LTL Syntax and Semantics

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Topics

- Syntax of Linear Time Logic (LTL). Basic and derived operators. Examples.
- Semantics of LTL: satisfaction of a formula by an infinite word. Examples.
- Semantics of LTL: satisfaction of a formula by a maximal path fragment of a transition system. Examples.
- Semantics of LTL: satisfaction of a formula by a transition system.
- Exercises on LTL formula semantics and satisfaction relations.

Material

Reading:

Chapter 5 of the book, pages 225-243.

More:

The slides in the following pages are taken from the material of the course "Introduction to Model Checking" held by Prof. Dr. Ir. Joost-Pieter Katoen at Aachen University.

Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation-Tree Logic

Equivalences and Abstraction

Temporal logics

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here: two propositional temporal logics:

LTL: linear temporal logic

CTL: computation tree logic

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where $a \in AP$

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where $a \in AP$ $\qquad \bigcirc \mathrel{\widehat{=}}$ next $\qquad \mathbf{U} \mathrel{\widehat{=}}$ until

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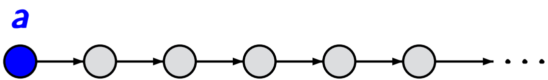
where $a \in AP$ $\qquad \bigcirc \ \widehat{=}$ next $\qquad \mathsf{U} \ \widehat{=}$ until

atomic proposition $a \in AP$



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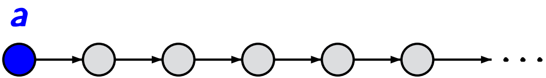
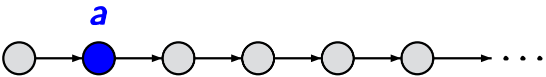


next operator $\bigcirc a$



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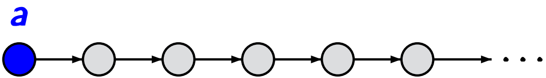

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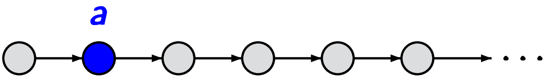
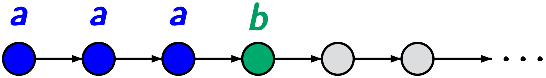


until operator $a \,\mathsf{U}\, b$



Derived operators in LTL

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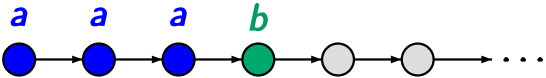
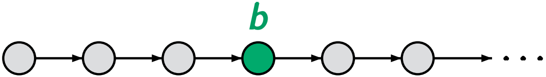


eventually
$\Diamond b$



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derived operators:

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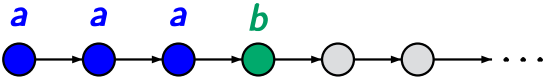
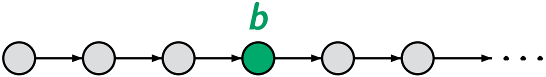
$\lozenge\varphi$	$\stackrel{\text{def}}{=}$	$\mathit{true}\,\mathsf{U}\,\varphi$	eventually
$\square\varphi$	$\stackrel{\text{def}}{=}$	$\neg\lozenge\neg\varphi$	always

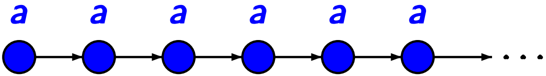
until operator
$a\,\mathsf{U}\,b$



eventually
$\lozenge b$

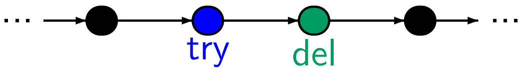


always
$\square a$



Next $\bigcirc$, until U and eventually $\Diamond$

$\Box\,(\text{try\_to\_send} \rightarrow \bigcirc\ \text{delivered})$



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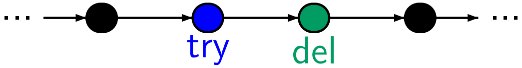
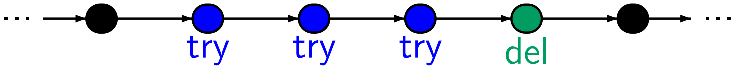
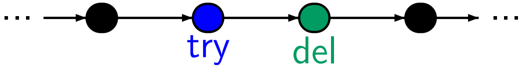


