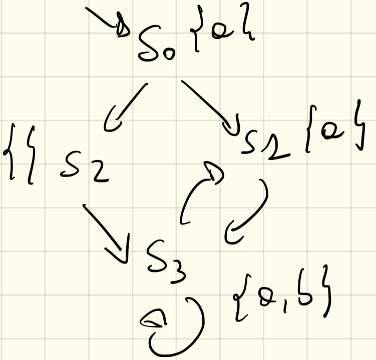


$AP = \{a, b\}$

$\pi_1 \quad s_0 s_2 s_3^{\omega}$

$\pi_2 \quad s_0 s_2 (s_3 s_2)^{\omega}$

$\pi_3 \quad s_0 (s_1 s_3)^{\omega}$

$\pi_4 \quad (s_0 s_2 s_3 \mid s_0 s_1 s_3) \left[s_3^* (s_1 s_3)^+ \right]^{\omega}$

$\pi_5 \quad (s_0 s_2 s_3 \mid s_0 s_1 s_3) \left[s_3^* (s_2 s_3)^+ \right]^+ s_3^{\omega}$

$Trace(\pi_1) = \{a\}\{\}\{a,b\}^{\omega}$ $\quad Trace(\pi_2) = \{a\}\{\}(\{a,b\}\{a\})^{\omega}$

$Trace(\pi_3) = \{a\}(\{a\}\{a,b\})^{\omega}$

$Trace(\pi_4) = (\{a\}\{\}\{a,b\} \mid \{a\}\{a\}\{a,b\}) \left(\{a,b\}^* (\{a\}\{a,b\})^+\right)^{\omega}$

$Trace(\pi_5) = (\{a\}\{\}\{a,b\} \mid \{a\}\{a\}\{a,b\}) \left(\{a,b\}^* (\{a\}\{a,b\})^+\right)^+ \{a,b\}^{\omega}$

$AP = \{x=0,\ x>1\}$

1) false $\quad E_1 = \{\}$

2) initially x is equal to zero $\quad E_2 = \{A_0 A_1 A_2 \dots \in (2^{AP})^\omega \mid x=0 \in A_0\}$

SAFETY

Bad Prefix (Minimal): $\{x>1\} \dots$

$\{\} \dots$

3) initially x differs from zero $\quad E_3 = \{A_0 A_1 A_2 \dots \in (2^{AP})^\omega \mid x=0 \notin A_0\}$

SAFETY

MBP: $\{x=0\} \dots$

4) initially x is equal to zero, but at some point x exceeds one

$E_4 = \{A_0 A_1 \dots \in (2^{AP})^\omega \mid x=0 \in A_0 \land \exists j \in \mathbb{N} : j>0 \land x>1 \in A_j\}$

E_4 is a MIXED property:

$$SAFE = \{ A_0 A_1 \ldots \mid x=0 \in A_0 \} \cap LIVE = \{ A_0 A_1 \ldots \mid \exists j \in \mathbb{N} : j>0 \land x>1 \in A_j \}$$

5) x exceeds one only finitely many times $E_5 = \{ A_0 A_1 \ldots \in (2^{AP})^\omega \mid *$

LIVENESS

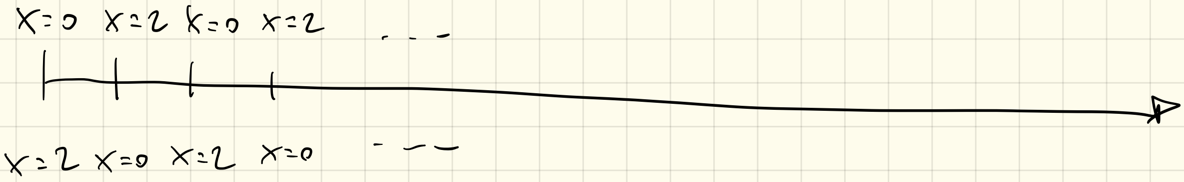
$$* \ \underbrace{\exists j \in \mathbb{N} : \forall i \in \mathbb{N} .\ i \geq j \Rightarrow x>1 \notin A_i}_{\overset{\infty}{\forall} i \in \mathbb{N} .\ x>1 \notin A_i} \}$$

(equivalent)

6) x exceeds one infinitely often $E_6 \{ A_0 A_1 \ldots \mid \overset{\infty}{\exists} i \in \mathbb{N} : x>1 \in A_i \}$

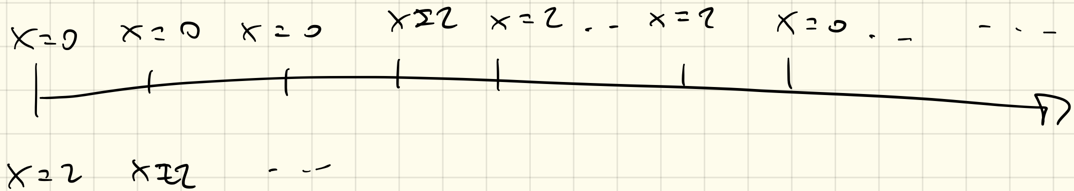
LIVENESS

7) The value of x alternates between zero and two



(a)

