CTL with Fairness

Luca Tesei

Reactive Systems Verification

MSc in Computer Science

University of Camerino

Topics

- Strong and Weak fairness conditions are not CTL formulas
- Definition of the variant of CTL satisfaction relation with fairness
- Algorithms for CTL model checking with fairness
- Examples
- Complexity of CTL model checking with fairness

Material

Reading:

Chapter 6 of the book: Section 6.5

More:

The slides in the following pages are taken from the material of the course "Introduction to Model Checking" held by Prof. Dr. Ir. Joost-Pieter Katoen at Aachen University.

Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation Tree Logic

- syntax and semantics of CTL
- expressiveness of CTL and LTL
- CTL model checking
- CTL with fairness ⟵
- counterexamples/witnesses, CTL^+ and CTL^*

Equivalences and Abstraction

Complexity of CTL and LTL model checking

Complexity of CTL and LTL model checking

LTL model checking problem:

PSPACE-complete and solvable in time

$$\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot \exp(|\varphi|))$$

CTL model checking problem:

solvable in polynomial time

$$\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot |\Phi|)$$

LTL model checking problem:

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solvable in polynomial time (even PTIME-complete)

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LTL with fairness: $\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot \exp(|\varphi| + |\mathit{fair}|))$

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solvable in polynomial time (even PTIME-complete)

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Complexity of CTL and LTL model checking

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LTL with fairness: $\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot \exp(|\varphi| + |\mathit{fair}|))$

CTL model checking problem:

solvable in polynomial time (even PTIME-complete)

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CTL with fairness: $\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot |\Phi| \cdot |\mathit{fair}|)$

Recall: LTL fairness assumptions

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are conjunctions of **LTL** formulas of the form

- unconditional fairness $\Box\Diamond\phi$
- strong fairness $\Box\Diamond\psi \rightarrow \Box\Diamond\phi$
- weak fairness $\Diamond\Box\psi \rightarrow \Box\Diamond\phi$

where ϕ, ψ are propositional formulas

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iff for all paths π in $\mathcal{T}$:

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iff for all paths π in $\mathcal{T}$:

$$\pi \models fair \rightarrow \varphi \equiv \Diamond\Box\neg a \vee \varphi$$

CTL fairness assumptions

conjunctions of "formulas" of the type

- unconditional fairness: $\Box\Diamond\Phi$
- strong fairness: $\Box\Diamond\Psi \rightarrow \Box\Diamond\Phi$
- weak fairness: $\Diamond\Box\Psi \rightarrow \Box\Diamond\Phi$

where Ψ, Φ are CTL state formulas conjunctions of "formulas" of the type

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note: CTL fairness assumptions

- are not CTL (state or path) formulas
- just a syntactic formalism to specify fairness assumptions

CTL fairness assumptions

conjunctions of "formulas" of the type

- unconditional fairness: $\Box\Diamond\Phi$
- strong fairness: $\Box\Diamond\Psi \rightarrow \Box\Diamond\Phi$
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where Ψ, Φ are CTL state formulas

e.g., a strong CTL fairness assumption has the form:

$$fair = \bigwedge_{1 \leq j \leq k} (\Box\Diamond\Psi_j \rightarrow \Box\Diamond\Phi_j)$$

where Ψ_j, Φ_j are CTL state formulas

Satisfaction relation for CTL with fairness

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$s \models_{fair} \mathit{true}$

$s \models_{fair} a$ iff $a \in L(s)$

$s \models_{fair} \neg\Phi$ iff $s \not\models_{fair} \Phi$

$s \models_{fair} \Phi_1 \wedge \Phi_2$ iff $s \models_{fair} \Phi_1$ and $s \models_{fair} \Phi_2$

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$s \models_{fair} \exists\varphi$ iff there exists $\pi \in Paths(s)$ with $\pi \models \mathit{fair}$ and $\pi \models_{fair} \varphi$

Satisfaction relation for CTL with fairness

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$s \models_{fair} \exists\varphi$	iff	there exists $\pi \in Paths(s)$ with $\pi \models fair$ and $\pi \models_{fair} \varphi$
$s \models_{fair} \forall\varphi$	iff	for all $\pi \in Paths(s)$: $\pi \models fair$ implies $\pi \models_{fair} \varphi$

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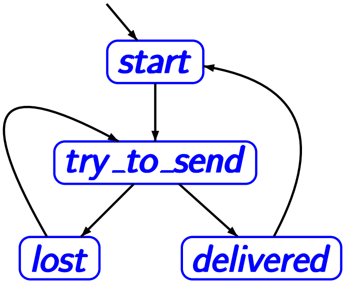
$s \models_{fair} \forall\varphi$ iff for all $\pi \in Paths(s)$: $\pi \models fair$ implies $\pi \models_{fair} \varphi$

e.g., $s_0\, s_1\, s_2 \ldots \models \Box\Diamond\Phi$ iff $\overset{\infty}{\exists}\, i \geq 0$ s.t. $s_i \models \Phi$