$\Box$ (try_to_send $\rightarrow$ try_to_send **U** delivered)



Next $\bigcirc$, until $\mathbf{U}$ and eventually $\Diamond$

$\square\,(\text{try\_to\_send} \rightarrow \bigcirc\ \text{delivered})$



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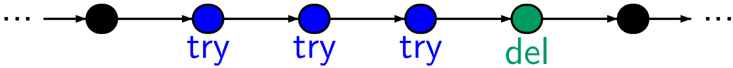

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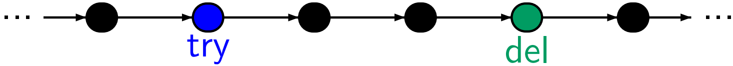

Examples for LTL formulas

$$\varphi ::= \mathit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

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Examples for LTL formulas:

mutual exclusion: $\Box(\neg \mathit{crit}_1 \vee \neg \mathit{crit}_2)$

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traffic light: $\Box(yellow \vee \bigcirc\neg red)$

Infinitely often and eventually forever

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e.g., unconditional fairness $\Box\Diamond \mathit{crit}_i$

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eventually forever		$\Diamond\Box\varphi$	

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interpretation of **LTL** formulas over traces, i.e.,
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$suffix(\sigma, j) = A_j\, A_{j+1}\, A_{j+2}\, \ldots \models \varphi_2$ and

$suffix(\sigma, i) = A_i\, A_{i+1}\, A_{i+2}\, \ldots \models \varphi_1$ for $0 \leq i < j$

Semantics of LTL over infinite words

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$\sigma \models \varphi_1 \mathsf{U} \varphi_2$	iff	there exists $j \geq 0$ such that

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LT property of formula φ:

$$Words(\varphi) \overset{\text{def}}{=} \{\sigma \in (2^{AP})^\omega : \sigma \models \varphi\}$$

LTL-semantics of derived operators $\lozenge$ and $\square$

LTL-semantics of derived operators $\Diamond$ and $\Box$

for $\sigma = A_0 A_1 A_2 \ldots \in (2^{AP})^\omega$:

$\vdots$

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$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that
		$A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have:
		$A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$

LTL semantics over TS

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given a TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

define satisfaction relation $\models$ for

- **LTL** formulas over AP
- the maximal path fragments and states of $\mathcal{T}$

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define satisfaction relation $\models$ for

- **LTL** formulas over AP
- the maximal path fragments and states of $\mathcal{T}$

assumption: $\mathcal{T}$ has no terminal states, i.e.,
all maximal path fragments in $\mathcal{T}$ are infinite

LTL semantics over paths of TS

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given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

LTL semantics over paths of TS

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LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

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$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$
$$\text{iff} \quad trace(\pi) \in Words(\varphi)$$

LTL semantics over paths of TS

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LTL formula φ over AP

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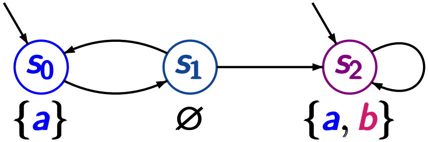
$$\text{iff} \quad trace(\pi) \in Words(\varphi)$$

remind: LT property of an LTL formula:

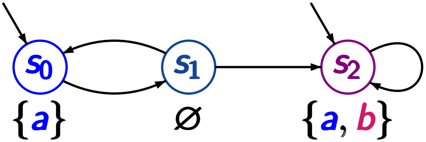
$$Words(\varphi) = \{\sigma \in (2^{AP})^\omega : \sigma \models \varphi\}$$

