mathcal{T}_1 \; ||| \; \mathcal{T}_2) \; \|_{Syn} \; \textit{Arbiter} \text{ where } \textit{Syn} = \{\text{request}, \text{release}\}$$

↑ "pure" interleaving for TS

↖ handshaking for actions request and release

Mutual exclusion with an arbiter

$(\mathcal{T}_1 \,|||\, \mathcal{T}_2) \parallel_{Syn} \textit{Arbiter}$ where $Syn = \{$request, release$\}$



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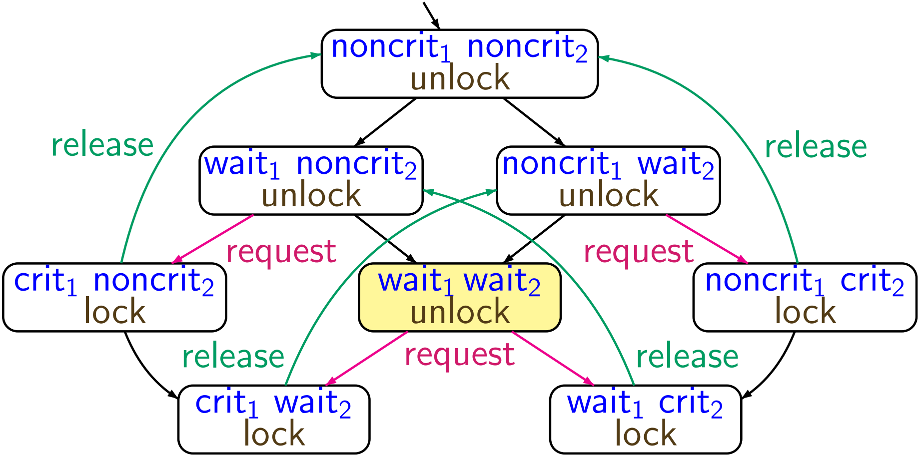

nondeterministic choice: who enters the critical section**?**

synchronization operator $\|_{Syn}$ for
three or more processes

Synchronous message passing

$$\left.\begin{array}{lcl} \mathcal{T}_1 & = & (S_1, Act_1, \rightarrow_1, \ldots) \\ \mathcal{T}_2 & = & (S_2, Act_2, \rightarrow_2, \ldots) \\ \mathcal{T}_3 & = & (S_3, Act_3, \rightarrow_3, \ldots) \\ \mathcal{T}_4 & = & (S_4, Act_4, \rightarrow_4, \ldots) \end{array}\right\} \text{transition systems}$$

$$\vdots \qquad\qquad \vdots$$

Synchronous message passing

$$
\left.
\begin{array}{lcl}
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\vdots & & \quad\vdots
\end{array}
\right\} \text{transition systems}
$$

for $Syn \subseteq Act_1 \cup Act_2 \cup Act_3 \cup Act_4 \cup \ldots$

$$
\mathcal{T}_1 \parallel_{Syn} \mathcal{T}_2 \parallel_{Syn} \mathcal{T}_3 \parallel_{Syn} \mathcal{T}_4 \parallel_{Syn} \ldots \stackrel{\text{def}}{=} \Big(\big((\mathcal{T}_1 \parallel_{Syn} \mathcal{T}_2) \parallel_{Syn} \mathcal{T}_3\big) \parallel_{Syn} \mathcal{T}_4\Big) \parallel_{Syn} \ldots
$$

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$$\mathcal{T}_1 \parallel_{Syn} \mathcal{T}_2 \parallel_{Syn} \mathcal{T}_3 \parallel_{Syn} \mathcal{T}_4 \parallel_{Syn} \ldots \stackrel{\text{def}}{=}$$

$$\Big(((\mathcal{T}_1 \parallel_{Syn} \mathcal{T}_2) \parallel_{Syn} \mathcal{T}_3) \parallel_{Syn} \mathcal{T}_4\Big) \parallel_{Syn} \ldots$$

or any other order of paranthesis

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where, e.g., $\mathcal{T}_1 \parallel_{Syn} \mathcal{T}_2 \stackrel{\text{def}}{=} \mathcal{T}_1 \parallel_H \mathcal{T}_2$

with $H = Syn \cap Act_1 \cap Act_2$

Parallel operator $\|$

$$\begin{aligned}
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&\vdots \qquad\qquad \vdots
\end{aligned}$$

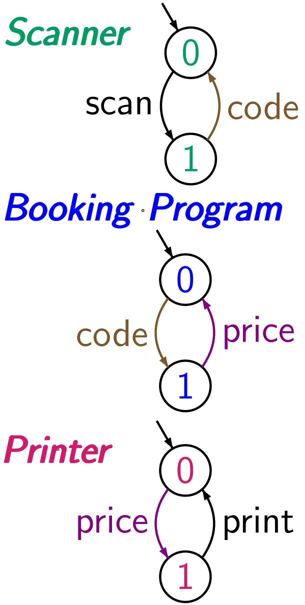
transition systems s.t. $Act_i \cap Act_j \cap Act_k = \emptyset$ if i, j, k are pairwise distinct

$$\mathcal{T}_1 \| \mathcal{T}_2 \| \mathcal{T}_3 \| \mathcal{T}_4 \| \ldots \overset{\text{def}}{=} (((\mathcal{T}_1 \|_{Syn_{1,2}} \mathcal{T}_2) \|_{Syn_{1,2,3}} \mathcal{T}_3) \|_{Syn_{1,2,3,4}} \mathcal{T}_4) \ldots$$