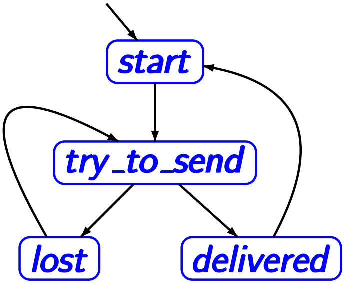
Simple communication protocol



CTL formula

$\Phi = \forall\Box\forall\Diamond \mathit{start}$

Simple communication protocol



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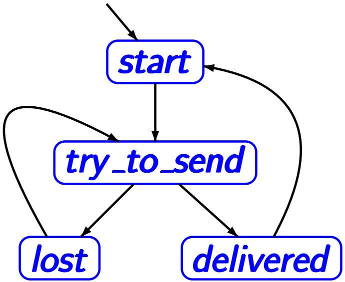
Simple communication protocol



CTL formula

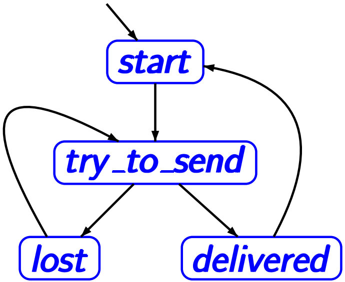
$\Phi = \forall\Box\forall\Diamond start$

$\mathcal{T} \not\models \Phi$

$\mathcal{T} \models_{ufair} \Phi$

unconditional CTL fairness assumption:

$ufair = \Box\Diamond delivered$



CTL formula

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$\mathcal{T} \models_{\mathit{ufair}} \Phi$

$\mathcal{T} \models_{\mathit{sfair}} \Phi$

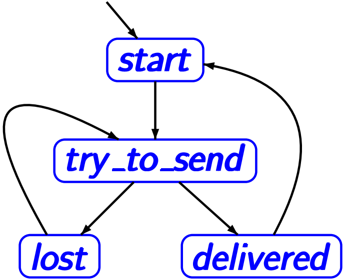
unconditional CTL fairness assumption:

$\mathit{ufair} = \Box\Diamond \mathit{delivered}$

strong CTL fairness assumption:

$\mathit{sfair} = \Box\Diamond \mathit{try\_to\_send} \rightarrow \Box\Diamond \mathit{delivered}$

Simple communication protocol

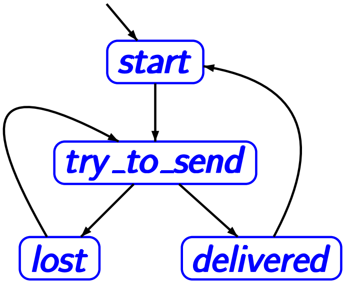


$\Phi = \forall\Box\forall\Diamond start$

$\mathcal{T} \models_{ufair} \Phi$?

unconditional fairness: $ufair = \Box\Diamond\, \exists\bigcirc start$

Simple communication protocol



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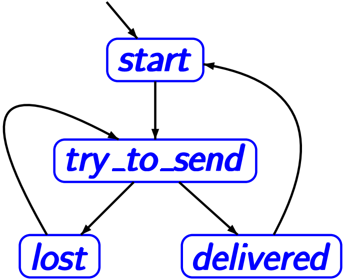
unconditional fairness: $ufair = \Box\Diamond \exists\bigcirc start$

$\uparrow$

$Sat(\exists\bigcirc start) = \{delivered\}$

Simple communication protocol



$\Phi = \forall\Box\forall\Diamond\, start$

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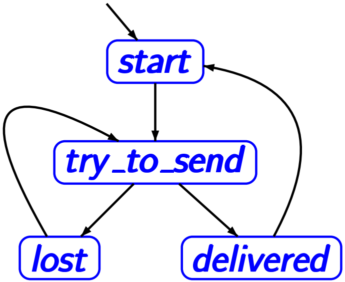

$\Phi = \forall\Box\forall\Diamond start$

$\mathcal{T} \models_{ufair} \Phi$ ✓

unconditional fairness: $ufair = \Box\Diamond\,\exists\bigcirc start$

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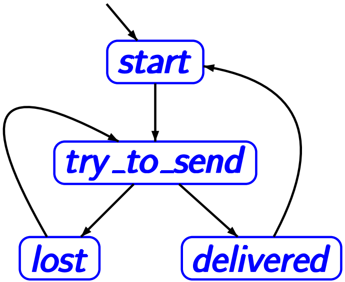

$\Phi = \forall\Box\forall\Diamond start$

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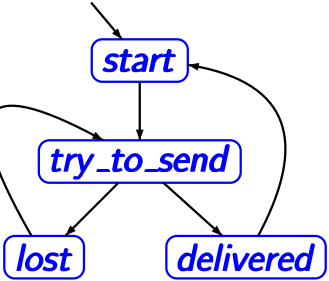
$\mathcal{T} \models_{wfair} \Phi$?

unconditional fairness: $ufair = \Box\Diamond\,\exists\bigcirc start$

weak fairness: $wfair = \Diamond\Box\,\exists\bigcirc delivered \rightarrow \Box\Diamond delivered$

Simple communication protocol



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$\mathcal{T} \models_{\textit{ufair}} \Phi$ ✓