Example: LTL-semantics over paths



$AP = \{a, b\}$

Example: LTL-semantics over paths



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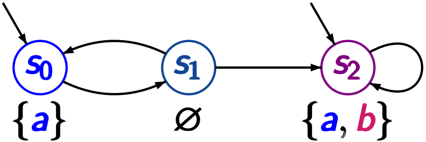

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$\pi \models a$

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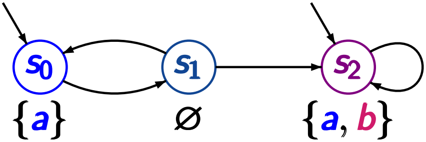

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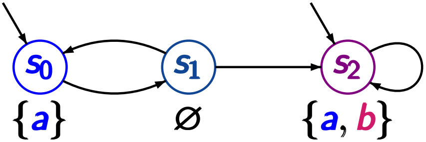

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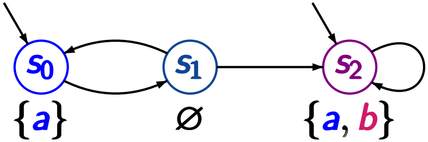

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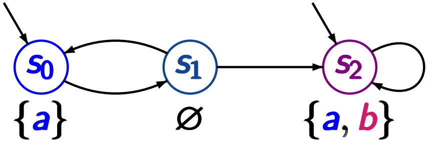

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$\pi \models a$, but $\pi \not\models b$	as $L(s_0) = \{a\}$
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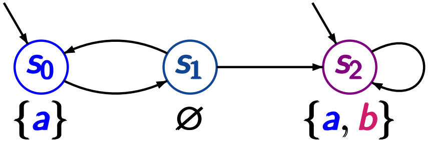

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path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$ as $L(s_0) = \{a\}$

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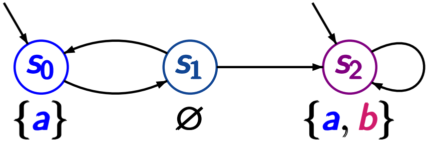

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$\pi \models a$, but $\pi \not\models b$	as $L(s_0) = \{a\}$
$\pi \models \bigcirc(\neg a \wedge \neg b)$	as $L(s_1) = \varnothing$
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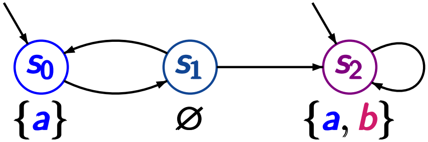

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$\pi \models (\neg b)\, \mathsf{U}\, (a \wedge b)$	as $s_0, s_1 \models \neg b$
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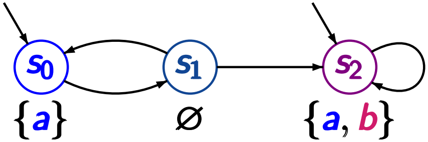

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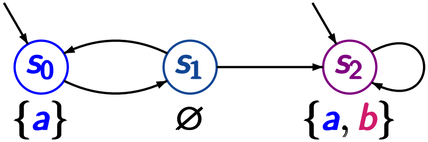
$\pi \models (\neg b) \,\mathsf{U}\, \Box(a \wedge b)$	and $s_2 \models a \wedge b$

Correct or wrong ?



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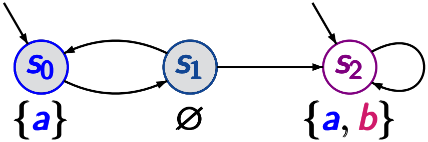

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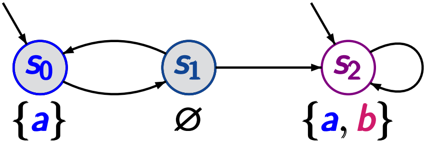

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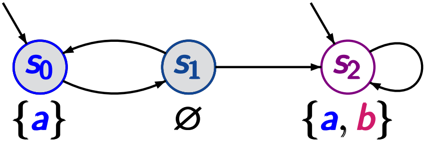