where

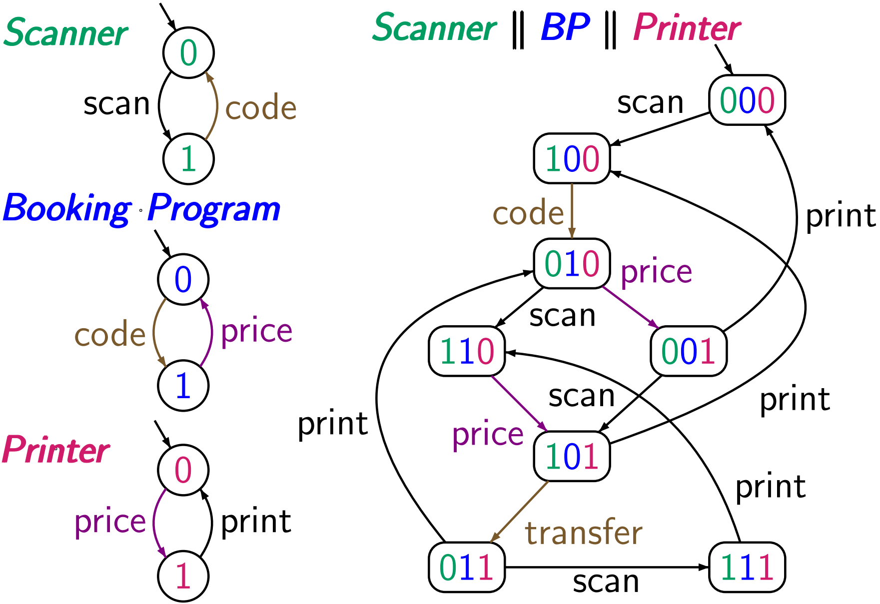
$$\begin{aligned}
Syn_{1,2} &= Act_1 \cap Act_2 \\
Syn_{1,2,3} &= (Act_1 \cup Act_2) \cap Act_3 \\
Syn_{1,2,3,4} &= (Act_1 \cup Act_2 \cup Act_3) \cap Act_4 \\
&\vdots \qquad\qquad \vdots
\end{aligned}$$

Booking system in supermarket



Scanner ∥ *BP* ∥ *Printer*

Booking system in supermarket



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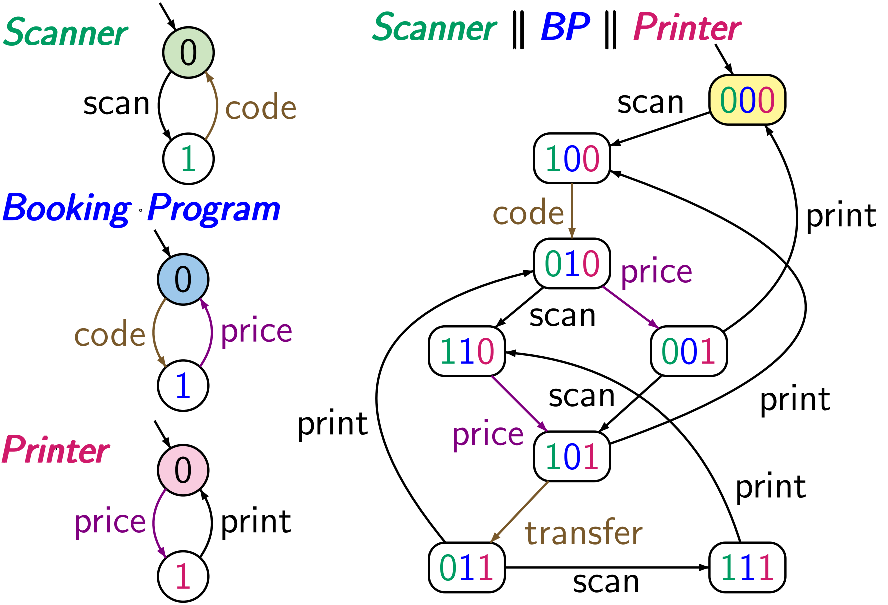

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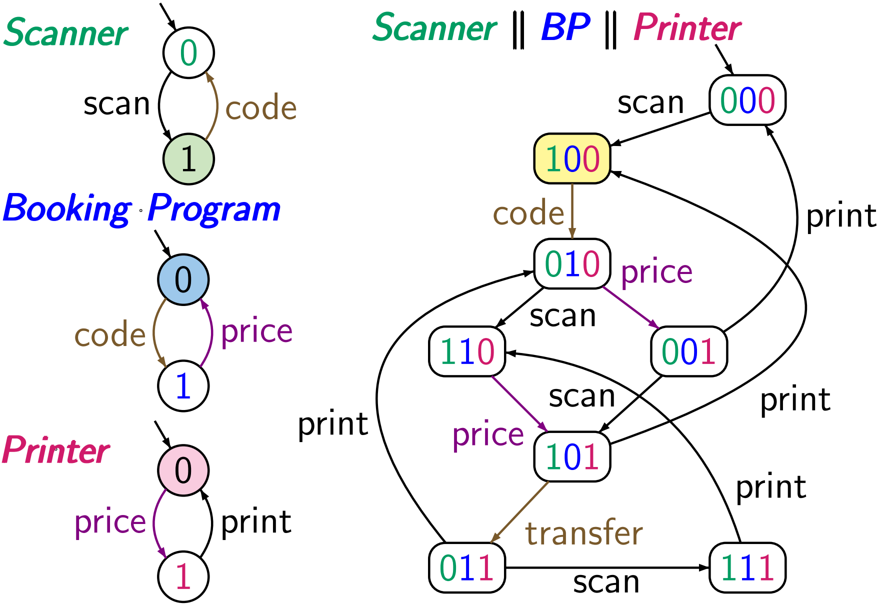