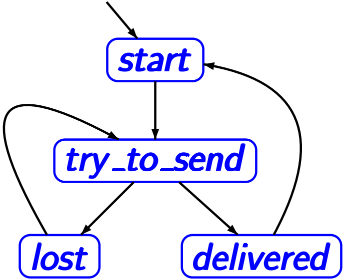
$\mathcal{T} \models_{\textit{wfair}} \Phi$ **?**

unconditional fairness: $\textit{ufair} = \Box\Diamond\, \exists\bigcirc \textit{start}$

weak fairness: $\textit{wfair} = \Diamond\Box\, \exists\bigcirc \textit{delivered} \longrightarrow \Box\Diamond \textit{delivered}$

$\textit{Sat}(\exists\bigcirc \textit{delivered}) = \{\textit{try\_to\_send}\}$

Simple communication protocol



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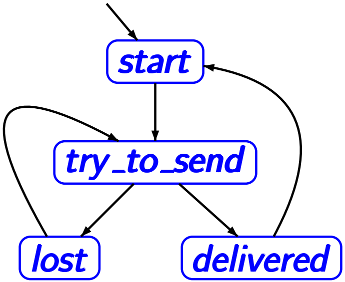

$\Phi = \forall\Box\forall\Diamond start$

$\mathcal{T} \models_{ufair} \Phi$ ✓

$\mathcal{T} \models_{wfair} \Phi$ wrong

unconditional fairness: $ufair = \Box\Diamond\, \exists\bigcirc start$

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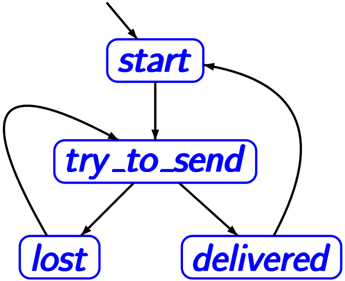

$\Phi = \forall\Box\forall\Diamond start$

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$\mathcal{T} \not\models_{wfair} \Phi$

$\mathcal{T} \models_{sfair} \Phi$?

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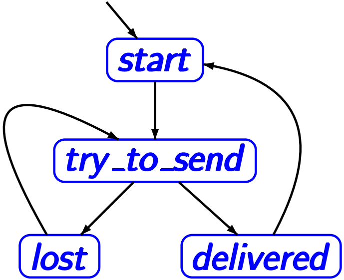
weak fairness: $wfair = \Diamond\Box\, \exists\bigcirc delivered \rightarrow \Box\Diamond delivered$

strong fairness: $sfair = \Box\Diamond\, \exists\bigcirc delivered \rightarrow \Box\Diamond delivered$

Simple communication protocol



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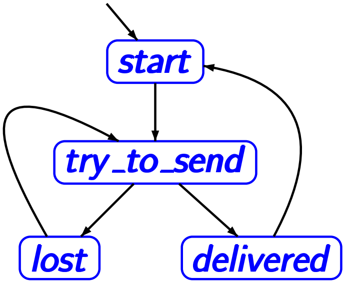

$\Phi = \forall\Box\forall\Diamond start$

$\mathcal{T} \models_{ufair} \Phi$ $\checkmark$

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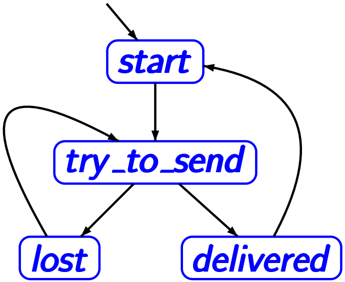

$\Phi = \forall\Box\forall\Diamond \textit{start}$

$\mathcal{T} \models_{\textit{ufair}} \Phi$ ✓

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$\mathcal{T} \models_{\textit{sfair}} \Phi$ ✓

unconditional fairness: $\textit{ufair} = \Box\Diamond\, \exists\bigcirc \textit{start}$

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strong fairness: $\textit{sfair} = \Box\Diamond\, \exists\bigcirc \textit{delivered} \rightarrow \Box\Diamond \textit{delivered}$

$\textit{Sat}(\exists\bigcirc \textit{delivered}) = \{\textit{try\_to\_send}\}$

$\textit{sfair} \,\widehat{=}\, \Box\Diamond \textit{try\_to\_send} \rightarrow \Box\Diamond \textit{delivered}$

Correct or wrong?

If $s \models \forall\Diamond a$ where $a \in AP$ then $s \models_{fair} \forall\Diamond a$

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correct.

Correct or wrong?

If $s \models \forall\Diamond a$ where $a \in AP$ then $s \models_{fair} \forall\Diamond a$

correct. Note that:

$$s \models \forall\varphi \quad \Longrightarrow \quad \text{for all } \pi \in Paths(s)\text{:} \;\; \pi \models \varphi$$

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$$
\begin{aligned}
s \models \forall\varphi \quad &\Longrightarrow \quad \text{for all } \pi \in Paths(s):\ \ \pi \models \varphi \\
&\Longrightarrow \quad \text{for all } \pi \in Paths(s): \\
&\qquad\qquad \pi \models fair \text{ implies } \pi \models \varphi \\
&\Longrightarrow \quad s \models_{fair} \forall\varphi
\end{aligned}
$$

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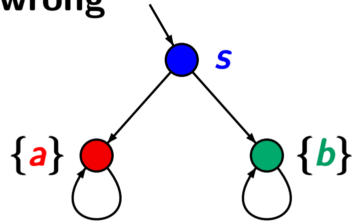
wrong



$fair = \Box\Diamond b$

Correct or wrong?

