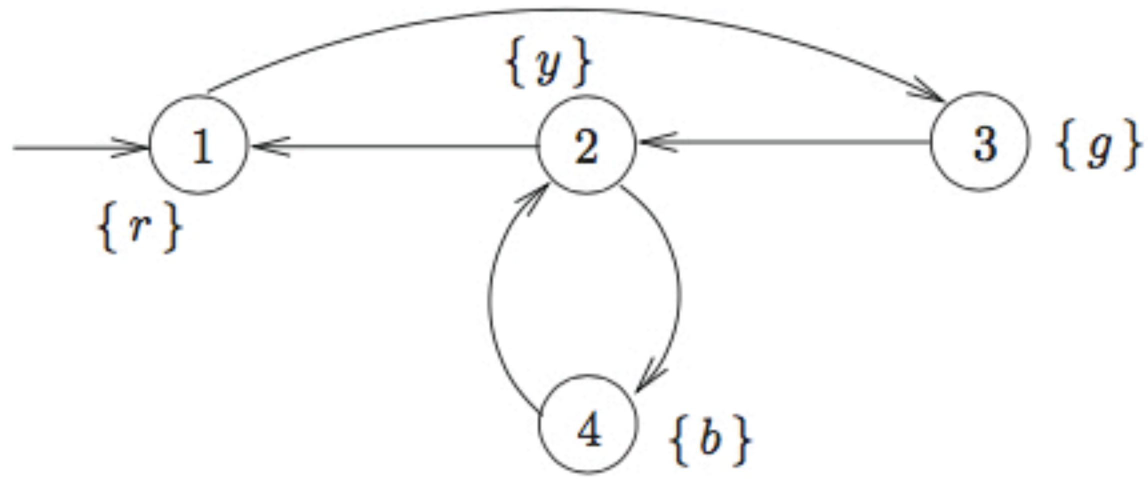
EXERCISE 6.1. Consider the following transition system over $AP = \{b, g, r, y\}$:

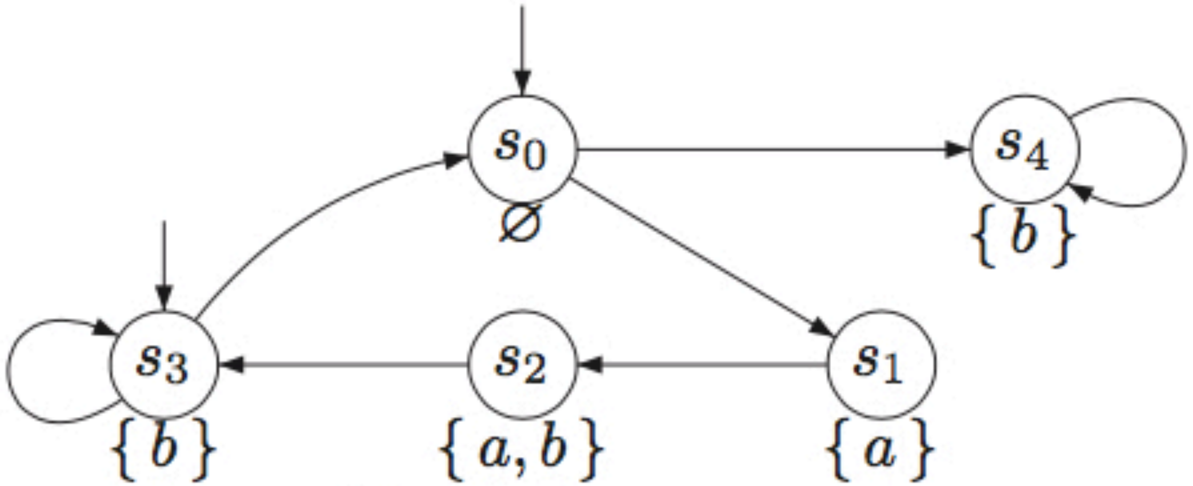



The following atomic propositions are used: $r$ (red), $y$ (yellow), $g$ (green), and $b$ (black). The model is intended to describe a traffic light that is able to blink yellow. You are requested to indicate for each of the following CTL formulae the set of states for which these formulae hold:

| | | | |
|---|---|---|---|
| (a) | $\forall\Diamond y$ | (g) | $\exists\Box\neg g$ |
| (b) | $\forall\Box y$ | (h) | $\forall(b \mathsf{U} \neg b)$ |
| (c) | $\forall\Box\forall\Diamond y$ | (i) | $\exists(b \mathsf{U} \neg b)$ |
| (d) | $\forall\Diamond g$ | (j) | $\forall(\neg b \mathsf{U} \exists\Diamond b)$ |
| (e) | $\exists\Diamond g$ | (k) | $\forall(g \mathsf{U} \forall(y \mathsf{U} r))$ |
| (f) | $\exists\Box g$ | (l) | $\forall(\neg b \mathsf{U} b)$ |