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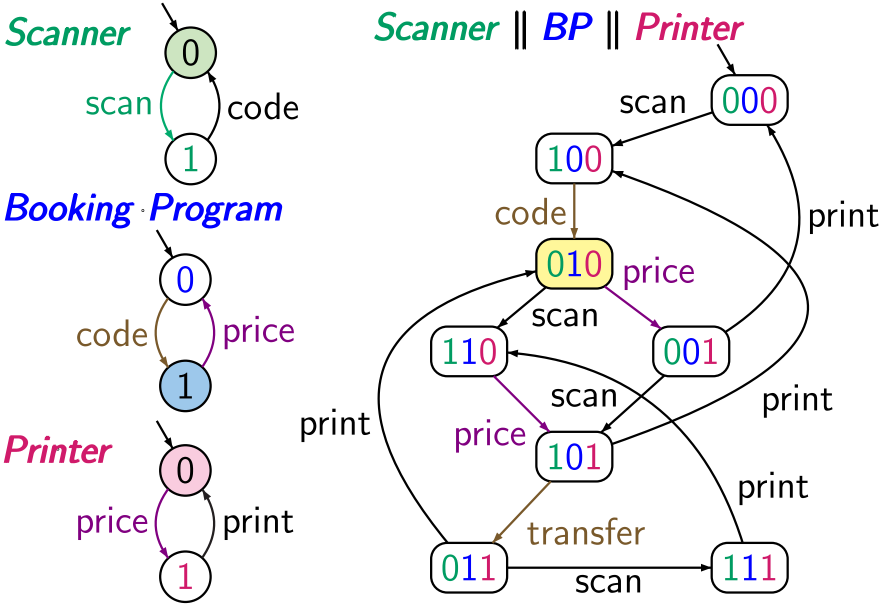
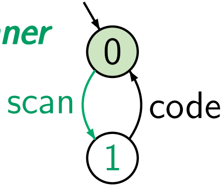
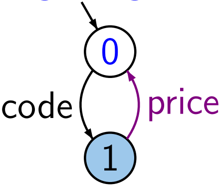


Interleaving

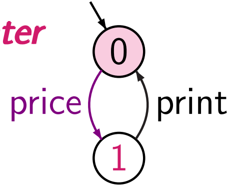
Scanner



Booking Program



Printer



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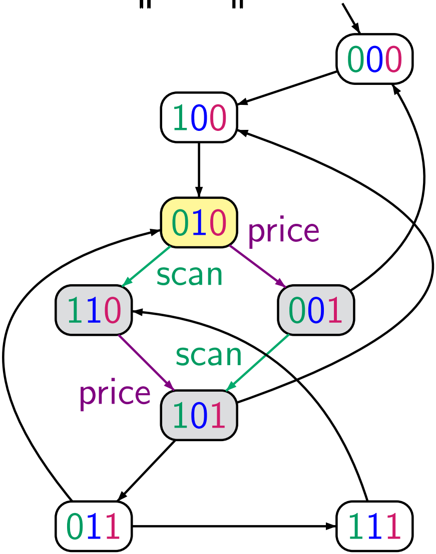

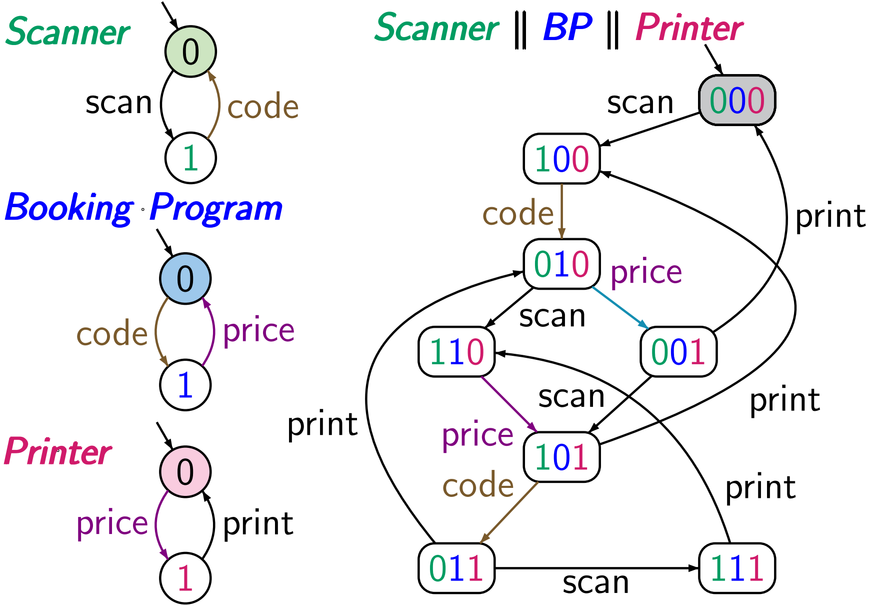

Scanner
0
scan
code
1
Booking Program
0
code
price
1
Printer
0
price
print
1
Scanner ∥ BP ∥ Printer
000
scan
100
code
010
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001
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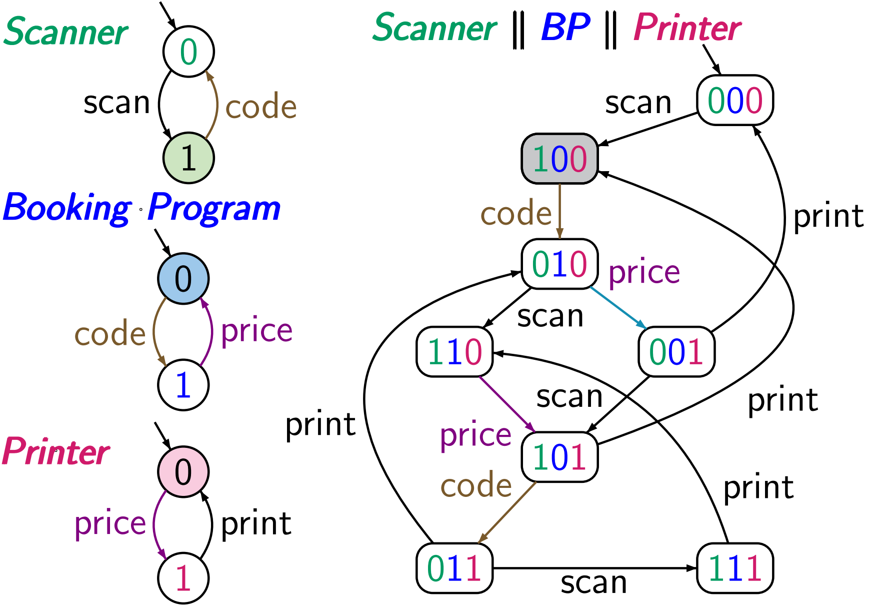