If $s \models \forall\Diamond a$ where $a \in AP$ then $s \models_{fair} \forall\Diamond a$

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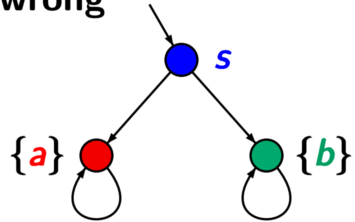
wrong



$fair = \Box\Diamond b$

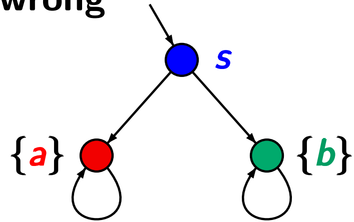
just one fair path ● ● ● ● . . .

If $s \models \forall\Diamond a$ where $a \in AP$ then $s \models_{fair} \forall\Diamond a$

correct.

If $s \models \exists\Diamond a$ where $a \in AP$ then $s \models_{fair} \exists\Diamond a$

wrong



$fair = \Box\Diamond b$

$s \not\models_{fair} \exists\Diamond a$

just one fair path

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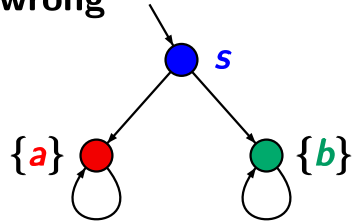


$fair = \Box\Diamond b$

$s \not\models_{fair} \exists\Diamond a$

$s \models \exists\Diamond a$

just one fair path

If $s \models \forall\Diamond a$ where $a \in AP$ then $s \models_{fair} \forall\Diamond a$

correct.

Does the same condition hold if a is replaced with an arbitrary state formula **?**

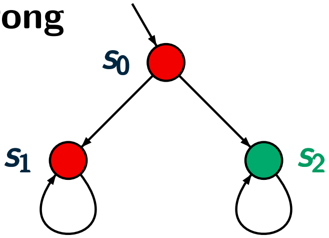
Correct or wrong?

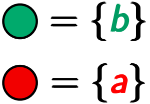
If $s \models \forall\Diamond\exists\Box a$ then $s \models_{fair} \forall\Diamond\exists\Box a$

Correct or wrong?

If $s \models \forall\Diamond\exists\Box a$ then $s \models_{fair} \forall\Diamond\exists\Box a$

wrong

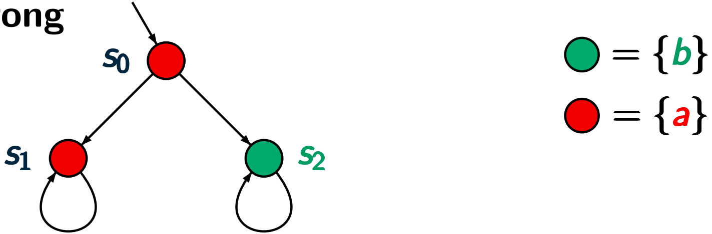




Correct or wrong?

If $s \models \forall\Diamond\exists\Box a$ then $s \models_{fair} \forall\Diamond\exists\Box a$

wrong



$$Sat(\exists\Box a) = \{s_0, s_1\}$$

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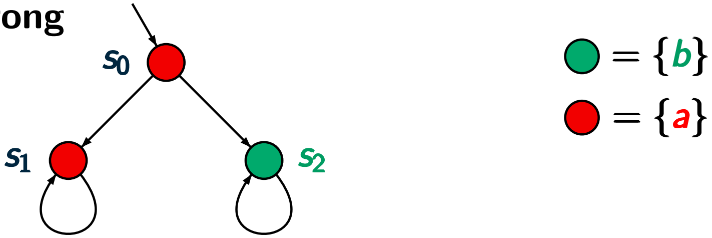

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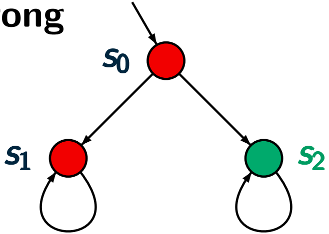

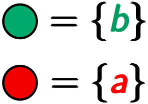

$fair = \Box\Diamond b$

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Correct or wrong?

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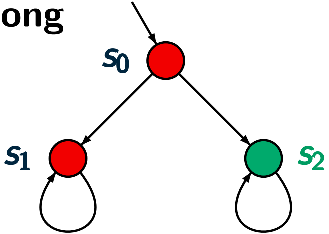


● $= \{b\}$ (green)

● $= \{a\}$ (red)

$fair = \Box\Diamond b$

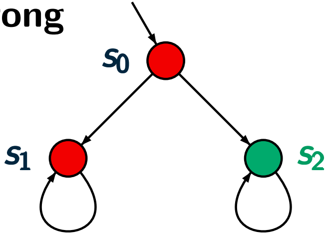
$Sat(\exists\Box a) = \{s_0, s_1\}$

$Sat_{fair}(\exists\Box a) = \varnothing$

$Sat(\forall\Diamond\exists\Box a) = \{s_0, s_1\}$

If $s \models \forall\Diamond\exists\Box a$ then $s \models_{fair} \forall\Diamond\exists\Box a$

wrong



$\bullet = \{b\}$ (green)

