Model Checking I
alias
Reactive Systems Verification

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Topics

- Synchronous Product
- Examples
- The state explosion problem

Material

Reading:

Chapter 2 of the book, pages 75–80.

More:

The slides in the following pages are taken from the material of the course “Introduction to Model Checking” held by Prof. Dr. Ir. Joost-Pieter Katoen at Aachen University.

Summary: parallel operators

PC2.2-39

(pure) interleaving for TS $\mathcal{T}_1 \;|||\; \mathcal{T}_2$

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channel systems: open $\mathcal{P}_1 | \ldots | \mathcal{P}_n$ or closed $[\mathcal{P}_1 | \ldots | \mathcal{P}_n]$

- interleaving, shared variables
- synchronous and asynchronous message passing

synchronous product for TS $\mathcal{T}_1 \otimes \mathcal{T}_2$

- no interleaving, "pure" synchronization

Synchronous product

for parallel systems with fully synchronized processes

$$\left.\begin{array}{l} \mathcal{T}_1 = (S_1, Act_1, \longrightarrow_1, \ldots) \\ \mathcal{T}_2 = (S_2, Act_2, \longrightarrow_2, \ldots) \end{array}\right\} \text{ two TS}$$

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if action names are irrelevant: $Act_1 = Act_2 = Act = \{\tau\}$

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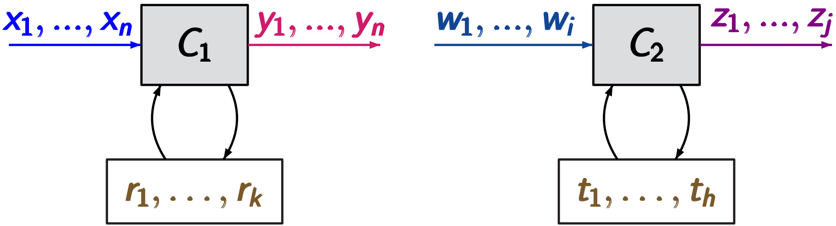
synchronous product:

$$\mathcal{T}_1 \otimes \mathcal{T}_2 \; = \; (S_1 \times S_2, Act, \longrightarrow, \ldots)$$

transition relation $\longrightarrow$:

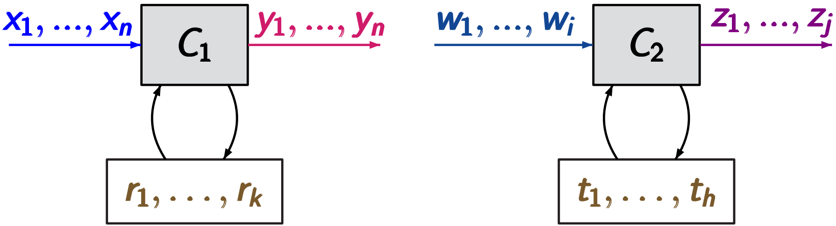
$$\frac{s_1 \xrightarrow{\alpha}_1 s_1' \;\wedge\; s_2 \xrightarrow{\beta}_2 s_2'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha * \beta} \langle s_1', s_2' \rangle}$$

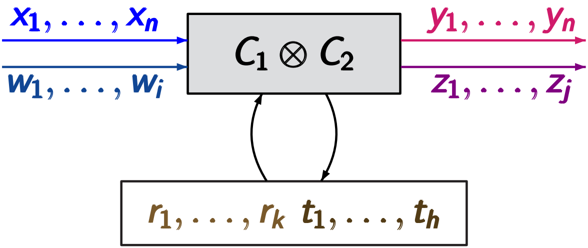
Synchronous product for composing circuits



2 sequential circuits

Synchronous product for composing circuits





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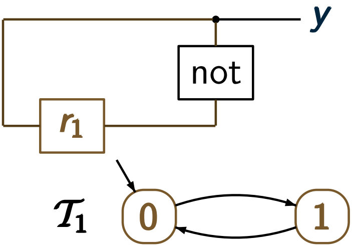

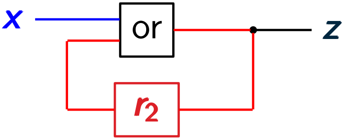

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$r_1 = 0$

transition function:
$\delta_{r_1} = \neg r_1$

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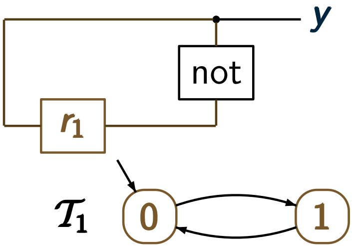

