LTL Syntax and Semantics

Luca Tesei

Reactive Systems Verification

MSc in Computer Science

University of Camerino

Topics

- Syntax of Linear Time Logic (LTL). Basic and derived operators. Examples.
- Semantics of LTL: satisfaction of a formula by an infinite word. Examples.
- Semantics of LTL: satisfaction of a formula by a maximal path fragment of a transition system. Examples.
- Semantics of LTL: satisfaction of a formula by a transition system.
- Exercises on LTL formula semantics and satisfaction relations.

Material

Reading:

Chapter 5 of the book, Sections 5.1.1, 5.1.2, 5.1.3

More:

The slides in the following pages are taken from the material of the course "Introduction to Model Checking" held by Prof. Dr. Ir. Joost-Pieter Katoen at Aachen University.

Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation-Tree Logic

Equivalences and Abstraction

Temporal logics

extend propositional or predicate logic by
temporal modalities

Temporal logics

extend propositional or predicate logic by temporal modalities, e.g.

$\Box\varphi$	"φ holds always", i.e.,	now and forever in the future
$\Diamond\varphi$	"φ holds now or eventually in the future"	

extend propositional or predicate logic by temporal modalities, e.g.

$\Box\varphi$	"φ holds always", i.e.,	now and forever in the future
$\Diamond\varphi$	"φ holds now or eventually in the future"	

here: two propositional temporal logics:

LTL: linear temporal logic

CTL: computation tree logic

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

- syntax and semantics of LTL ⟵
- automata-based LTL model checking
- complexity of LTL model checking

Computation-Tree Logic

Equivalences and Abstraction

Linear Temporal Logic (LTL)

Linear Temporal Logic (LTL)

$$\varphi ::= \textit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi$$

where $a \in AP$

$$\varphi \ ::= \ \textit{true} \ \big| \ a \ \big| \ \varphi_1 \wedge \varphi_2 \ \big| \ \neg\varphi \ \big| \ \bigcirc \varphi$$

where $a \in AP$ $\qquad \bigcirc \mathrel{\widehat{=}}$ next

Linear Temporal Logic (LTL)

$$\varphi \ ::= \ \mathit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \, \mathsf{U} \, \varphi_2$$

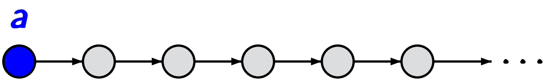
where $a \in AP$ $\qquad$ $\bigcirc \mathrel{\widehat{=}}$ next $\qquad$ $\mathsf{U} \mathrel{\widehat{=}}$ until

Linear Temporal Logic (LTL)

$$\varphi \ ::= \ \textit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

where $a \in AP$ $\qquad \bigcirc \mathrel{\widehat{=}}$ next $\qquad \mathsf{U} \mathrel{\widehat{=}}$ until

atomic proposition $a \in AP$

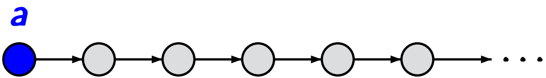


Linear Temporal Logic (LTL)

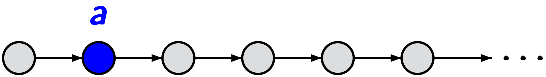
$$\varphi \ ::= \ \mathit{true} \ \mid \ a \ \mid \ \varphi_1 \wedge \varphi_2 \ \mid \ \neg\varphi \ \mid \ \bigcirc\varphi \ \mid \ \varphi_1 \,\mathsf{U}\, \varphi_2$$

where $a \in AP$ $\qquad$ $\bigcirc \ \widehat{=}$ next $\qquad$ $\mathsf{U} \ \widehat{=}$ until

atomic proposition $a \in AP$



next operator $\bigcirc a$

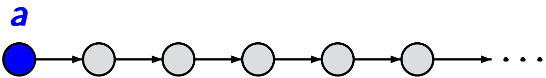


Linear Temporal Logic (LTL)

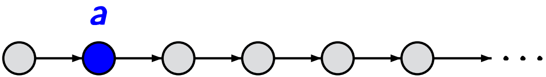
$$\varphi \ ::= \ \textit{true} \ \mid \ a \ \mid \ \varphi_1 \wedge \varphi_2 \ \mid \ \neg\varphi \ \mid \ \bigcirc \varphi \ \mid \ \varphi_1 \, \mathsf{U} \, \varphi_2$$

where $a \in AP$ $\quad \bigcirc \ \widehat{=}$ next $\quad \mathsf{U} \ \widehat{=}$ until

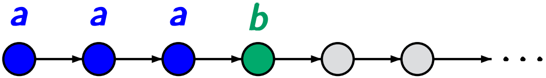
atomic proposition $a \in AP$



next operator $\bigcirc a$



until operator $a \, \mathsf{U} \, b$



Derived operators in LTL

$$\varphi \ ::= \ \textit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \, \mathsf{U} \, \varphi_2$$

derived operators:

$\vee, \rightarrow, \ldots$ as usual

Derived operators in LTL

$$\varphi ::= true \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \mathsf{U} \varphi_2$$

derived operators:

$\vee, \rightarrow, \ldots$ as usual

$\Diamond\varphi \overset{\text{def}}{=} true \, \mathsf{U} \, \varphi$ eventually

Derived operators in LTL

$$\varphi ::= \textit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi_1 \mathsf{U} \varphi_2$$

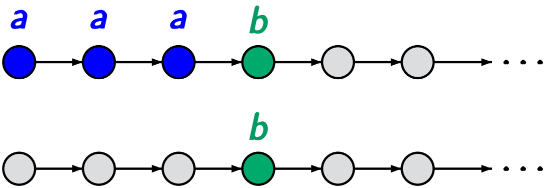
derived operators:

$\vee, \rightarrow, \ldots$ as usual

$\Diamond\varphi \overset{\text{def}}{=} \textit{true}\,\mathsf{U}\,\varphi$ eventually

until operator $a \,\mathsf{U}\, b$

eventually $\Diamond b$



Derived operators in LTL

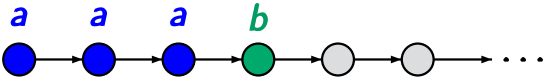
$$\varphi \;::=\; \mathit{true} \;\big|\; a \;\big|\; \varphi_1 \wedge \varphi_2 \;\big|\; \neg\varphi \;\big|\; \bigcirc\varphi \;\big|\; \varphi_1 \,\mathsf{U}\, \varphi_2$$

derived operators:

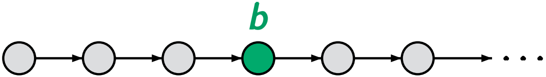
$\vee, \rightarrow, \ldots$ as usual

$\Diamond\varphi$	$\stackrel{\text{def}}{=}$	$\mathit{true}\,\mathsf{U}\,\varphi$	eventually
$\Box\varphi$	$\stackrel{\text{def}}{=}$	$\neg\Diamond\neg\varphi$	always