$\bullet = \{a\}$ (red)

$fair = \Box\Diamond b$

$Sat(\exists\Box a) = \{s_0, s_1\}$ $\quad$ $Sat_{fair}(\exists\Box a) = \varnothing$

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$Sat_{fair}(\exists\Box true) = ?$

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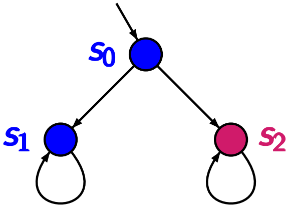
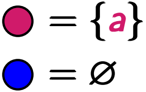
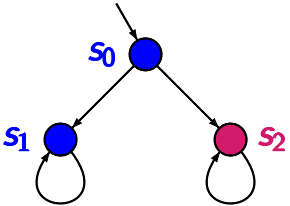




$fair = \Box\Diamond a$

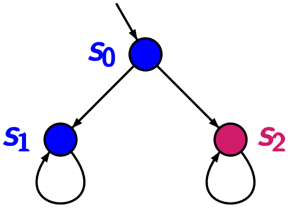


$\bullet = \{a\}$ (red state)

$\bullet = \varnothing$ (blue state)

$fair = \Box\Diamond a$

$Sat_{fair}(\exists\Box true) = ?$



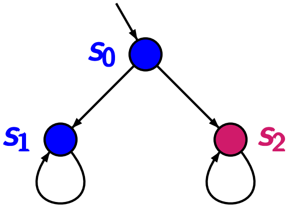
(red) $= \{a\}$

(blue) $= \varnothing$

$fair = \Box\Diamond a$

$$Sat_{fair}(\exists\Box true) = \{s_0, s_2\}$$

$Sat_{fair}(\exists\Box true) = ?$



$\bullet = \{a\}$ (magenta)

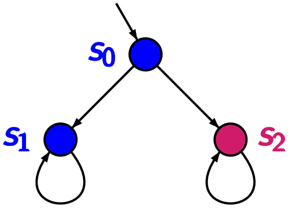
$\bullet = \varnothing$ (blue)

$fair = \Box\Diamond a$

$Sat_{fair}(\exists\Box true) = \{s_0, s_2\}$

$Sat_{fair}(\exists\Box true)$ = set of states s that have at least one fair path

$Sat_{fair}(\exists\Box true) = ?$



$\bullet = \{a\}$

$\bullet = \varnothing$

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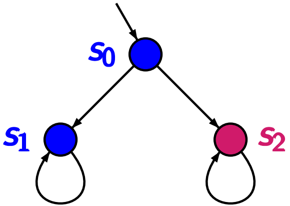
$$= \{s : \exists\pi \in Paths(s) \text{ s.t. } \pi \models fair\}$$

$Sat_{fair}(\exists\Box true) = ?$



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$$Sat_{fair}(\exists\Box true) = \text{set of states } s \text{ that have at least one fair path}$$
$$= \{s : \exists\pi \in Paths(s) \text{ s.t. } \pi \models fair\}$$

fair is realizable iff

$$Sat_{fair}(\exists\Box true) \supseteq \text{set of all reachable states}$$

Model checking problem for FairCTL

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given: finite transition system $\mathcal{T}$
CTL formula Φ
CTL fairness assumption *fair*

question: does $\mathcal{T} \models_{fair} \Phi$ hold ?

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for simplicity:

we suppose that Φ is in existential normal form,
i.e., a $\forall$-free CTL formula with temporal modalities

$$\exists\bigcirc, \ \exists\mathsf{U}, \ \exists\Box$$

Preprocessing of FairCTL model checking

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CTL formula Φ in $\exists$-normal form
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Idea of FairCTL model checking

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   - ***true***, ***a*** $\in$ ***AP***, $\wedge$, $\neg$: as for standard **CTL**

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   - $\exists\bigcirc$, $\exists$**U**: via standard **CTL** model checking
   - $\exists\Box$: via analysis of **SCCs**

CTL model checking with fairness

recursive computation of the fair satisfaction sets:

$$Sat_{fair}(\Psi) \;=\; \{ s \in S : s \models_{fair} \Psi \}$$

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simple cases: $\Psi = \textit{true}$ or $\Psi = a \in AP$ or the outer most operator of Ψ is a negation or conjunction:

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$$Sat_{fair}(\Psi) = \{ s \in S : s \models_{fair} \Psi \}$$

simple cases: $\Psi = true$ or $\Psi = a \in AP$ or the outer most operator of Ψ is a negation or conjunction:

$$\begin{array}{lcl} Sat_{fair}(true) & = & S \\ Sat_{fair}(a) & = & \{ s \in S : \; a \in L(s) \} \\ Sat_{fair}(\neg\Psi) & = & S \setminus Sat_{fair}(\Psi) \\ Sat_{fair}(\Psi_1 \wedge \Psi_2) & = & Sat_{fair}(\Psi_1) \cap Sat_{fair}(\Psi_2) \end{array}$$

Idea of FairCTL model checking

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CTL formula Φ in $\exists$-normal form
CTL fairness assumption *fair*

question: does $\mathcal{T} \models_{fair} \Phi$ hold ?