OR



(b)

$$E_7^{(a)} = \{ A_0 A_1 \ldots \in (2^{AP})^\omega \mid \forall i \in \mathbb{N} \, (x=0 \in A_{2i} \wedge x>1 \in A_{2i+1}) \vee (x \geq 1 \in A_{2i} \wedge x=0 \in A_{2i+1}) \}$$

$\downarrow$

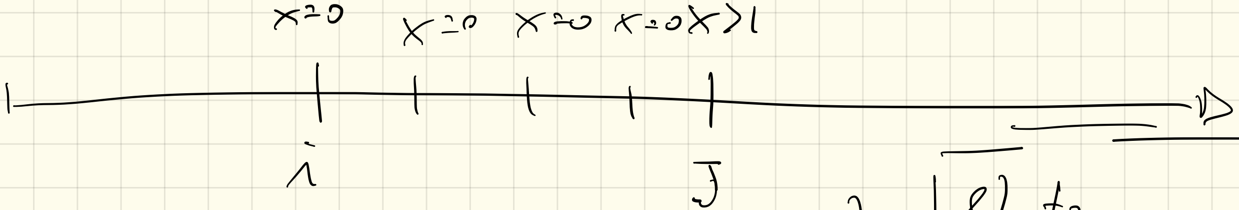
SAFETY

BP: $\{x=0\}\{x=0\} \ldots$

(b)

$E_7 = \{ A_0 A_1 \ldots \in (2^{AP})^\omega \mid \forall i \in \mathbb{N} \; [(x=0 \in A_i) \Rightarrow (\exists j \in \mathbb{N} \; j > i \wedge x > 1 \in A_j \wedge (\forall k \in \mathbb{N} : i < k < j \Rightarrow x = 0 \in A_k))] \wedge$

$[(x>1) \in A_i \Rightarrow (\exists j \in \mathbb{N} : j > i \wedge x = 0 \in A_j \wedge (\forall k \in \mathbb{N} : *$



$* \; i < k < j \Rightarrow x > 1 \in A_k)))] \}$

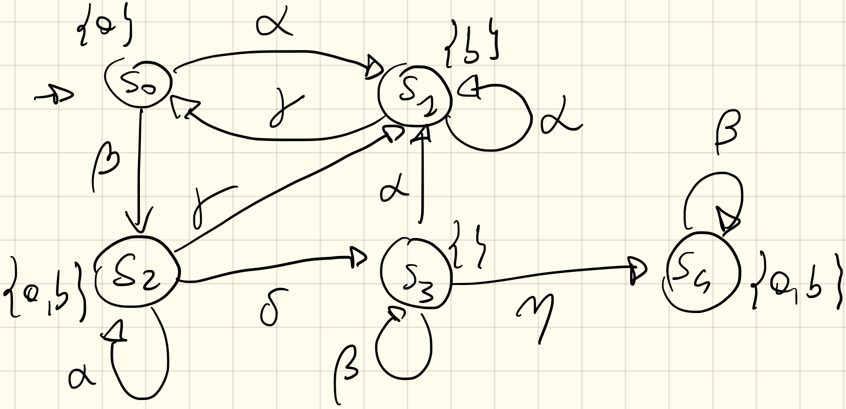
SAFETY BP: $\{x=0\}\{\}\{\}$

8) true

$E_8 = (2^{AP})^\omega$

SAFETY AND LIVENESS (the only one)

P: $\overset{\infty}{\exists} k \in \mathbb{N}: A_k = \{a,b\} \wedge \exists n \geq 0: \forall k > n. (a \in A_k \Rightarrow b \in A_{k+1})$



$F_1 = \{\{\{\alpha\}\}, \{\{\beta\}, \{\delta,\gamma\}, \{\eta\}\}, \emptyset\}$

$F_2 = \{\{\{\alpha\}\}, \{\{\beta\}, \{\gamma\}\}, \{\{\eta\}\}\}$

$T \models_{F_1} P$

1) any run ending in s_4^ω is not fair due to α-ucond

2) any run ending in $(s_0 s_2 s_1)^\omega$ is not fair " "

3) " " " " s_3^ω is not fair " "

4) any run passing infinitely many times in s_3 is not fair (due to (η) strong)

5) any run ending in s_2^ω or s_1^ω is not fair due to strong fairness on {δ, γ}

6) runs ending in $(s_0 s_2)^\omega$ are not fair due to strong fairness of β

The only fair runs are those that pass through s_0, s_2, s_1 infinitely many times, but pass through s_3 only finitely many times

We have that P is true under the fairness assumption F_1.

F_2: The runs in which we pass through s_0, s_2, s_3 and s_1 infinitely many times are fair

P is not satisfied because from s_2 to s_3 we have $a \in L(s_2)$ and $b \notin L(s_3)$