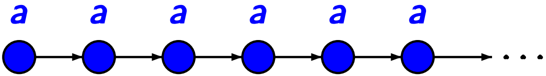
until operator
$a\,\mathsf{U}\,b$



eventually
$\Diamond b$

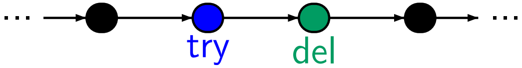


always
$\Box a$



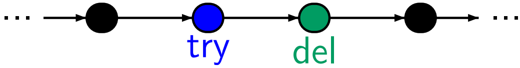
Next $\bigcirc$, until U and eventually $\Diamond$

$\Box\,(\text{try\_to\_send} \rightarrow \bigcirc\ \text{delivered})$

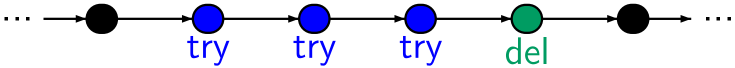


Next $\bigcirc$, until U and eventually $\Diamond$

$\Box\,(\mathsf{try\_to\_send} \longrightarrow \bigcirc\ \mathsf{delivered})$

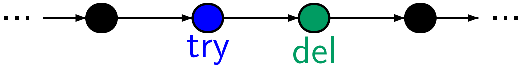


$\Box\,(\mathsf{try\_to\_send} \longrightarrow \mathsf{try\_to\_send}\ \mathbf{U}\ \mathsf{delivered})$

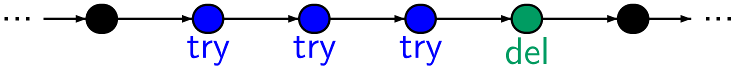


Next $\bigcirc$, until U and eventually $\Diamond$

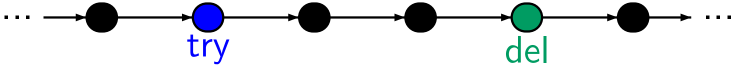
$\square\,(\text{try\_to\_send} \rightarrow \bigcirc\ \text{delivered})$



$\square\,(\text{try\_to\_send} \rightarrow \text{try\_to\_send}\ \mathsf{U}\ \text{delivered})$



$\square\,(\text{try\_to\_send} \rightarrow \Diamond\ \text{delivered})$



Examples for LTL formulas

$$\varphi \ ::= \ \mathit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \, \mathsf{U} \, \varphi_2$$

eventually

$$\Diamond\varphi \stackrel{\text{def}}{=} \mathit{true} \, \mathsf{U} \, \varphi$$

always

$$\Box\varphi \stackrel{\text{def}}{=} \neg\Diamond\neg\varphi$$

Examples for LTL formulas

$$\varphi \ ::= \ \textit{true} \ \big| \ a \ \big| \ \varphi_1 \wedge \varphi_2 \ \big| \ \neg\varphi \ \big| \ \bigcirc\varphi \ \big| \ \varphi_1 \,\mathsf{U}\, \varphi_2$$

eventually

$$\Diamond\varphi \stackrel{\text{def}}{=} \textit{true}\,\mathsf{U}\,\varphi$$

always

$$\Box\varphi \stackrel{\text{def}}{=} \neg\Diamond\neg\varphi$$

Examples for LTL formulas:

mutual exclusion: $\Box\big(\neg\mathit{crit}_1 \vee \neg\mathit{crit}_2\big)$

Examples for LTL formulas

$$\varphi \ ::= \ \mathit{true} \ \big| \ a \ \big| \ \varphi_1 \wedge \varphi_2 \ \big| \ \neg\varphi \ \big| \ \bigcirc\varphi \ \big| \ \varphi_1 \, \mathsf{U} \, \varphi_2$$

eventually

$$\Diamond\varphi \overset{\text{def}}{=} \mathit{true} \, \mathsf{U} \, \varphi$$

always

$$\Box\varphi \overset{\text{def}}{=} \neg\Diamond\neg\varphi$$

Examples for LTL formulas:

mutual exclusion: $\Box(\neg \mathit{crit}_1 \vee \neg \mathit{crit}_2)$

railroad-crossing: $\Box(\mathit{train\_is\_near} \rightarrow \mathit{gate\_is\_closed})$

$$\varphi \ ::= \ \mathit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \, \mathsf{U} \, \varphi_2$$

eventually

$$\Diamond\varphi \stackrel{\text{def}}{=} \mathit{true} \, \mathsf{U} \, \varphi$$

always

$$\Box\varphi \stackrel{\text{def}}{=} \neg\Diamond\neg\varphi$$

Examples for LTL formulas:

mutual exclusion: $\Box(\neg \mathit{crit}_1 \vee \neg \mathit{crit}_2)$

railroad-crossing: $\Box(\mathit{train\_is\_near} \rightarrow \mathit{gate\_is\_closed})$

progress property: $\Box(\mathit{request} \rightarrow \Diamond \mathit{response})$

Examples for LTL formulas

$$\varphi \ ::= \ \mathit{true} \ \big| \ a \ \big| \ \varphi_1 \wedge \varphi_2 \ \big| \ \neg\varphi \ \big| \ \bigcirc \varphi \ \big| \ \varphi_1 \,\mathsf{U}\, \varphi_2$$

eventually

$$\Diamond\varphi \stackrel{\text{def}}{=} \mathit{true}\,\mathsf{U}\,\varphi$$

always

$$\Box\varphi \stackrel{\text{def}}{=} \neg\Diamond\neg\varphi$$

Examples for LTL formulas:

mutual exclusion: $\Box(\neg \mathit{crit}_1 \vee \neg \mathit{crit}_2)$

railroad-crossing: $\Box(\mathit{train\_is\_near} \rightarrow \mathit{gate\_is\_closed})$

progress property: $\Box(\mathit{request} \rightarrow \Diamond \mathit{response})$

traffic light: $\Box(\mathit{yellow} \vee \bigcirc\neg \mathit{red})$

$$\varphi \;::=\; \mathit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

eventually $\Diamond\varphi \stackrel{\text{def}}{=} \mathit{true}\,\mathsf{U}\,\varphi$

always $\Box\varphi \stackrel{\text{def}}{=} \neg\Diamond\neg\varphi$

Infinitely often and eventually forever

$$\varphi \ ::= \ \mathit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \, \mathsf{U} \, \varphi_2$$

eventually $\Diamond\varphi \stackrel{\text{def}}{=} \mathit{true} \, \mathsf{U} \, \varphi$

always $\Box\varphi \stackrel{\text{def}}{=} \neg\Diamond\neg\varphi$

infinitely often $\Box\Diamond\varphi$

Infinitely often and eventually forever

$$\varphi \ ::= \ \mathit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

eventually $\Diamond\varphi \overset{\text{def}}{=} \mathit{true}\,\mathsf{U}\,\varphi$

always $\Box\varphi \overset{\text{def}}{=} \neg\Diamond\neg\varphi$

infinitely often $\Box\Diamond\varphi$

e.g., unconditional fairness $\Box\Diamond \mathit{crit}_i$

strong fairness $\Box\Diamond \mathit{wait}_i \rightarrow \Box\Diamond \mathit{crit}_i$