1. ... preprocessing ...
2. Build the parse tree of Φ and process it in bottom-up-manner. Treatment of:
   - *true*, $a \in AP$, $\wedge$, $\neg$: as for standard CTL
   - $\exists\bigcirc$, $\exists\mathsf{U}$: via standard **CTL** model checking
   - $\exists\Box$: via analysis of SCCs

FairCTL model checking: treatment of $\exists\bigcirc$

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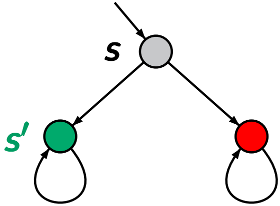
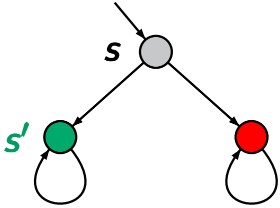


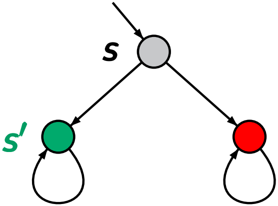
$fair = \Box\Diamond red$

FairCTL model checking: treatment of $\exists\bigcirc$



$fair = \Box\Diamond red$

$s \not\models_{fair} \exists\bigcirc green$



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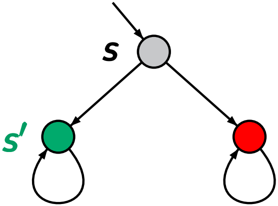
as $s' \not\models_{fair} \exists\Box true$



$fair = \Box\Diamond red$

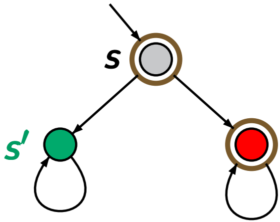
$s \not\models_{fair} \exists\bigcirc green$

as $s' \not\models_{fair} \exists\Box true$

introduce an additional atomic proposition a_{fair} s.t. for all states s:

$$a_{fair} \in L(s) \;\text{ iff }\; s \models_{fair} \exists\Box true$$

FairCTL model checking: treatment of $\exists\bigcirc$



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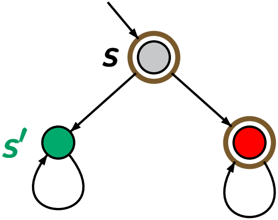

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$$a_{fair} \in L(s) \quad \text{iff} \quad s \models_{fair} \exists\Box true$$

This yields that for all $b \in AP$ and all states s:

$$s \models_{fair} \exists\bigcirc b \quad \text{iff} \quad s \models \exists\bigcirc(b \wedge a_{fair})$$

introduce an additional atomic proposition a_{fair} s.t.

$$a_{fair} \in L(s) \quad \text{iff} \quad s \models_{fair} \exists\Box true$$

This yields that for all $b, c \in AP$ and all states s:

$$s \models_{fair} \exists\bigcirc b \quad \text{iff} \quad s \models \exists\bigcirc(b \wedge a_{fair})$$

$$s \models_{fair} \exists(c \, U \, b) \quad \text{iff} \quad ?$$

introduce an additional atomic proposition a_{fair} s.t.

$$a_{fair} \in L(s) \quad \text{iff} \quad s \models_{fair} \exists\Box true$$

This yields that for all $b, c \in AP$ and all states s:

$$s \models_{fair} \exists\bigcirc b \quad \text{iff} \quad s \models \exists\bigcirc(b \wedge a_{fair})$$
$$s \models_{fair} \exists(c \,U\, b) \quad \text{iff} \quad s \models \exists(c \,U(b \wedge a_{fair}))$$

FairCTL model checking: $\exists\bigcirc$ and $\exists\mathsf{U}$

introduce an additional atomic proposition a_{fair} s.t.

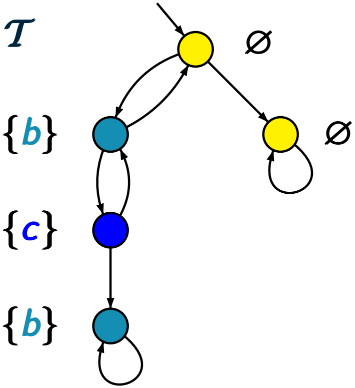
$$a_{fair} \in L(s) \quad \text{iff} \quad s \models_{fair} \exists\Box true$$

This yields that for all $b, c \in AP$ and all states s:

$$s \models_{fair} \exists\bigcirc b \quad \text{iff} \quad s \models \exists\bigcirc(b \wedge a_{fair})$$
$$s \models_{fair} \exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists(c \,\mathsf{U}(b \wedge a_{fair}))$$

hence: treatment of $\exists\bigcirc$ and $\exists\mathsf{U}$ for FairCTL via

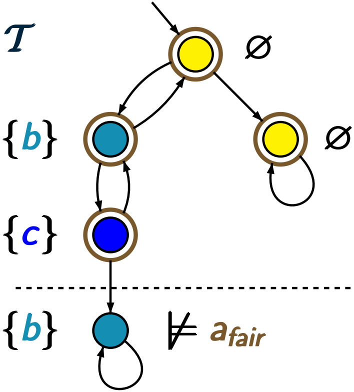
- special methods to compute $Sat_{fair}(\exists\Box true)$
- standard CTL model checking for $\exists\bigcirc$ and $\exists\mathsf{U}$