EXERCISE 6.2. Consider the following CTL formulae and the transition system $TS$ outlined on the right:

$$\Phi_1 = \forall(a \,\mathsf{U}\, b) \vee \exists \bigcirc (\forall\Box\, b)$$
$$\Phi_2 = \forall\Box\,\forall(a \,\mathsf{U}\, b)$$
$$\Phi_3 = (a \wedge b) \to \exists\Box\,\exists\bigcirc\,\forall(b \,\mathsf{W}\, a)$$
$$\Phi_4 = (\forall\Box\,\exists\Diamond\,\Phi_3)$$




Determine the satisfaction sets $Sat(\Phi_i)$ and decide whether $TS \models \Phi_i$ $(1 \leqslant i \leqslant 4)$.

**Exercise 6.3.** Which of the following assertions are correct? Provide a proof or a counterexample.

(a) If $s \models \exists\Box\, a$, then $s \models \forall\Box\, a$.

(b) If $s \models \forall\Box\, a$, then $s \models \exists\Box\, a$.

(c) If $s \models \forall\Diamond\, a \vee \forall\Diamond\, b$, then $s \models \forall\Diamond\,(a \vee b)$.

(d) If $s \models \forall\Diamond\,(a \vee b)$, then $s \models \forall\Diamond\, a \vee \forall\Diamond\, b$.

**Exercise 6.4.** Let $\Phi$ and $\Psi$ be arbitrary CTL formulae. Which of the following equivalences for CTL formulae are correct?

(a) $\forall\bigcirc\forall\Diamond\Phi \equiv \forall\Diamond\forall\bigcirc\Phi$

(b) $\exists\bigcirc\exists\Diamond\Phi \equiv \exists\Diamond\exists\bigcirc\Phi$

(c) $\forall\bigcirc\forall\Box\Phi \equiv \forall\Box\forall\bigcirc\Phi$

(d) $\exists\bigcirc\exists\Box\Phi \equiv \exists\Box\exists\bigcirc\Phi$

(e) $\exists\Diamond\exists\Box\Phi \equiv \exists\Box\exists\Diamond\Phi$

(f) $\forall\Box(\Phi \Rightarrow (\neg\Psi \wedge \exists\bigcirc\Phi)) \equiv (\Phi \Rightarrow \neg\forall\Diamond\Psi)$

(g) $\forall\Box(\Phi \Rightarrow \Psi) \equiv (\exists\bigcirc\Phi \Rightarrow \exists\bigcirc\Psi)$

(h) $\neg\forall(\Phi \mathsf{U} \Psi) \equiv \exists(\Phi \mathsf{U} \neg\Psi)$

(i) $\exists((\Phi\wedge\Psi) \mathsf{U} (\neg\Phi\wedge\Psi)) \equiv \exists(\Phi \mathsf{U} (\neg\Phi\wedge\Psi))$

(j) $\forall(\Phi \mathsf{W} \Psi) \equiv \neg\exists(\neg\Phi \mathsf{W} \neg\Psi)$

(k) $\exists(\Phi \mathsf{U} \Psi) \equiv \exists(\Phi \mathsf{U} \Psi) \wedge \exists\Diamond\Psi$

(l) $\exists(\Psi \mathsf{W} \neg\Psi) \vee \forall(\Psi \mathsf{U} \text{false}) \equiv \exists\bigcirc\Phi \vee \forall\bigcirc\neg\Phi$

(m) $\forall\Box\Phi \wedge (\neg\Phi \vee \exists\bigcirc\exists\Diamond\neg\Phi) \equiv \exists X\neg\Phi \wedge \forall\bigcirc\Phi$