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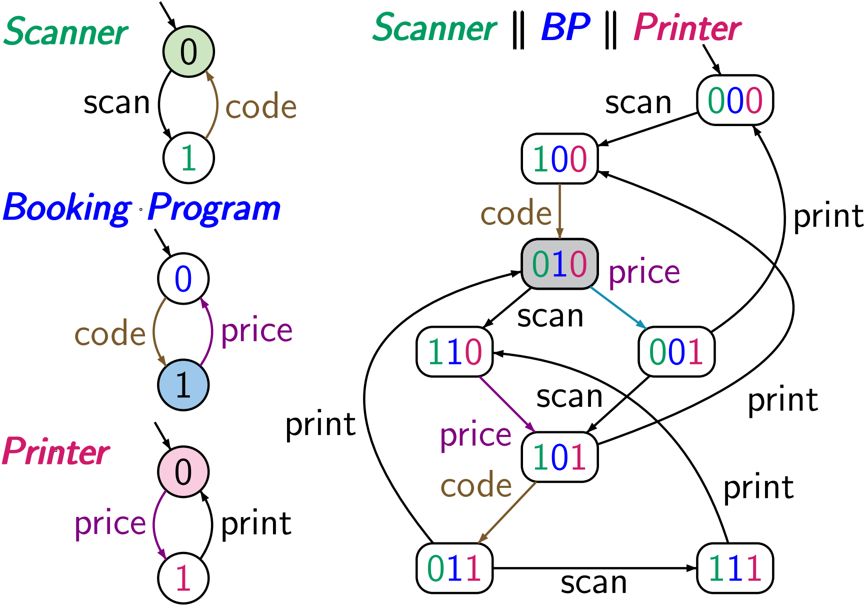

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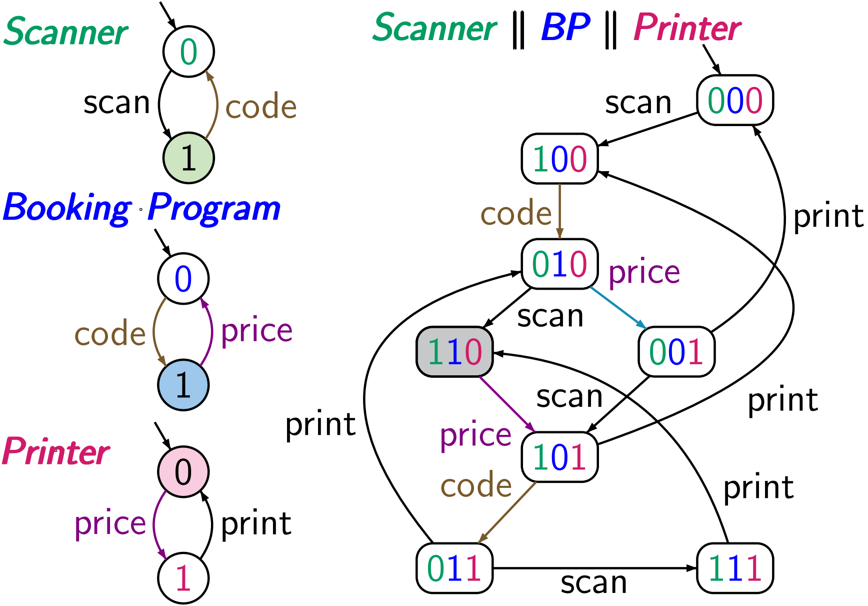

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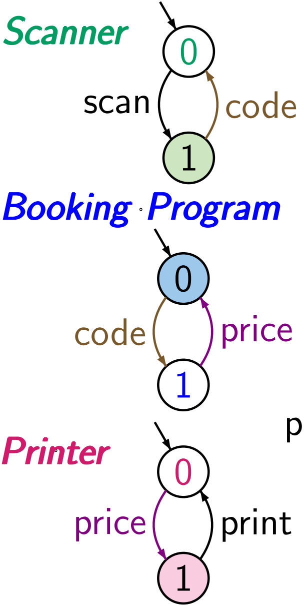