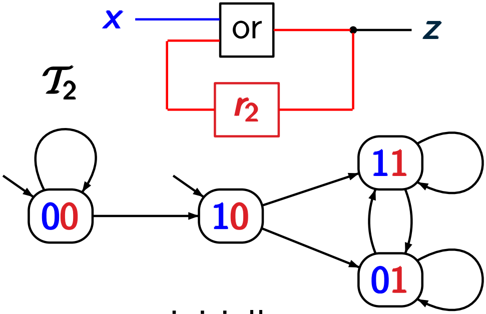

initially:
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initially:
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transition function:
$\delta_{r_2} = r_2 \vee x$

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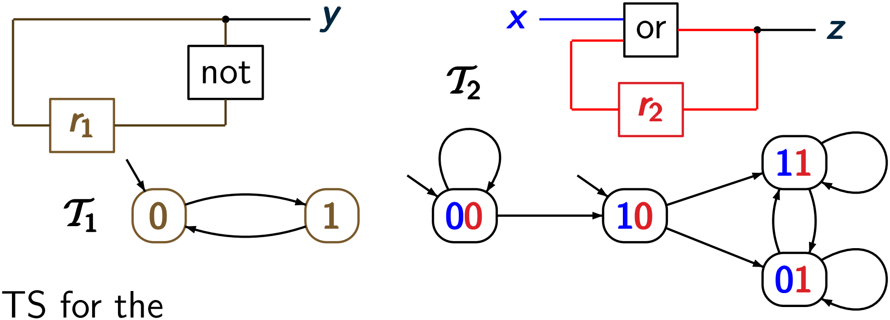
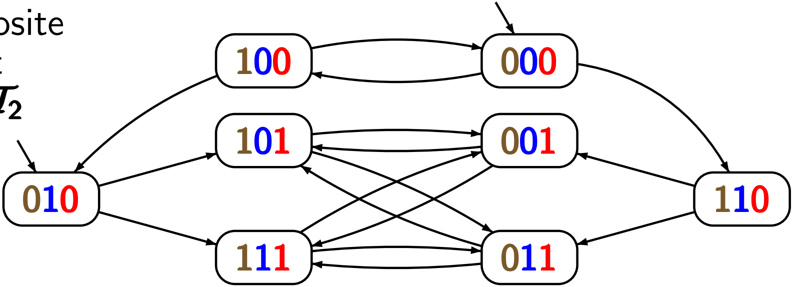


TS for the composite circuit $\mathcal{T}_1 \otimes \mathcal{T}_2$



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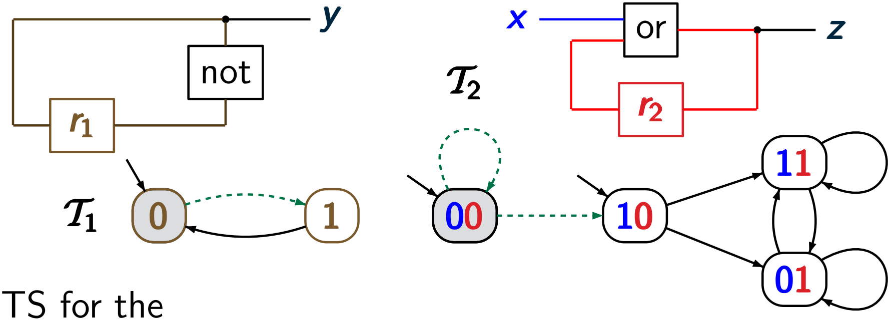

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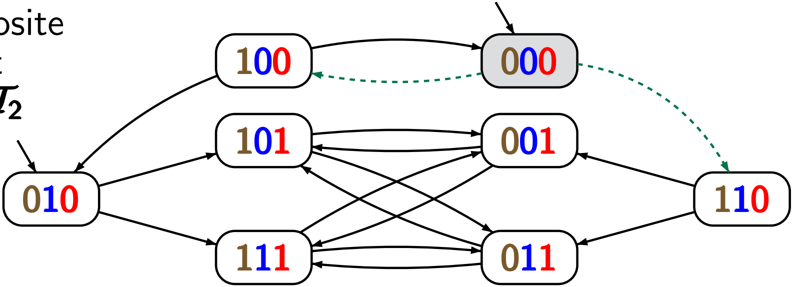