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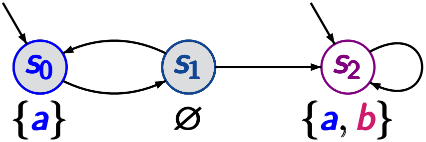

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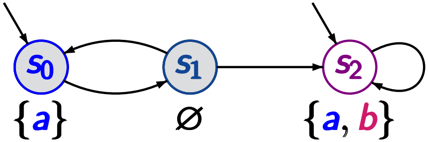

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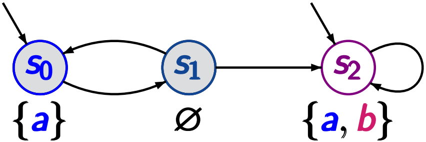

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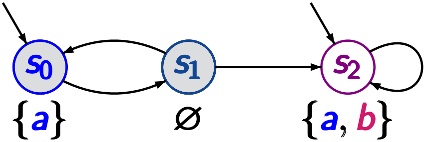

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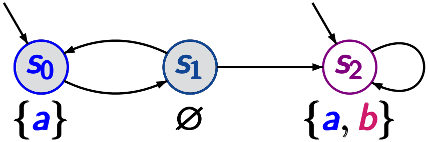

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$\pi \models \Box a$?

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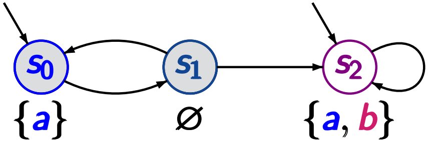

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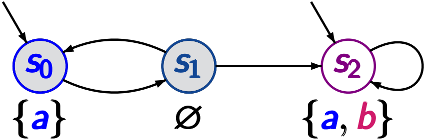

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$\pi \models \Box\Diamond a$?

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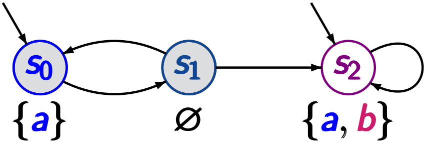

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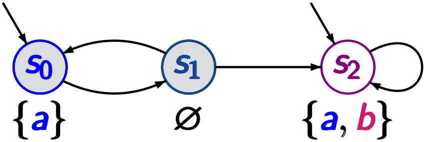

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$\pi \not\models a \,\mathsf{U}\, b$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

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$\pi \models \bigcirc \bigcirc \neg b$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ as $s_1 \not\models a$

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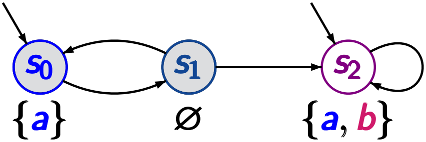

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$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\qquad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ $\qquad$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ $\qquad$ as $s_1 \not\models a$

$\pi \models \Box\Diamond a$ $\qquad$ as $\Box\Diamond \mathrel{\widehat{=}}$ infinitely often

$\pi \not\models \Diamond\Box a$ $\qquad$ as $\Diamond\Box \mathrel{\widehat{=}}$ eventually forever

LTL-semantics of derived operators

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

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$\sigma \models \Diamond\varphi$ iff there exists $j \geq 0$ such that $A_j \, A_{j+1} \, A_{j+2} \, \ldots \models \varphi$

$\sigma \models \Box\varphi$ iff for all $j \geq 0$ we have: $A_j \, A_{j+1} \, A_{j+2} \, \ldots \models \varphi$

LTL-semantics of derived operators

for $\sigma = A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$:

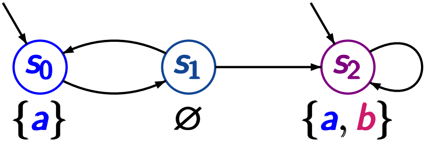
$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have: $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\Diamond\varphi$	iff	there are infinitely many $j \geq 0$ s.t. $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$