Infinitely often and eventually forever

$$\varphi ::= true \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \mathsf{U} \varphi_2$$

eventually $\Diamond\varphi \overset{\text{def}}{=} true \,\mathsf{U}\, \varphi$

always $\Box\varphi \overset{\text{def}}{=} \neg\Diamond\neg\varphi$

infinitely often $\Box\Diamond\varphi$

eventually forever $\Diamond\Box\varphi$

e.g., unconditional fairness $\Box\Diamond crit_i$

strong fairness $\Box\Diamond wait_i \to \Box\Diamond crit_i$

weak fairness $\Diamond\Box wait_i \to \Box\Diamond crit_i$

LTL-semantics

LTLSF3.1-6A

LTL-semantics

interpretation of **LTL** formulas over traces, i.e., infinite words over 2^{AP}

interpretation of **LTL** formulas over traces, i.e., infinite words over 2^{AP}

formalized by a satisfaction relation $\models$ for

- LTL formulas and
- infinite words $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^\omega$

Semantics of LTL over infinite words

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

for $\sigma = A_0 \, A_1 \, A_2 \, \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \mathit{true}$

Semantics of LTL over infinite words

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \mathit{true}$

$\sigma \models a$ iff $A_0 \models a$,i.e., $a \in A_0$

Semantics of LTL over infinite words

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \mathit{true}$

$\sigma \models a$ iff $A_0 \models a$,i.e., $a \in A_0$

$\sigma \models \varphi_1 \wedge \varphi_2$ iff $\sigma \models \varphi_1$ and $\sigma \models \varphi_2$

Semantics of LTL over infinite words

for $\sigma = A_0 A_1 A_2 \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \mathit{true}$		
$\sigma \models a$	iff	$A_0 \models a$,i.e., $a \in A_0$
$\sigma \models \varphi_1 \wedge \varphi_2$	iff	$\sigma \models \varphi_1$ and $\sigma \models \varphi_2$
$\sigma \models \neg\varphi$	iff	$\sigma \not\models \varphi$

Semantics of LTL over infinite words

for $\sigma = A_0 \, A_1 \, A_2 \, \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \mathit{true}$		
$\sigma \models a$	iff	$A_0 \models a$,i.e., $a \in A_0$
$\sigma \models \varphi_1 \wedge \varphi_2$	iff	$\sigma \models \varphi_1$ and $\sigma \models \varphi_2$
$\sigma \models \neg\varphi$	iff	$\sigma \not\models \varphi$
$\sigma \models \bigcirc\varphi$	iff	$\mathit{suffix}(\sigma, 1) = A_1 \, A_2 \, A_3 \, \ldots \models \varphi$

Semantics of LTL over infinite words

for $\sigma = A_0\, A_1\, A_2 \, \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \mathit{true}$		
$\sigma \models a$	iff	$A_0 \models a$,i.e., $a \in A_0$
$\sigma \models \varphi_1 \wedge \varphi_2$	iff	$\sigma \models \varphi_1$ and $\sigma \models \varphi_2$
$\sigma \models \neg\varphi$	iff	$\sigma \not\models \varphi$
$\sigma \models \bigcirc\varphi$	iff	$\mathit{suffix}(\sigma, 1) = A_1\, A_2\, A_3 \, \ldots \models \varphi$
$\sigma \models \varphi_1 \,\mathsf{U}\, \varphi_2$	iff	there exists $j \geq 0$ such that

$\mathit{suffix}(\sigma, j) = A_j\, A_{j+1}\, A_{j+2} \, \ldots \models \varphi_2$ and

$\mathit{suffix}(\sigma, i) = A_i\, A_{i+1}\, A_{i+2} \, \ldots \models \varphi_1$ for $0 \leq i < j$

Semantics of LTL over infinite words

for $\sigma = A_0\, A_1\, A_2 \ldots \in (2^{AP})^\omega$:

$\sigma \models \mathit{true}$		
$\sigma \models a$	iff	$A_0 \models a$,i.e., $a \in A_0$
$\sigma \models \varphi_1 \wedge \varphi_2$	iff	$\sigma \models \varphi_1$ and $\sigma \models \varphi_2$
$\sigma \models \neg\varphi$	iff	$\sigma \not\models \varphi$
$\sigma \models \bigcirc\varphi$	iff	$\mathit{suffix}(\sigma, 1) = A_1\, A_2\, A_3 \ldots \models \varphi$
$\sigma \models \varphi_1 \,\mathsf{U}\, \varphi_2$	iff	there exists $j \geq 0$ such that

$\mathit{suffix}(\sigma, j) = A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi_2$ and

$\mathit{suffix}(\sigma, i) = A_i\, A_{i+1}\, A_{i+2} \ldots \models \varphi_1$ for $0 \leq i < j$

LT property of LTL formulas

LT property of LTL formulas

interpretation of **LTL** formulas over traces, i.e., infinite words over 2^{AP}

formalized by a satisfaction relation $\models$ for

- LTL formulas and
- infinite words $\sigma = A_0\,A_1\,A_2\,\ldots \in \left(2^{AP}\right)^{\omega}$

LT property of LTL formulas

interpretation of **LTL** formulas over traces, i.e., infinite words over 2^{AP}

formalized by a satisfaction relation $\models$ for

- LTL formulas and
- infinite words $\sigma = A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$

LT property of formula φ:

$$Words(\varphi) \stackrel{\text{def}}{=} \left\{\sigma \in \left(2^{AP}\right)^{\omega} : \sigma \models \varphi\right\}$$

LTL-semantics of derived operators $\Diamond$ and $\Box$

LTL-semantics of derived operators $\Diamond$ and $\Box$

for $\sigma = A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$:

$\vdots$

$\sigma \models \varphi_1 \,\mathsf{U}\, \varphi_2$ iff there exists $j \geq 0$ such that

$A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi_2$ and

$A_i\, A_{i+1}\, A_{i+2} \ldots \models \varphi_1$ for $0 \leq i < j$

LTL-semantics of derived operators $\Diamond$ and $\Box$

for $\sigma = A_0 A_1 A_2 \ldots \in (2^{AP})^\omega$:

$$\vdots$$

$\sigma \models \varphi_1 \mathsf{U} \varphi_2$ iff there exists $j \geq 0$ such that
$A_j A_{j+1} A_{j+2} \ldots \models \varphi_2$ and
$A_i A_{i+1} A_{i+2} \ldots \models \varphi_1$ for $0 \leq i < j$

$\sigma \models \Diamond\varphi$ iff there exists $j \geq 0$ such that
$A_j A_{j+1} A_{j+2} \ldots \models \varphi$

LTL-semantics of derived operators $\Diamond$ and $\Box$

for $\sigma = A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^\omega$:

$$\vdots$$

$\sigma \models \varphi_1 \,\mathsf{U}\, \varphi_2$	iff	there exists $j \geq 0$ such that
		$A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi_2$ and
		$A_i\, A_{i+1}\, A_{i+2} \ldots \models \varphi_1$ for $0 \leq i < j$
$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that
		$A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have:
		$A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$

LTL semantics over TS

LTL semantics over TS

given a TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

define satisfaction relation $\models$ for

- **LTL** formulas over AP
- the maximal path fragments and states of $\mathcal{T}$