CTL formula $\exists\Diamond c$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

Example: treatment of $\exists\Diamond$



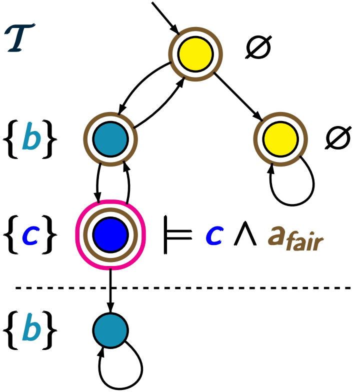
CTL formula $\exists\Diamond c$

$\downarrow$

$\exists\Diamond(c \wedge a_{fair})$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

Example: treatment of $\exists\Diamond$



CTL formula $\exists\Diamond c$

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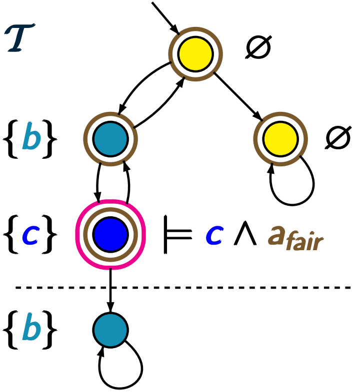


CTL formula $\exists\Diamond c$

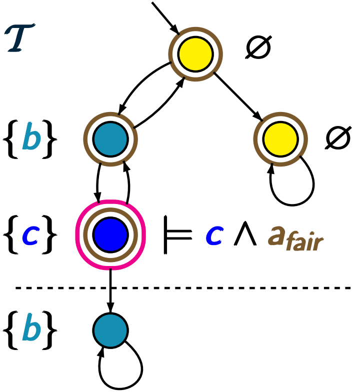
$\downarrow$

$\exists\Diamond (c \wedge a_{fair})$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$\mathcal{T} \models \exists\Diamond(c \wedge a_{fair})$

Example: treatment of $\exists\Diamond$



CTL formula $\exists\Diamond c$

$\downarrow$

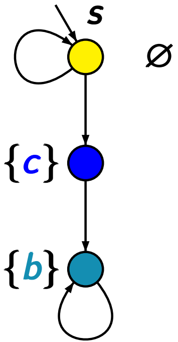
$\exists\Diamond\,(c \wedge a_{fair})$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$$\mathcal{T} \models \exists\Diamond(c \wedge a_{fair}) \implies \mathcal{T} \models_{fair} \exists\Diamond c$$

Example: treatment of ∃U

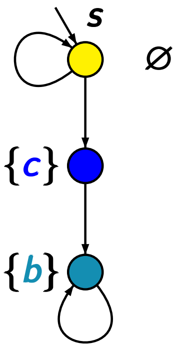
$\mathcal{T}$:



$$\mathcal{T} \models \exists(\neg b \, \mathsf{U} \, c)$$

Example: treatment of ∃U

$\mathcal{T}$:



strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$$\mathcal{T} \models \exists(\neg b \,\mathsf{U}\, c)$$

Example: treatment of ∃U

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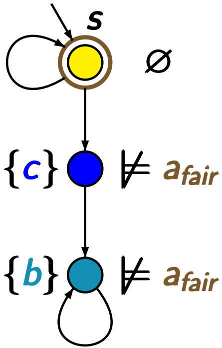

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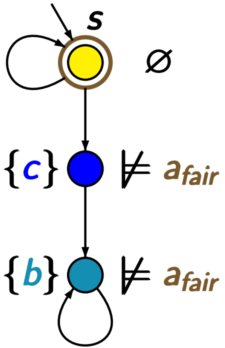


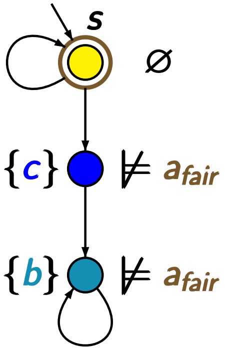
$Sat(c \wedge a_{fair}) = \varnothing$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$\mathcal{T} \models \exists(\neg b \, \mathsf{U} \, c)$

Example: treatment of ∃U

$\mathcal{T}$:



$$s \not\models \exists(\neg b \,\mathsf{U}\, (c \wedge a_{fair}))$$

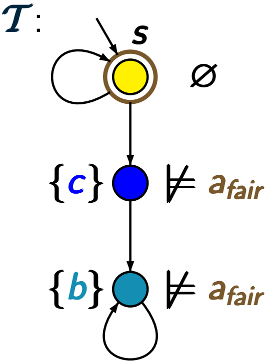
$$\Uparrow$$

$$Sat(c \wedge a_{fair}) = \varnothing$$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$$\mathcal{T} \models \exists(\neg b \,\mathsf{U}\, c)$$

Example: treatment of ∃U



$$s \not\models_{fair} \exists(\neg b \,\mathsf{U}\, c)$$

$$\Uparrow$$

$$s \not\models \exists(\neg b \,\mathsf{U}(c \wedge a_{fair}))$$

$$\Uparrow$$

$$Sat(c \wedge a_{fair}) = \varnothing$$

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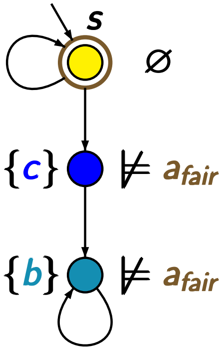
Example: treatment of ∃U