LTL-semantics of derived operators

for $\sigma = A_0 A_1 A_2 \ldots \in \left(2^{AP}\right)^\omega$:

$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that $A_j A_{j+1} A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have: $A_j A_{j+1} A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\Diamond\varphi$	iff	there are infinitely many $j \geq 0$ s.t. $A_j A_{j+1} A_{j+2} \ldots \models \varphi$
$\sigma \models \Diamond\Box\varphi$	iff	for almost all $j \geq 0$ we have: $A_j A_{j+1} A_{j+2} \ldots \models \varphi$

LTL-semantics over paths



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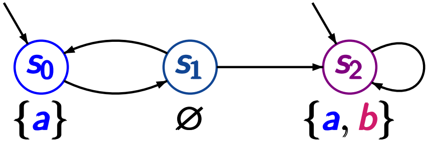
LTL-semantics over paths



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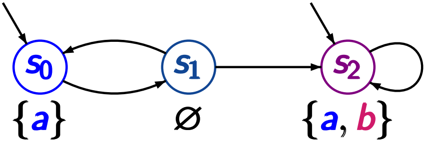

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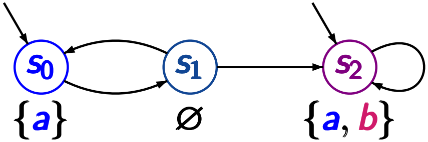
$\pi \models \bigcirc((\neg a \wedge \neg b) \,\mathsf{U}\, (a \wedge b))$?



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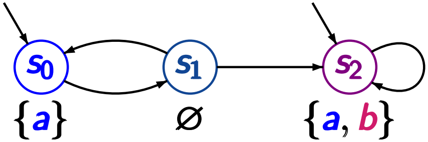
LTL-semantics over paths



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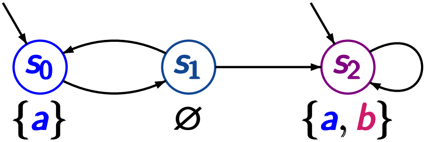
$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U}(a \wedge b))$ as $s_1 \models \neg a \wedge \neg b$

$s_2 \models a \wedge b$

$\pi \models \bigcirc\Box(a \leftrightarrow b)$?

LTL-semantics over paths



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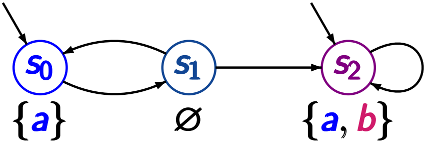
LTL-semantics over paths



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

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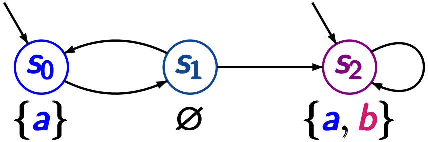

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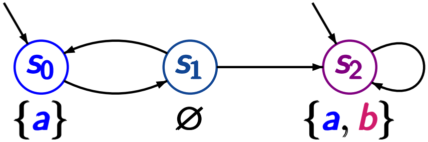

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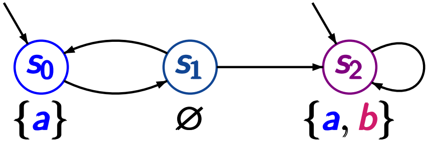

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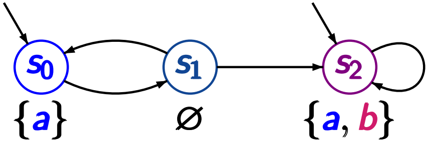

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$\pi \models \Box(\neg b \rightarrow \bigcirc a)$?