LTL semantics over TS

given a TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

define satisfaction relation $\models$ for

- **LTL** formulas over AP
- the maximal path fragments and states of $\mathcal{T}$

assumption: $\mathcal{T}$ has no terminal states, i.e., all maximal path fragments in $\mathcal{T}$ are infinite

LTL semantics over paths of TS

LTL semantics over paths of TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

LTL semantics over paths of TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

LTL semantics over paths of TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states
LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$
$$\text{iff} \quad trace(\pi) \in Words(\varphi)$$

LTL semantics over paths of TS

given: TS $\mathcal{T} = (S, Act, \to, S_0, AP, L)$
without terminal states
LTL formula φ over AP

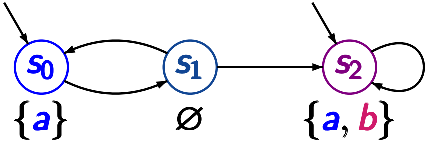
interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$
$$\text{iff} \quad trace(\pi) \in Words(\varphi)$$

remind: LT property of an LTL formula:

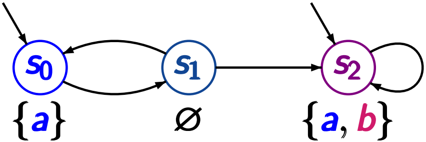
$$Words(\varphi) = \{\sigma \in (2^{AP})^\omega : \sigma \models \varphi\}$$

Example: LTL-semantics over paths



$AP = \{a, b\}$

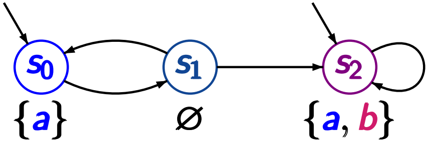
Example: LTL-semantics over paths



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$

Example: LTL-semantics over paths

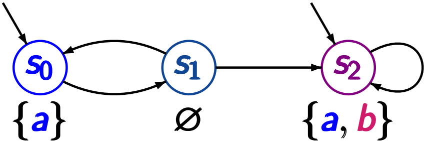


$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\qquad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$

Example: LTL-semantics over paths

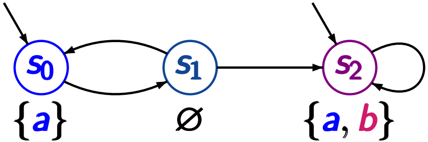


$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$ $\quad$ as $L(s_0) = \{a\}$

Example: LTL-semantics over paths



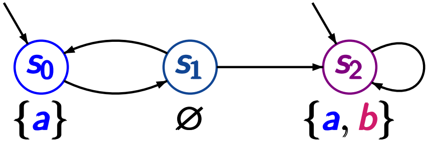
$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$ $\quad$ as $L(s_0) = \{a\}$

$\pi \models \bigcirc(\neg a \wedge \neg b)$

Example: LTL-semantics over paths



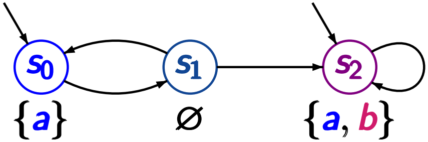
$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$ $\quad$ as $L(s_0) = \{a\}$

$\pi \models \bigcirc(\neg a \wedge \neg b)$ $\quad$ as $L(s_1) = \varnothing$

Example: LTL-semantics over paths



$AP = \{a, b\}$

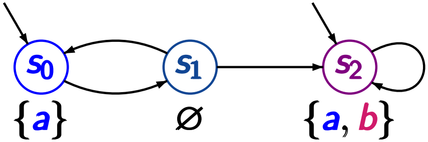
path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\qquad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$ $\qquad$ as $L(s_0) = \{a\}$

$\pi \models \bigcirc(\neg a \wedge \neg b)$ $\qquad$ as $L(s_1) = \varnothing$

$\pi \models \bigcirc \bigcirc (a \wedge b)$

Example: LTL-semantics over paths



$AP = \{a, b\}$

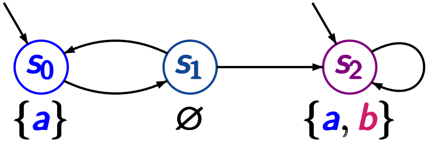
path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\qquad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$ $\qquad$ as $L(s_0) = \{a\}$

$\pi \models \bigcirc(\neg a \wedge \neg b)$ $\qquad$ as $L(s_1) = \varnothing$

$\pi \models \bigcirc \bigcirc (a \wedge b)$ $\qquad$ as $L(s_2) = \{a, b\}$

Example: LTL-semantics over paths

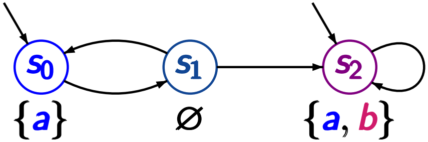


$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\qquad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$	as $L(s_0) = \{a\}$
$\pi \models \bigcirc(\neg a \wedge \neg b)$	as $L(s_1) = \varnothing$
$\pi \models \bigcirc\bigcirc(a \wedge b)$	as $L(s_2) = \{a, b\}$
$\pi \models (\neg b)\, \mathsf{U}\, (a \wedge b)$	

Example: LTL-semantics over paths

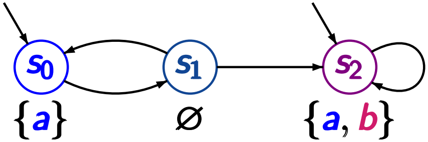


$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\qquad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$	as $L(s_0) = \{a\}$
$\pi \models \bigcirc(\neg a \wedge \neg b)$	as $L(s_1) = \varnothing$
$\pi \models \bigcirc \bigcirc (a \wedge b)$	as $L(s_2) = \{a, b\}$
$\pi \models (\neg b) \, \mathsf{U} \, (a \wedge b)$	as $s_0, s_1 \models \neg b$
	and $s_2 \models a \wedge b$

Example: LTL-semantics over paths

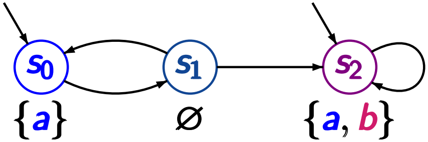


$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$	as $L(s_0) = \{a\}$
$\pi \models \bigcirc(\neg a \wedge \neg b)$	as $L(s_1) = \varnothing$
$\pi \models \bigcirc \bigcirc (a \wedge b)$	as $L(s_2) = \{a, b\}$
$\pi \models (\neg b)\, \mathsf{U}\, (a \wedge b)$	as $s_0, s_1 \models \neg b$
$\pi \models (\neg b)\, \mathsf{U}\, \Box(a \wedge b)$	and $s_2 \models a \wedge b$

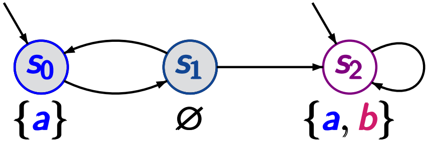
Correct or wrong ?