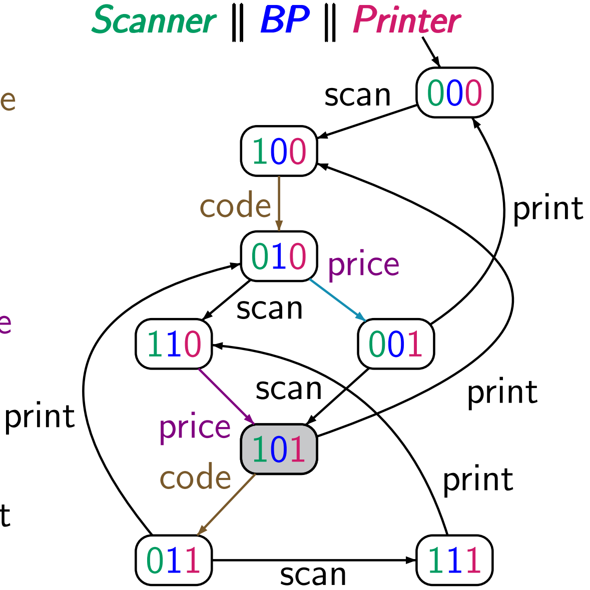

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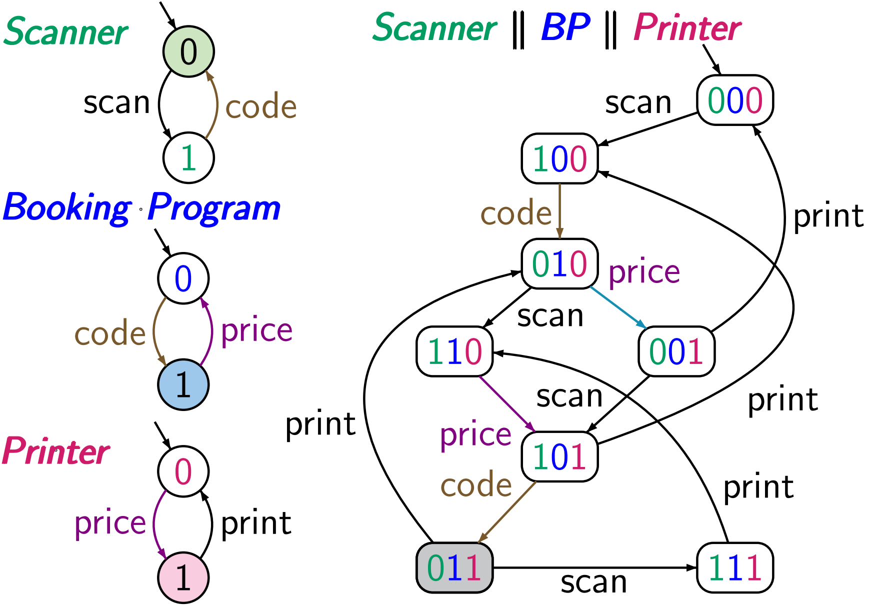
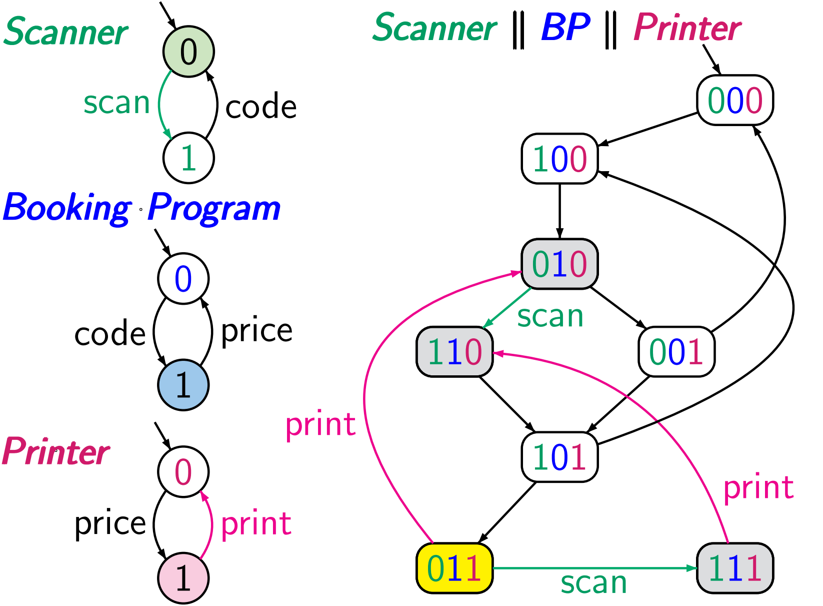
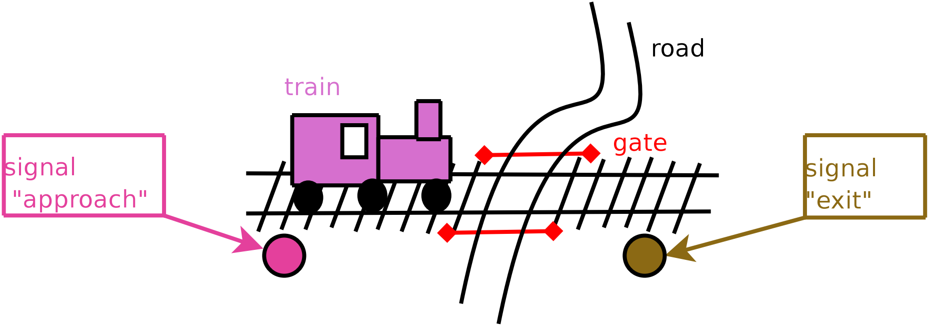