(n) $\forall\Box\forall\Diamond\Phi \equiv \Phi \wedge (\forall\bigcirc\forall\Box\forall\Diamond\Phi) \vee \forall\bigcirc(\forall\Diamond\Phi \wedge \forall\Box\forall\Diamond\Phi)$

(o) $\forall\Box\Phi \equiv \Phi \vee \forall\bigcirc\forall\Box\Phi$

EXERCISE 6.7. Transform the following CTL formulae into ENF and PNF. Show all intermediate steps.

$$\Phi_1 = \forall( (\neg a) \mathsf{W} (b \rightarrow \forall \bigcirc c) )$$

$$\Phi_2 = \forall \bigcirc ( \exists((\neg a) \mathsf{U} (b \wedge \neg c)) \vee \exists \Box \forall \bigcirc a )$$

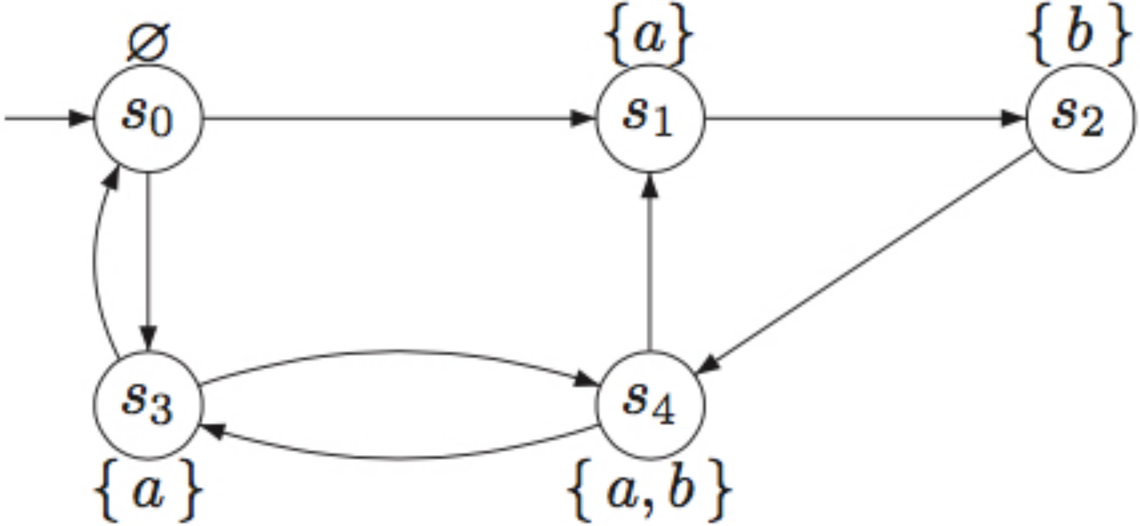
EXERCISE 6.9. Consider the CTL formula

$$\Phi = \forall\Box\left(a \rightarrow \forall\Diamond\left(b \wedge \neg a\right)\right)$$

and the following CTL fairness assumption:

$$\mathit{fair} = \forall\Diamond\,\forall\bigcirc\left(a \wedge \neg b\right) \rightarrow \forall\Diamond\,\forall\bigcirc\left(b \wedge \neg a\right) \wedge \Diamond\Box\,\exists\Diamond\, b \rightarrow \Box\Diamond\, b.$$

Prove that $TS \models_{fair} \Phi$ where transition system $TS$ is depicted below.


∅
{a}
{b}
$s_0$
$s_1$
$s_2$
$s_3$
$s_4$
{a}
{a, b}

EXERCISE 6.14. Check for each of the following formula pairs $(\Phi_i, \varphi_i)$ whether the CTL formula $\Phi_i$ is equivalent to the LTL formula $\varphi_i$. Prove the equivalence or provide a counterexample that illustrates why $\Phi_i \not\equiv \varphi_i$.