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1\, \ldots$

Correct or wrong ?

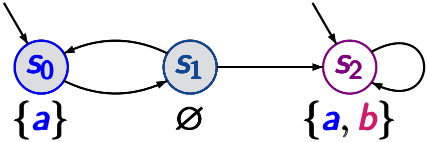


$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1\, \ldots$

$trace(\pi) = (\{a\}\, \varnothing)^\omega$

Correct or wrong ?



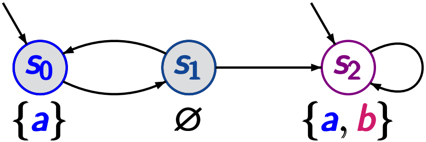
$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$

$trace(\pi) = (\{a\}\, \varnothing)^\omega$

$\pi \models a \,\mathsf{U}\, b$?

Correct or wrong ?

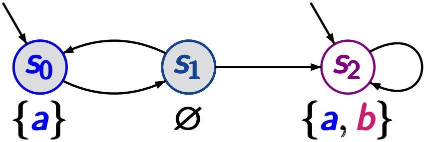


$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad$ $trace(\pi) = (\{a\}\,\varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

Correct or wrong ?



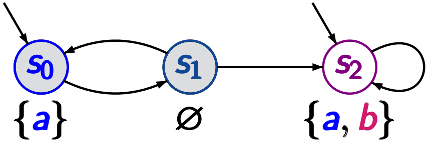
$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad$ $trace(\pi) = (\{a\}\, \varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \lozenge b \rightarrow (a \,\mathsf{U}\, b)$?

Correct or wrong ?



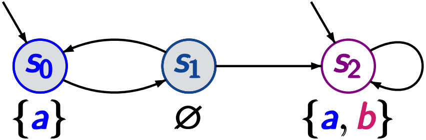
$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad$ $trace(\pi) = (\{a\}\,\varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\qquad$ as $\pi \not\models \Diamond b$

Correct or wrong ?



$AP = \{a, b\}$

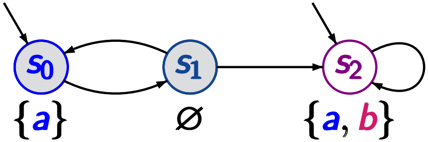
path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad$ $trace(\pi) = (\{a\}\, \varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\qquad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$?

Correct or wrong ?



$AP = \{a, b\}$

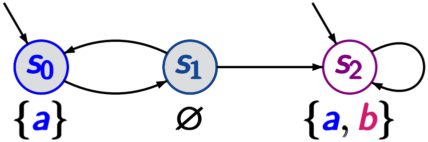
path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad$ $trace(\pi) = (\{a\}\,\varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\qquad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ $\qquad$ as $s_0 \models \neg b$

Correct or wrong ?



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad trace(\pi) = (\{a\}\, \varnothing)^\omega$

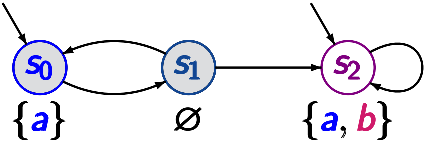
$\pi \not\models a \, \mathsf{U} \, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \, \mathsf{U} \, b)$ $\qquad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ $\qquad$ as $s_0 \models \neg b$

$\pi \models \Box a$?

Correct or wrong ?



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad$ $trace(\pi) = (\{a\}\,\varnothing)^\omega$

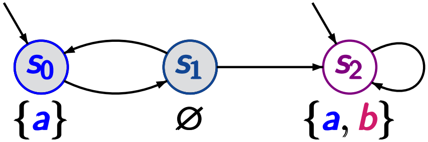
$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\qquad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ $\qquad$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ $\qquad$ as $s_1 \not\models a$

Correct or wrong ?



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad$ $trace(\pi) = (\{a\}\, \varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

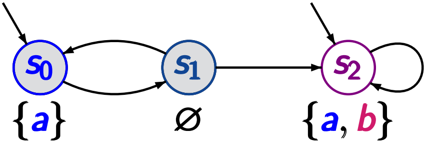
$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\qquad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ $\qquad$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ $\qquad$ as $s_1 \not\models a$

$\pi \models \Box \Diamond a$?

Correct or wrong ?



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad$ $trace(\pi) = (\{a\}\,\varnothing)^\omega$

$\pi \not\models a \, \mathsf{U} \, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

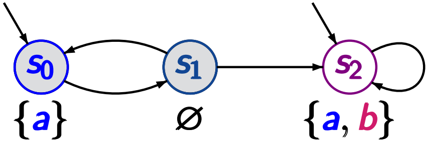
$\pi \models \Diamond b \rightarrow (a \, \mathsf{U} \, b)$ $\qquad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ $\qquad$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ $\qquad$ as $s_1 \not\models a$

$\pi \models \Box\Diamond a$ $\qquad$ as $\Box\Diamond \mathrel{\widehat{=}}$ infinitely often

Correct or wrong ?



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1\, \ldots$ $\quad$ $\mathit{trace}(\pi) = (\{a\}\, \varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ as $\pi \not\models \Diamond b$

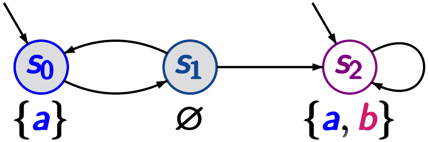
$\pi \models \bigcirc \bigcirc \neg b$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ as $s_1 \not\models a$

$\pi \models \Box\Diamond a$ as $\Box\Diamond \mathrel{\widehat{=}}$ infinitely often

$\pi \models \Diamond\Box a$?

Correct or wrong ?



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\quad$ $trace(\pi) = (\{a\}\, \varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ $\quad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\quad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ $\quad$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ $\quad$ as $s_1 \not\models a$

$\pi \models \Box\Diamond a$ $\quad$ as $\Box\Diamond \mathrel{\widehat{=}}$ infinitely often

$\pi \not\models \Diamond\Box a$ $\quad$ as $\Diamond\Box \mathrel{\widehat{=}}$ eventually forever

LTL-semantics of derived operators

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

LTL-semantics of derived operators

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that $A_j\, A_{j+1}\, A_{j+2}\, \ldots \models \varphi$
$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have: $A_j\, A_{j+1}\, A_{j+2}\, \ldots \models \varphi$

LTL-semantics of derived operators

for $\sigma = A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$:

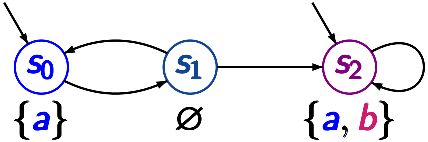
$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have: $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\Diamond\varphi$	iff	there are infinitely many $j \geq 0$ s.t. $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$

LTL-semantics of derived operators

for $\sigma = A_0 A_1 A_2 \ldots \in (2^{AP})^\omega$:

$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that $A_j A_{j+1} A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have: $A_j A_{j+1} A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\Diamond\varphi$	iff	there are infinitely many $j \geq 0$ s.t. $A_j A_{j+1} A_{j+2} \ldots \models \varphi$
$\sigma \models \Diamond\Box\varphi$	iff	for almost all $j \geq 0$ we have: $A_j A_{j+1} A_{j+2} \ldots \models \varphi$

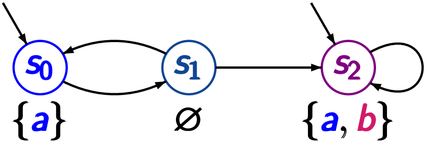
LTL-semantics over paths



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$

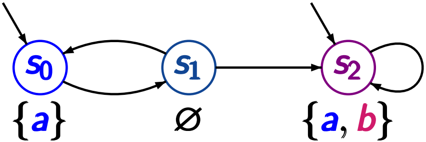
LTL-semantics over paths



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

LTL-semantics over paths

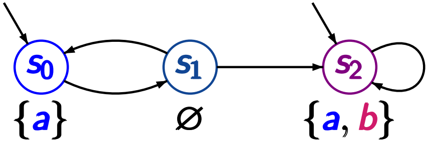


$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U}\, (a \wedge b))$?

LTL-semantics over paths



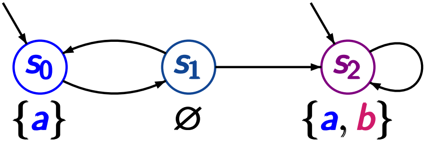
$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\qquad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U}\, (a \wedge b))$ as $s_1 \models \neg a \wedge \neg b$

$s_2 \models a \wedge b$

LTL-semantics over paths



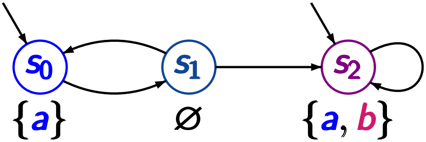
$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^{\omega}$

$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U}(a \wedge b))$ as $s_1 \models \neg a \wedge \neg b$

$s_2 \models a \wedge b$

$\pi \models \bigcirc\Box(a \leftrightarrow b)$?



$AP = \{a, b\}$

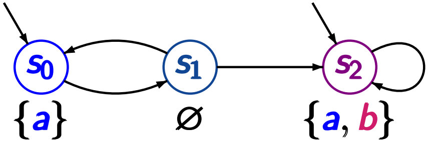
path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U}\, (a \wedge b))$ $\quad$ as $s_1 \models \neg a \wedge \neg b$

$s_2 \models a \wedge b$

$\pi \models \bigcirc\Box(a \leftrightarrow b)$ $\quad$ as $s_1, s_2 \models a \leftrightarrow b$

LTL-semantics over paths



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

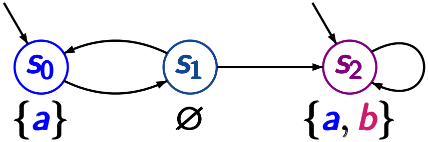
$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U}\, (a \wedge b))$ as $s_1 \models \neg a \wedge \neg b$

$s_2 \models a \wedge b$

$\pi \models \bigcirc\Box(a \leftrightarrow b)$ as $s_1, s_2 \models a \leftrightarrow b$

$\pi \models a\, \mathsf{U}\, (\neg b\, \mathsf{U}\, a)$?

LTL-semantics over paths



$AP = \{a, b\}$

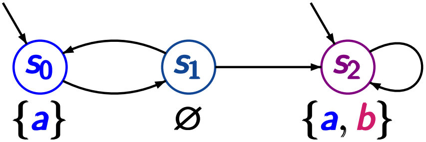
path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U}\, (a \wedge b))$ as $s_1 \models \neg a \wedge \neg b$
$s_2 \models a \wedge b$

$\pi \models \bigcirc\Box(a \leftrightarrow b)$ as $s_1, s_2 \models a \leftrightarrow b$

$\pi \models a\, \mathsf{U}\, (\neg b\, \mathsf{U}\, a)$ as $s_0, s_2 \models a$, $s_1 \models \neg b$

LTL-semantics over paths



$AP = \{a, b\}$

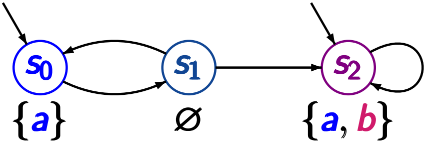
path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U}\, (a \wedge b))$ as $s_1 \models \neg a \wedge \neg b$, $\; s_2 \models a \wedge b$

$\pi \models \bigcirc \Box (a \leftrightarrow b)$ as $s_1, s_2 \models a \leftrightarrow b$

$\pi \models a\, \mathsf{U}\, (\neg b\, \mathsf{U}\, a)$ as $s_0, s_2 \models a$, $s_1 \models \neg b$

$\pi \models \Diamond \Box (\neg a \rightarrow \Diamond \neg b)$?



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

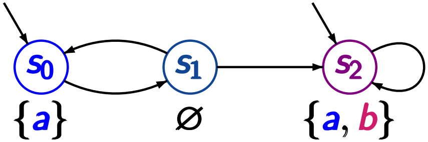
$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U}\, (a \wedge b))$ as $s_1 \models \neg a \wedge \neg b$
$s_2 \models a \wedge b$

$\pi \models \bigcirc\Box(a \leftrightarrow b)$ as $s_1, s_2 \models a \leftrightarrow b$

$\pi \models a\, \mathsf{U}\, (\neg b\, \mathsf{U}\, a)$ as $s_0, s_2 \models a,\ s_1 \models \neg b$

$\pi \models \Diamond\Box(\neg a \rightarrow \Diamond\neg b)$ as $s_2\, s_2\, s_2 \ldots \models \neg a \rightarrow \Diamond\neg b$

LTL-semantics over paths



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a,b\}^\omega$

$\pi \models \bigcirc((\neg a \wedge \neg b)\,\mathsf{U}\,(a \wedge b))$ as $s_1 \models \neg a \wedge \neg b$, $s_2 \models a \wedge b$

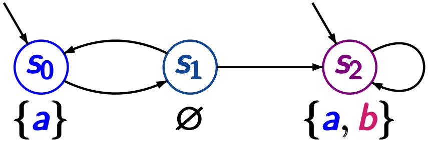
$\pi \models \bigcirc\Box(a \leftrightarrow b)$ as $s_1, s_2 \models a \leftrightarrow b$

$\pi \models a\,\mathsf{U}\,(\neg b\,\mathsf{U}\,a)$ as $s_0, s_2 \models a$, $s_1 \models \neg b$

$\pi \models \Diamond\Box(\neg a \rightarrow \Diamond\neg b)$ as $s_2\, s_2\, s_2 \ldots \models \neg a \rightarrow \Diamond\neg b$

$\pi \models \Box(\neg b \rightarrow \bigcirc a)$?