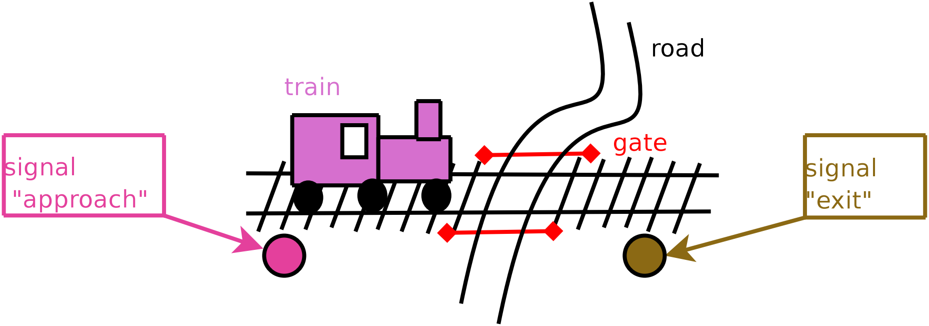
Interleaving



Railroad crossing



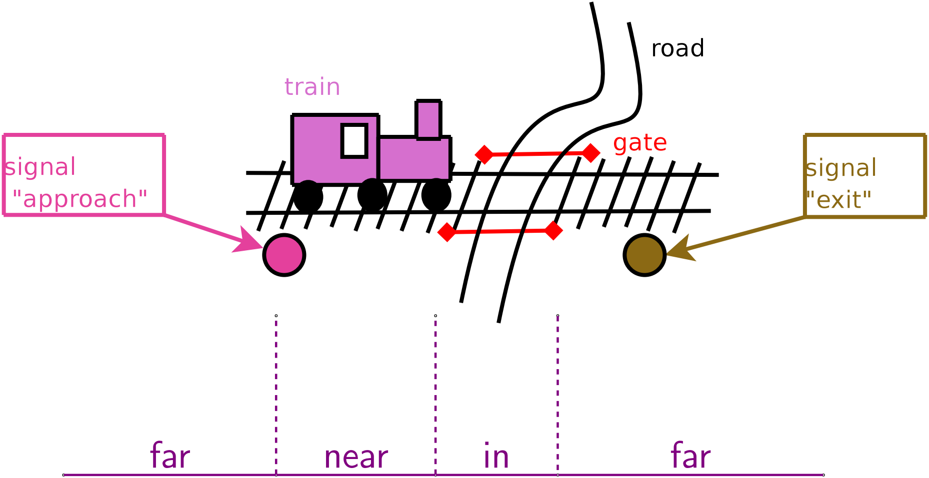
Railroad crossing



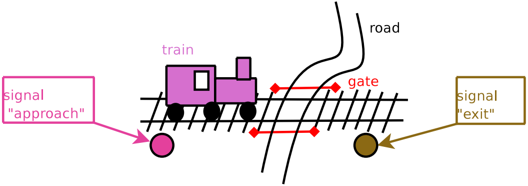
modeling by a transition system with **3** processes:

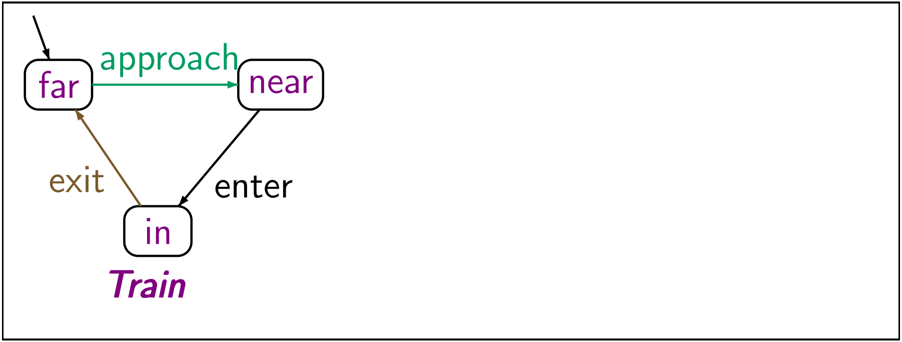
Train || *Controller* || *Gate*

Modeling the behavior of trains

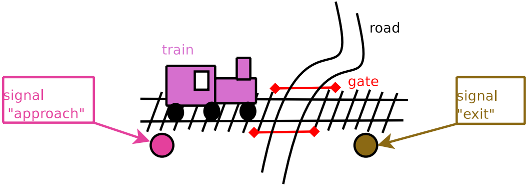


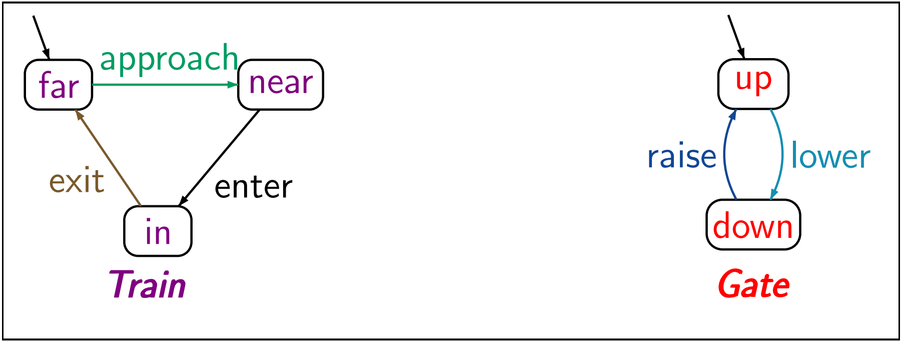
TS for the trains



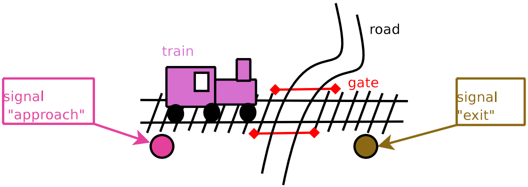


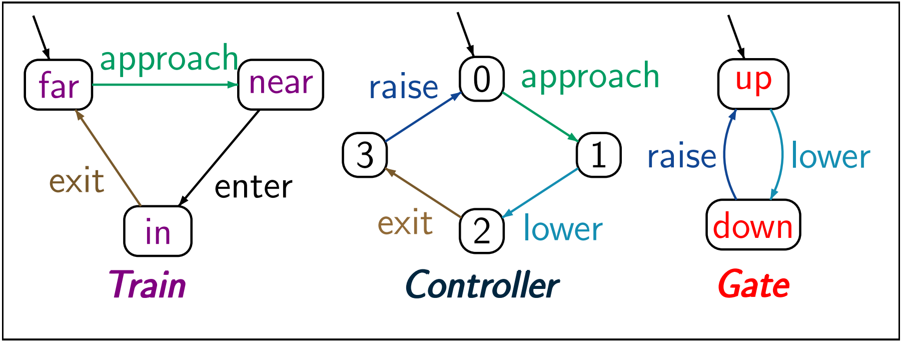
TS for the trains and gate





TS for the trains, controller and gate





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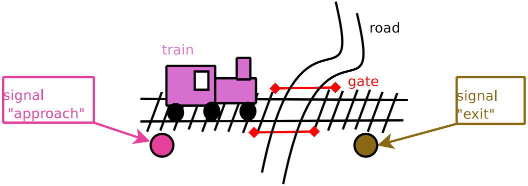
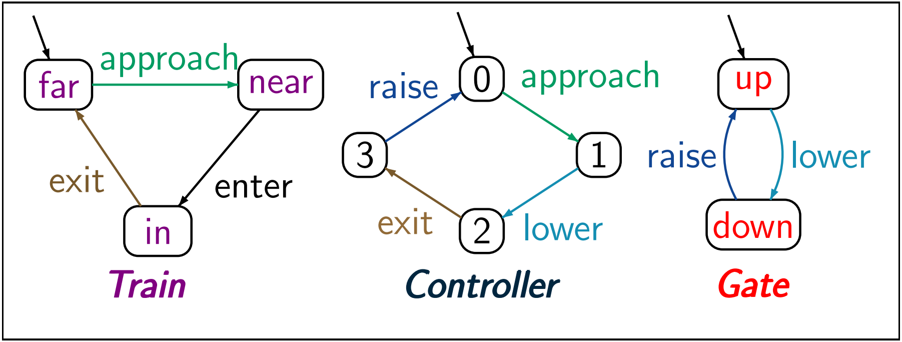
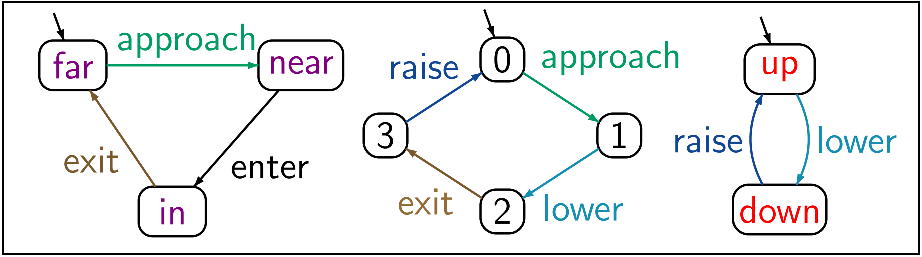




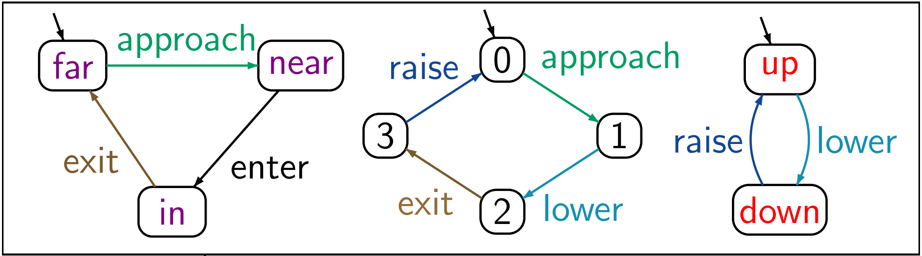
transition system *Train* ‖ *Controller* ‖ *Gate*

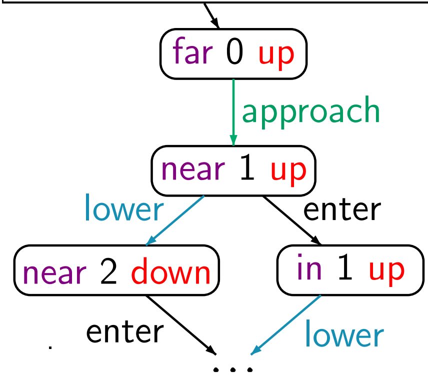
TS for railroad crossing



reachable fragment
of the transition system
Train || ***Controller*** || ***Gate***

TS for railroad crossing





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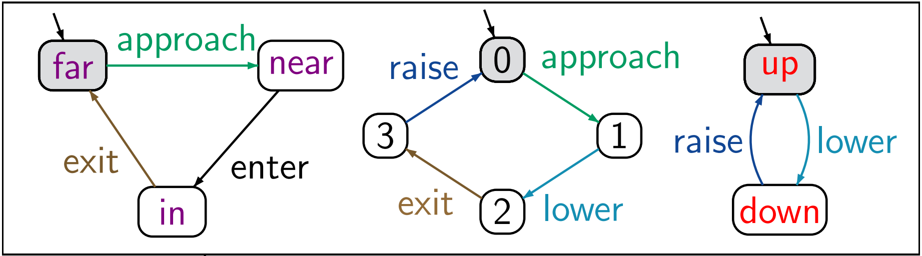
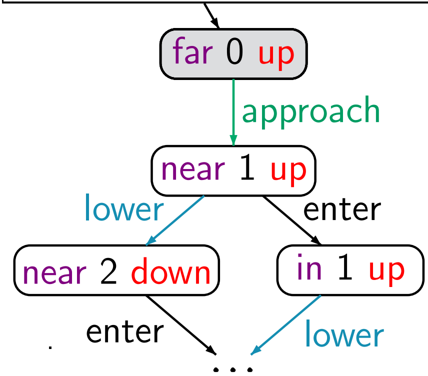
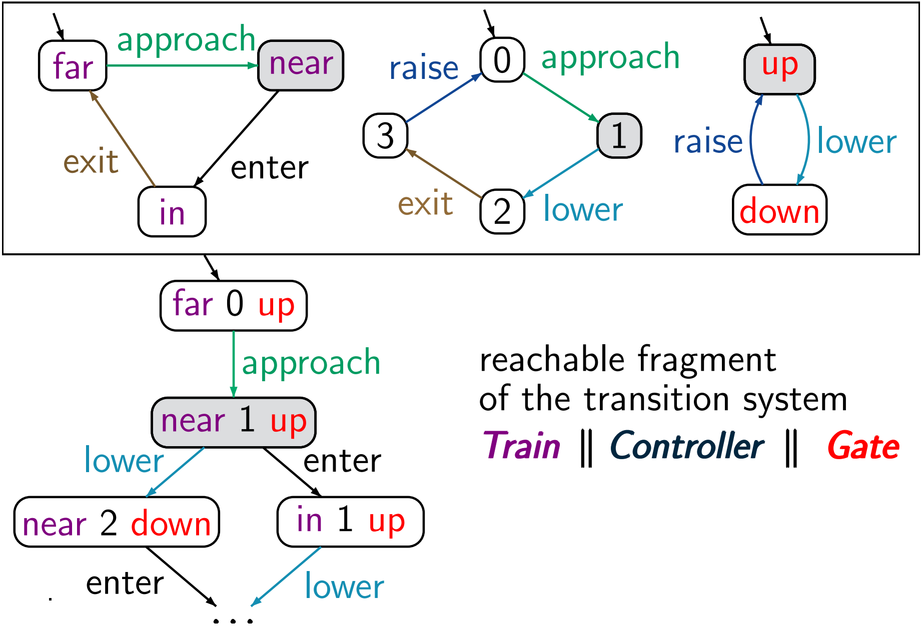
TS for railroad crossing