(a) $\Phi_1 = \forall\Box\forall\bigcirc a.$ and $\varphi_1 = \Box\bigcirc a$

(b) $\Phi_2 = \forall\Diamond\forall\bigcirc a$ and $\varphi_2 = \Diamond\bigcirc a.$

(c) $\Phi_3 = \forall\Diamond(a \land \exists\bigcirc a)$ and $\varphi_3 = \Diamond(a \land \bigcirc a).$

(d) $\Phi_4 = \forall\Diamond a \lor \forall\Diamond b$ and $\varphi_4 = \Diamond(a \lor b).$

(e) $\Phi_5 = \forall\Box(a \rightarrow \forall\Diamond b)$ and $\varphi_5 = \Box(a \rightarrow \Diamond b).$

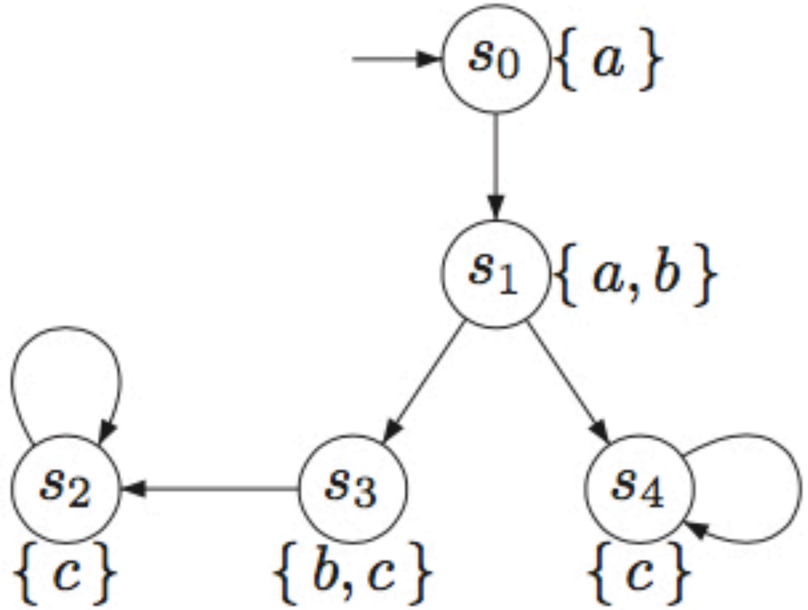
(f) $\Phi_6 = \forall(b \,\mathsf{U}\, (a \land \forall\Box b))$ and $\varphi_6 = \Diamond a \land \Box b.$

EXERCISE 6.16.

Consider the following CTL formulae

$$\Phi_1 = \exists\Diamond\,\forall\Box\, c \quad \text{and} \quad \Phi_2 = \forall(a \,\mathsf{U}\, \forall\Diamond\, c)$$

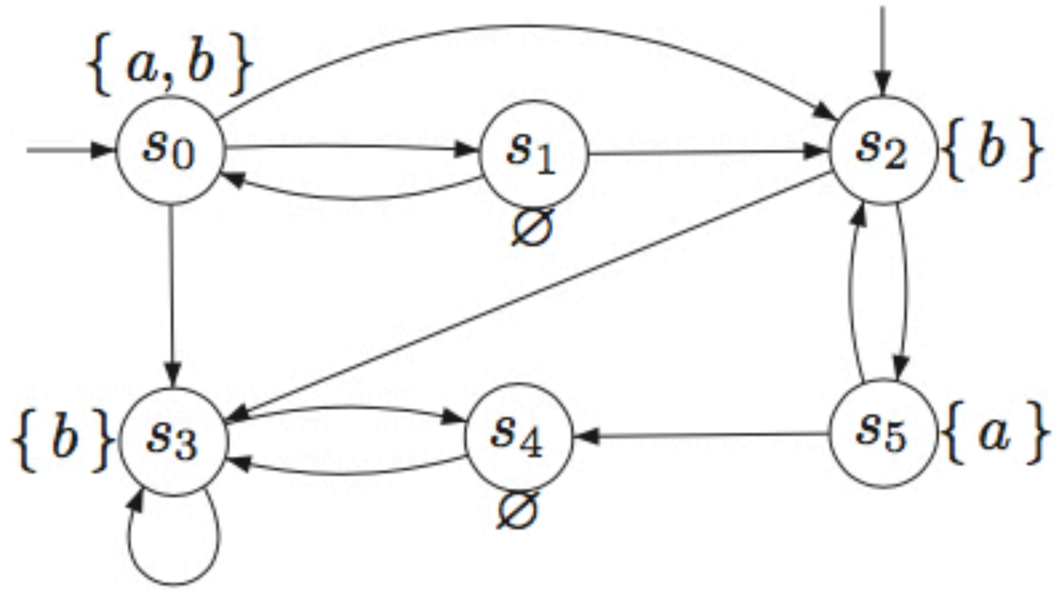
and the transition system $TS$ outlined on the right. Decide whether $TS \models \Phi_i$ for $i = 1, 2$ using the CTL model-checking algorithm. Sketch its main steps.