LTL-semantics over paths



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\qquad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U}\, (a \wedge b))$	as $s_1 \models \neg a \wedge \neg b$ $s_2 \models a \wedge b$
$\pi \models \bigcirc\Box(a \leftrightarrow b)$	as $s_1, s_2 \models a \leftrightarrow b$
$\pi \models a\, \mathsf{U}\, (\neg b\, \mathsf{U}\, a)$	as $s_0, s_2 \models a$, $s_1 \models \neg b$
$\pi \models \Diamond\Box(\neg a \rightarrow \Diamond\neg b)$	as $s_2\, s_2\, s_2 \ldots \models \neg a \rightarrow \Diamond\neg b$
$\pi \not\models \Box(\neg b \rightarrow \bigcirc a)$	as $s_0 \models \neg b$, $s_1 \not\models a$

LTL semantics over the states of a TS

LTL semantics over the states of a TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

interpretation of φ over states:

$$s \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(s)$$

LTL semantics over the states of a TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

interpretation of φ over states:

$$s \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(s)$$
$$\quad\quad \text{iff} \quad s \models Words(\varphi)$$

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

interpretation of φ over states:

$$s \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(s)$$

$$\text{iff} \quad s \models Words(\varphi)$$

↑

satisfaction relation for LT properties

LTL semantics over the states of a TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

interpretation of φ over states:

$$\begin{aligned} s \models \varphi \quad & \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(s) \\ & \text{iff} \quad s \models Words(\varphi) \\ & \text{iff} \quad Traces(s) \subseteq Words(\varphi) \end{aligned}$$

Interpretation of LTL formulas over TS

Interpretation of LTL formulas over TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

$\mathcal{T} \models \varphi$ iff $s_0 \models \varphi$ for all $s_0 \in S_0$

Interpretation of LTL formulas over TS

given: TS $\mathcal{T} = (S, Act, \to, S_0, AP, L)$
without terminal states

LTL formula φ over AP

$$\mathcal{T} \models \varphi \quad \text{iff} \quad s_0 \models \varphi \text{ for all } s_0 \in S_0$$

$$\quad \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(\mathcal{T})$$

Interpretation of LTL formulas over TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

$\mathcal{T} \models \varphi$ iff $s_0 \models \varphi$ for all $s_0 \in S_0$

iff $trace(\pi) \models \varphi$ for all $\pi \in Paths(\mathcal{T})$

iff $Traces(\mathcal{T}) \subseteq Words(\varphi)$

Interpretation of LTL formulas over TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

$\mathcal{T} \models \varphi$ iff $s_0 \models \varphi$ for all $s_0 \in S_0$

iff $trace(\pi) \models \varphi$ for all $\pi \in Paths(\mathcal{T})$

iff $Traces(\mathcal{T}) \subseteq Words(\varphi)$

iff $\mathcal{T} \models Words(\varphi)$

Interpretation of LTL formulas over TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

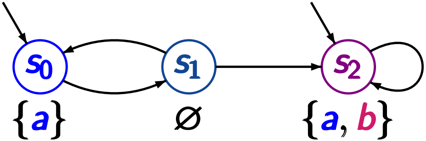
LTL formula φ over AP

$$
\begin{aligned}
\mathcal{T} \models \varphi \quad & \text{iff} \quad s_0 \models \varphi \text{ for all } s_0 \in S_0 \\
& \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(\mathcal{T}) \\
& \text{iff} \quad Traces(\mathcal{T}) \subseteq Words(\varphi) \\
& \text{iff} \quad \mathcal{T} \models Words(\varphi)
\end{aligned}
$$

↑

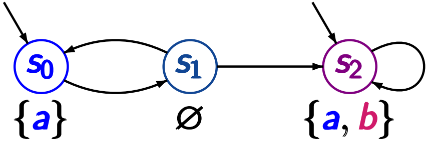
satisfaction relation for LT properties

Which formulas hold for $\mathcal{T}$?



$$AP = \{a, b\}$$

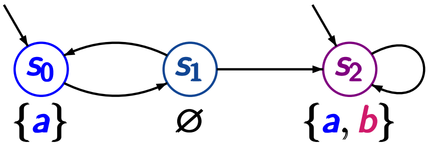
Which formulas hold for $\mathcal{T}$?



$AP = \{a, b\}$

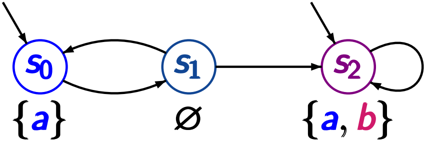
$\mathcal{T} \models a$

Which formulas hold for $\mathcal{T}$?



$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

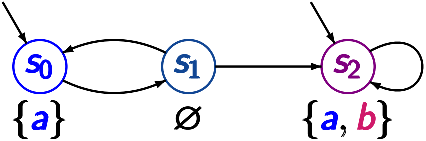


$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \models \Diamond\Box a$

Which formulas hold for $\mathcal{T}$?

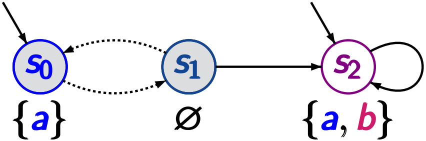


$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \not\models \Diamond\Box a$

Which formulas hold for $\mathcal{T}$?

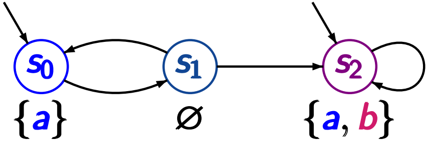


$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \not\models \Diamond\Box a$ as $s_0\, s_1\, s_0\, s_1 \ldots \not\models \Diamond\Box a$

Which formulas hold for $\mathcal{T}$?



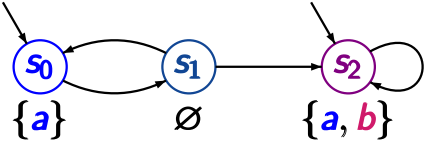
$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \not\models \Diamond\Box a$ as $s_0\, s_1\, s_0\, s_1 \ldots \not\models \Diamond\Box a$

$\mathcal{T} \models \Diamond\Box b \vee \Box\Diamond(\neg a \wedge \neg b)$

Which formulas hold for $\mathcal{T}$?



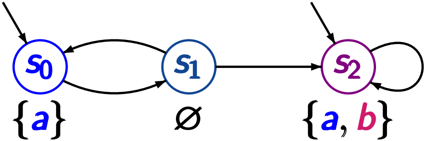
$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \not\models \Diamond\Box a$ as $s_0\, s_1\, s_0\, s_1 \ldots \not\models \Diamond\Box a$

$\mathcal{T} \models \Diamond\Box b \vee \Box\Diamond(\neg a \wedge \neg b)$ as $s_2 \models b$, $s_1 \not\models a, b$

Which formulas hold for $\mathcal{T}$?



$AP = \{a, b\}$

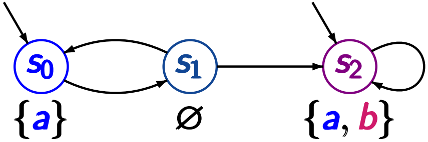
$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \not\models \Diamond\Box a$ as $s_0\, s_1\, s_0\, s_1 \ldots \not\models \Diamond\Box a$

$\mathcal{T} \models \Diamond\Box b \lor \Box\Diamond(\neg a \land \neg b)$ as $s_2 \models b$, $s_1 \not\models a, b$

$\mathcal{T} \models \Box(a \rightarrow (\bigcirc\neg a \lor b))$

Which formulas hold for $\mathcal{T}$?