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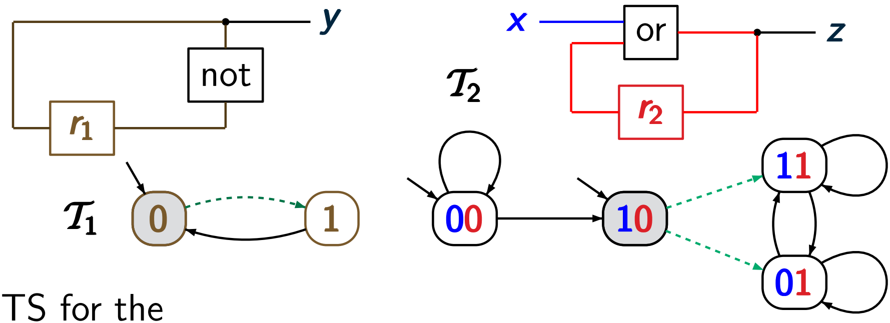

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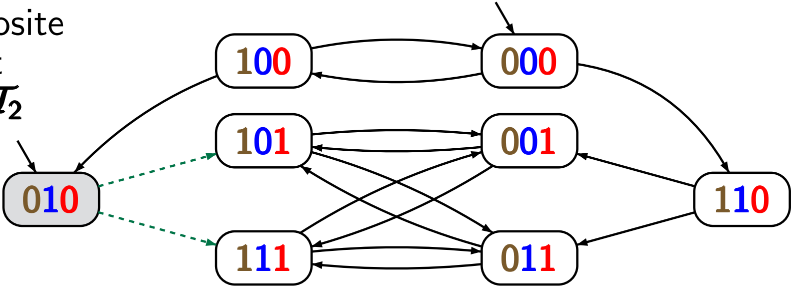

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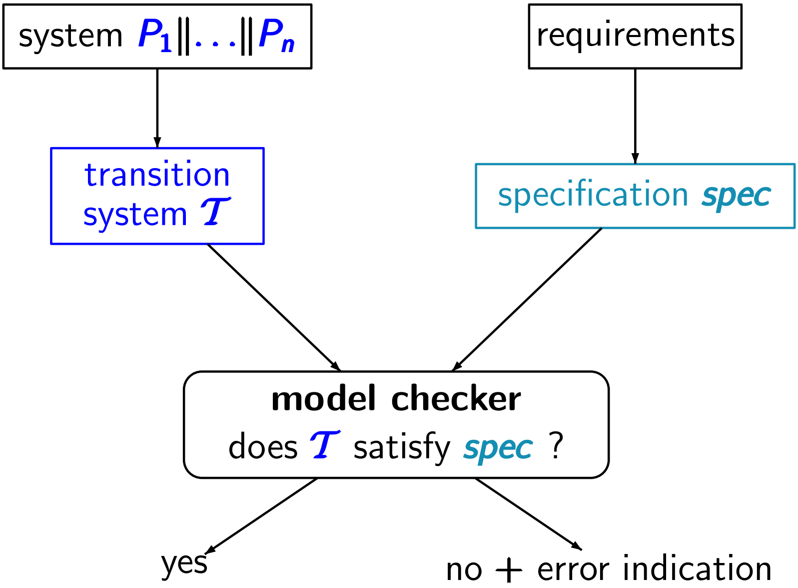
e.g., for channel systems: size of the state space is

$$|Loc_1| \cdot \ldots \cdot |Loc_n| \cdot \prod_{x \in Var} |Dom(x)| \cdot \prod_{c \in Chan} |Dom(c)|^{cap(c)}$$

Model checking



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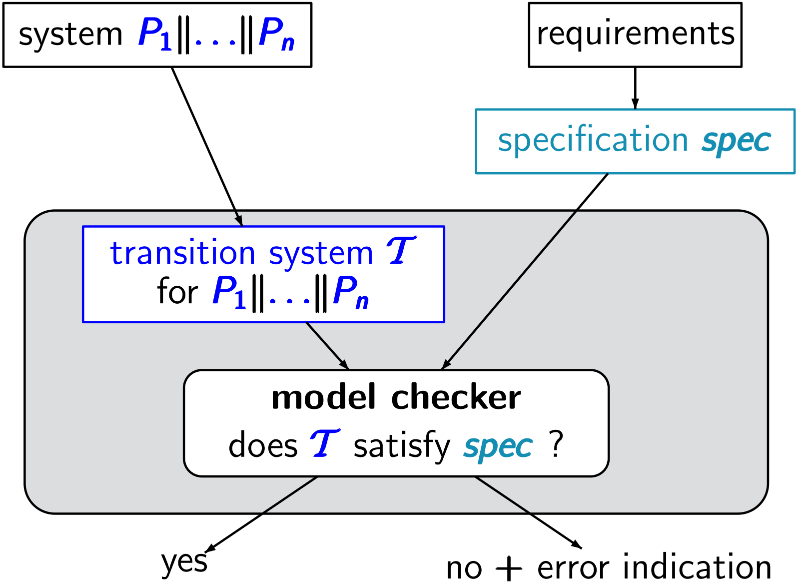

Model checking