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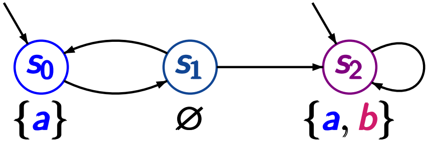

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$\pi \models a\, \mathsf{U}\, (\neg b\, \mathsf{U}\, a)$	as $s_0, s_2 \models a$, $s_1 \models \neg b$
$\pi \models \Diamond\Box(\neg a \rightarrow \Diamond\neg b)$	as $s_2\, s_2\, s_2 \ldots \models \neg a \rightarrow \Diamond\neg b$
$\pi \not\models \Box(\neg b \rightarrow \bigcirc a)$	as $s_0 \models \neg b$, $s_1 \not\models a$

LTL semantics over the states of a TS

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given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

interpretation of φ over states:

$$s \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(s)$$

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$\mathcal{T} \models \varphi$	iff	$s_0 \models \varphi$ for all $s_0 \in S_0$
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	iff	$Traces(\mathcal{T}) \subseteq Words(\varphi)$
	iff	$\mathcal{T} \models Words(\varphi)$

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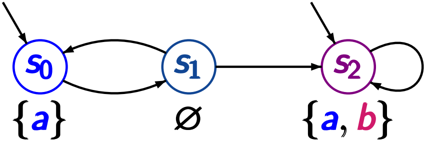
↑ satisfaction relation for LT properties

Which formulas hold for $\mathcal{T}$?



$AP = \{a, b\}$

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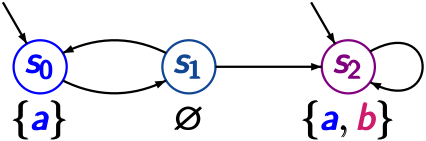

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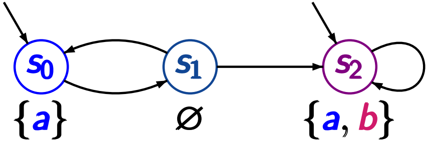
$\mathcal{T} \models a$

Which formulas hold for $\mathcal{T}$?



$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

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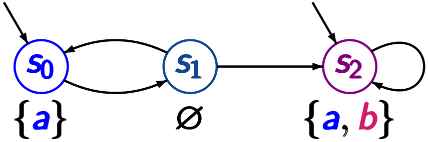
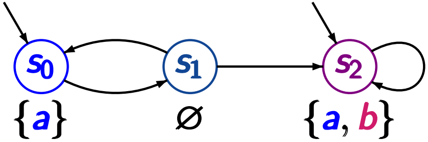


$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \models \Diamond\Box a$

Which formulas hold for $\mathcal{T}$?



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$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \not\models \Diamond\Box a$

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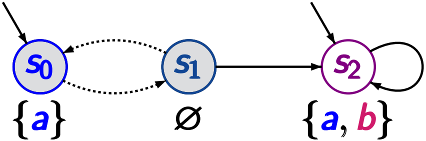

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$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

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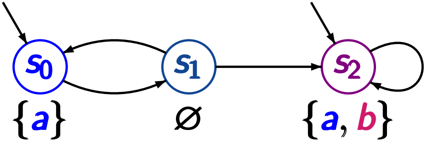

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$\mathcal{T} \models \Diamond\Box b \lor \Box\Diamond(\neg a \land \neg b)$

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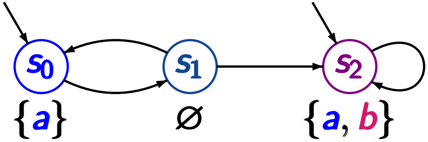

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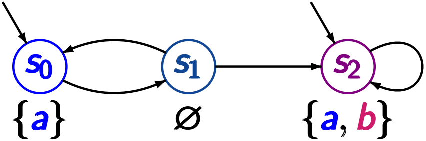
Which formulas hold for $\mathcal{T}$?



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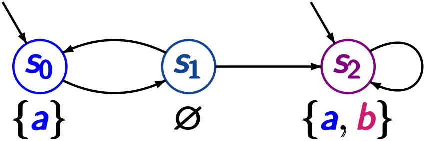

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