$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \not\models \Diamond\Box a$ as $s_0\, s_1\, s_0\, s_1 \ldots \not\models \Diamond\Box a$

$\mathcal{T} \models \Diamond\Box b \vee \Box\Diamond(\neg a \wedge \neg b)$ as $s_2 \models b$, $s_1 \not\models a, b$

$\mathcal{T} \models \Box(a \rightarrow (\bigcirc\neg a \vee b))$ as $s_2 \models b$, $s_0 \models \bigcirc\neg a$

Correct or wrong?

For each path π we have: $\pi \models \varphi$ or $\pi \models \neg\varphi$

Correct or wrong?

For each path π we have: $\pi \models \varphi$ or $\pi \models \neg\varphi$

correct, since $\pi \models \neg\varphi$ iff $\pi \not\models \varphi$

Correct or wrong?

For each path π we have: $\pi \models \varphi$ or $\pi \models \neg\varphi$

correct, since $\pi \models \neg\varphi$ iff $\pi \not\models \varphi$

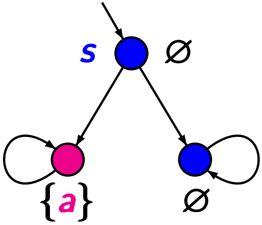
For each state s we have: $s \models \varphi$ or $s \models \neg\varphi$

For each path π we have: $\pi \models \varphi$ or $\pi \models \neg\varphi$

correct, since $\pi \models \neg\varphi$ iff $\pi \not\models \varphi$

For each state s we have: $s \models \varphi$ or $s \models \neg\varphi$

wrong.



$s \not\models \Diamond a$ and $s \not\models \neg\Diamond a$

LTL-formulas for MUTEX protocols

LTL-formulas for MUTEX protocols

LTL formulas over $AP = \{wait_1, crit_1, wait_2, crit_2\}$

- the mutual exclusion property

$$\varphi_{mutex} = ?$$

LTL-formulas for MUTEX protocols

LTL formulas over $AP = \{\mathsf{wait}_1, \mathsf{crit}_1, \mathsf{wait}_2, \mathsf{crit}_2\}$

- the mutual exclusion property

$$\varphi_{mutex} = \Box(\neg \mathsf{crit}_1 \vee \neg \mathsf{crit}_2)$$

LTL-formulas for MUTEX protocols

LTL formulas over $AP = \{\mathsf{wait}_1, \mathsf{crit}_1, \mathsf{wait}_2, \mathsf{crit}_2\}$

- the mutual exclusion property

$$\varphi_{mutex} \;=\; \Box(\neg\mathsf{crit}_1 \vee \neg\mathsf{crit}_2)$$

- "every process enters the critical section infinitely often"

$$\varphi_{live} \;=\; ?$$

LTL-formulas for MUTEX protocols

LTL formulas over $AP = \{wait_1, crit_1, wait_2, crit_2\}$

- the mutual exclusion property

$$\varphi_{mutex} = \Box(\neg crit_1 \vee \neg crit_2)$$

- "every process enters the critical section infinitely often"

$$\varphi_{live} = \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

LTL-formulas for MUTEX protocols

LTL formulas over $AP = \{wait_1, crit_1, wait_2, crit_2\}$

- the mutual exclusion property

$$\varphi_{mutex} \;=\; \Box(\neg crit_1 \vee \neg crit_2)$$

- "every process enters the critical section infinitely often"

$$\varphi_{live} \;=\; \Box\Diamond crit_1 \;\wedge\; \Box\Diamond crit_2$$

- starvation freedom
  "every waiting process finally enters its critical section"

$$\varphi_{sf} \;=\; ?$$

LTL-formulas for MUTEX protocols

LTL formulas over $AP = \{wait_1, crit_1, wait_2, crit_2\}$

- the mutual exclusion property

$$\varphi_{mutex} \;=\; \Box(\neg crit_1 \vee \neg crit_2)$$

- "every process enters the critical section infinitely often"

$$\varphi_{live} \;=\; \Box\Diamond crit_1 \;\wedge\; \Box\Diamond crit_2$$

- starvation freedom
  "every waiting process finally enters its critical section"

$$\varphi_{sf} \;=\; \Box(wait_1 \rightarrow \Diamond crit_1) \;\wedge\; \Box(wait_2 \rightarrow \Diamond crit_2)$$

Provide an LTL formula over $AP = \{a, b\}$ for ...

- set of all words $A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^\omega$ such that:

$$\forall i \geq 0.\ (\ a \in A_i \implies i \geq 1 \land b \in A_{i-1}\ )$$

- set of all words $A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$ such that:

$$\forall i \geq 0.\ (\ a \in A_i \implies i \geq 1 \wedge b \in A_{i-1}\ )$$

$$\forall j \geq 0.\ (\ b \in A_j \ \vee\ a \notin A_{j+1})$$

Provide an LTL formula over $AP = \{a, b\}$ for ...

- set of all words $A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^\omega$ such that:

$$\forall i \geq 0.\ (\ a \in A_i \implies i \geq 1 \wedge b \in A_{i-1}\ )$$

$$\forall j \geq 0.\ (\ b \in A_j \ \vee \ a \notin A_{j+1})$$

$$\widehat{=} \ \ Words(\ \Box(b \vee \bigcirc \neg a)\ )$$

- set of all words $A_0\, A_1\, A_2 \ldots \in (2^{AP})^\omega$ such that:

$$\forall i \geq 0. \;(\; a \in A_i \implies i \geq 1 \wedge b \in A_{i-1} \;)$$

$$\forall j \geq 0. \;(\; b \in A_j \;\vee\; a \notin A_{j+1})$$

$$\widehat{=} \;\; \mathit{Words}(\; \Box(b \vee \bigcirc \neg a) \;)$$

- set of all words of the form

$$\{b\}^{n_1}\,\{a\}\,\{b\}^{n_2}\,\{a\}\,\{b\}^{n_3}\,\{a\} \ldots$$

where $n_1, n_2, n_3, \ldots \geq 0$

Provide an LTL formula over $AP = \{a, b\}$ for ...

- set of all words $A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$ such that:

  $$\forall i \geq 0.\ (\ a \in A_i \implies i \geq 1 \wedge b \in A_{i-1}\ )$$

  $$\forall j \geq 0.\ (\ b \in A_j \vee a \notin A_{j+1})$$

  $$\widehat{=}\quad Words(\ \Box(b \vee \bigcirc \neg a)\ )$$

- set of all words of the form

  $$\{b\}^{n_1}\,\{a\}\,\{b\}^{n_2}\,\{a\}\,\{b\}^{n_3}\,\{a\}\ldots$$

  where $n_1, n_2, n_3, \ldots \geq 0$

  $$\widehat{=}\quad Words(\ \Box((b \wedge \neg a)\ \mathsf{U}\ (a \wedge \neg b))\ )$$