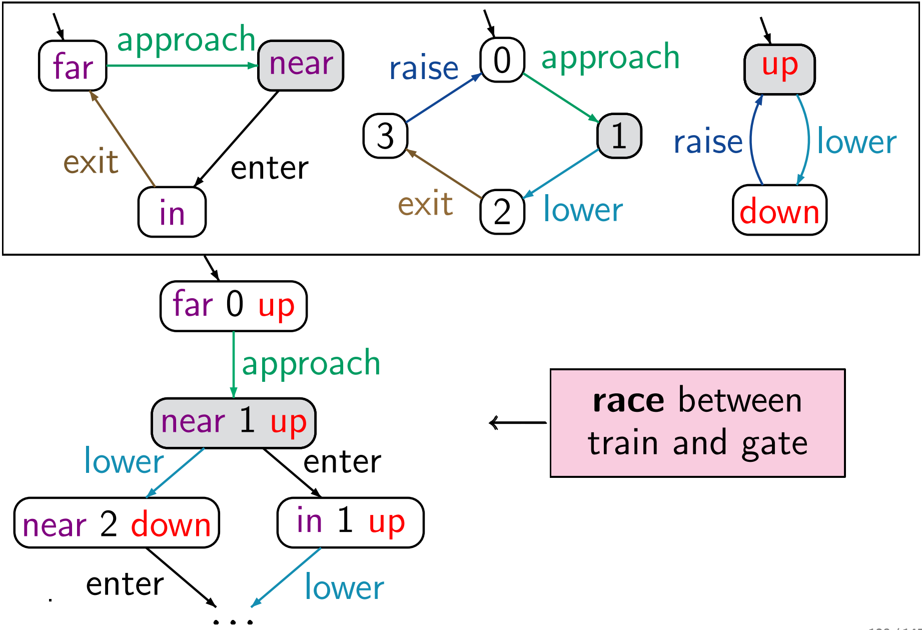
reachable fragment
of the transition system
Train || ***Controller*** || ***Gate***

TS for railroad crossing



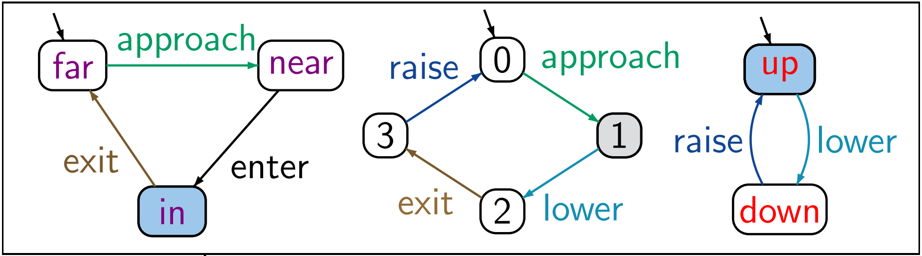
reachable fragment
of the transition system
Train || ***Controller*** || ***Gate***

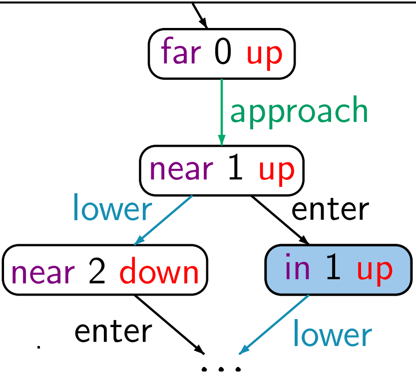
TS for railroad crossing



race between train and gate

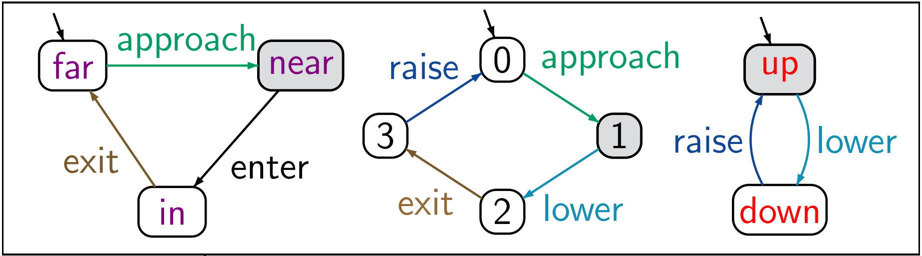
TS for railroad crossing

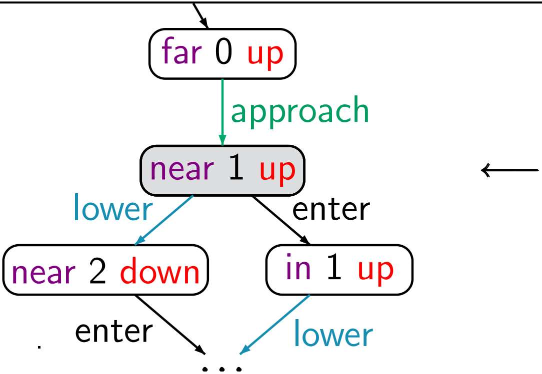




gate is open,
while train is
crossing the road

TS for railroad crossing





interleaving is time-abstract