EXERCISE 6.21. Consider the CTL formula $\Phi$ and the strong fairness assumption *sfair*:

$$\begin{aligned}\Phi &= \forall\Box\,\forall\Diamond\, a\\ sfair &= \Box\Diamond\,\underbrace{(b \wedge \neg a)}_{\Phi_1} \rightarrow \Box\Diamond\,\underbrace{\exists(b\,\mathsf{U}\,(a \wedge \neg b))}_{\Psi_1}\end{aligned}$$

and transition system $TS$ over $AP = \{a, b\}$ which is given by



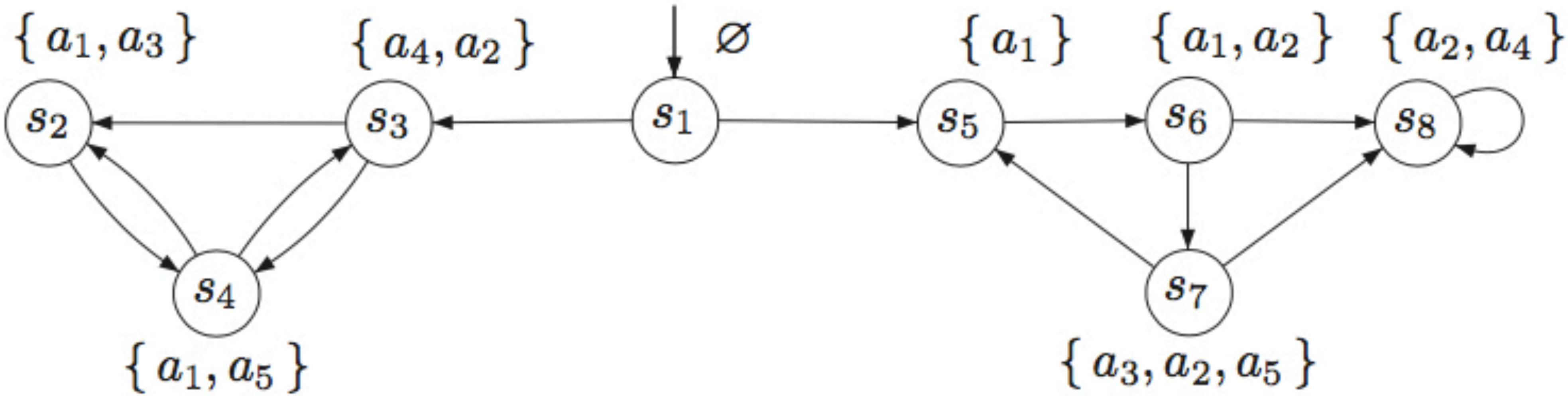

Questions:

(a) Determine $Sat(\Phi_1)$ and $Sat(\Psi_1)$ (without fairness).

(b) Determine $Sat_{sfair}(\exists\Box\ \text{true})$.

(c) Determine $Sat_{sfair}(\Phi)$.

EXERCISE 6.23. Consider the following transition system $TS$ over $AP = \{ a_1, \ldots, a_6 \}$.



Let $\Phi = \exists \bigcirc (a_1 \rightarrow \exists(a_1 \mathsf{U}\, a_2))$ and $\mathit{sfair} = \mathit{sfair}_1 \wedge \mathit{sfair}_2 \wedge \mathit{sfair}_3$ a strong CTL fairness assumption where

$$
\begin{aligned}
\mathit{sfair}_1 &= \Box\Diamond\, \forall\Diamond(a_1 \vee a_3) \longrightarrow \Box\Diamond a_4 \\
\mathit{sfair}_2 &= \Box\Diamond(a_3 \wedge \neg a_4) \longrightarrow \Box\Diamond a_5 \\
\mathit{sfair}_3 &= \Box\Diamond(a_2 \wedge a_5) \longrightarrow \Box\Diamond a_6
\end{aligned}
$$

Sketch the main steps for computing the satisfaction sets $\mathit{Sat}_{\mathit{sfair}}(\exists\Box\text{true})$ and $\mathit{Sat}_{\mathit{sfair}}(\Phi)$.