$\mathcal{T}$:



$$s \not\models_{fair} \exists(\neg b \,\mathsf{U}\, c)$$

$$\Uparrow$$

$$s \not\models \exists(\neg b \,\mathsf{U}(c \wedge a_{fair}))$$

$$\Uparrow$$

$$Sat(c \wedge a_{fair}) = \varnothing$$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$\mathcal{T} \models \exists(\neg b \,\mathsf{U}\, c)$, but $\mathcal{T} \not\models_{fair} \exists(\neg b \,\mathsf{U}\, c)$

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c\,\mathsf{U}\,b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c\,\mathsf{U}(b \wedge a_{fair}))$$

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}\,(b \wedge a_{fair}))$$

correct.

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}(b \wedge a_{fair}))$$

correct. Note that:

if $s_0\, s_1 \ldots s_{n-1} s_n$ is a path fragment from $s_0 = s$ s.t. $s_n \models a_{fair}$ then $s_0, s_1, \ldots, s_{n-1} \models a_{fair}$.

$$s \models_{fair} \exists \bigcirc \exists (c \, \mathsf{U} \, b) \quad \text{iff} \quad s \models \exists \bigcirc \exists (c \, \mathsf{U} (b \wedge a_{fair}))$$

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$$\begin{array}{rll} & s \models & \exists \bigcirc \exists (c \, \mathsf{U} (b \wedge a_{fair})) \\ \Longleftrightarrow & s \models & \exists \bigcirc \exists ((c \wedge a_{fair}) \, \mathsf{U} (b \wedge a_{fair})) \end{array}$$

$$s \models_{fair} \exists\bigcirc\exists(c\,\mathsf{U}\,b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c\,\mathsf{U}(b \wedge a_{fair}))$$

correct. Note that:

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$$\begin{aligned} & s \models && \exists\bigcirc\exists(c\,\mathsf{U}(b \wedge a_{fair})) \\ \Longleftrightarrow\; & s \models && \exists\bigcirc\exists((c \wedge a_{fair})\,\mathsf{U}(b \wedge a_{fair})) \\ \Longleftrightarrow\; & s \models && \exists\bigcirc\big(\exists(c\,\mathsf{U}(b \wedge a_{fair})) \wedge a_{fair}\big) \end{aligned}$$

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}(b \wedge a_{fair}))$$

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if $s_0\, s_1 \ldots s_{n-1} s_n$ is a path fragment from $s_0 = s$ s.t. $s_n \models a_{fair}$ then $s_0, s_1, \ldots, s_{n-1} \models a_{fair}$. Hence:

$$\begin{aligned}
& s \models && \exists\bigcirc\exists(c \,\mathsf{U}(b \wedge a_{fair})) \\
\Longleftrightarrow\;& s \models && \exists\bigcirc\exists((c \wedge a_{fair})\,\mathsf{U}(b \wedge a_{fair})) \\
\Longleftrightarrow\;& s \models && \exists\bigcirc\big(\exists(c \,\mathsf{U}(b \wedge a_{fair})) \wedge a_{fair}\big) \\
\Longleftrightarrow\;& s \models_{fair} && \exists\bigcirc\exists(c \,\mathsf{U}\, b)
\end{aligned}$$

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \, \mathsf{U} \, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \, \mathsf{U}(b \wedge a_{fair}))$$

correct.

$$s \models_{fair} \exists\bigcirc\exists(c \, \mathsf{U} \, b) \quad \text{iff} \quad s \models \exists\bigcirc(\exists(c \, \mathsf{U} \, b) \wedge a_{fair})$$

Correct or wrong?

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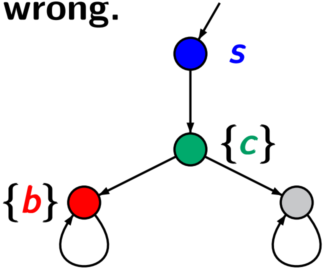
wrong.



$fair = \Box\Diamond gray$

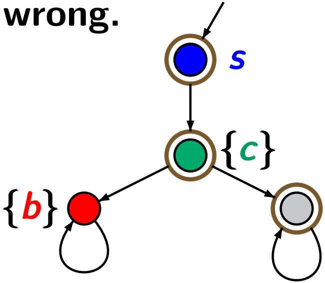
Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}(b \wedge a_{fair}))$$

correct.

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc(\exists(c \,\mathsf{U}\, b) \wedge a_{fair})$$

wrong.



$fair = \Box\Diamond gray$

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}(b \wedge a_{fair}))$$

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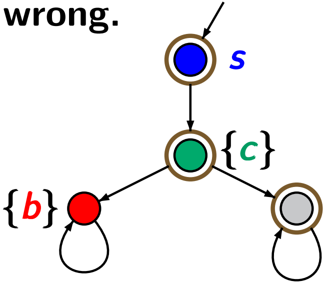

$fair = \Box\Diamond gray$

$Sat_{fair}(\exists(c \,\mathsf{U}\, b)) = \varnothing$

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}(b \wedge a_{fair}))$$

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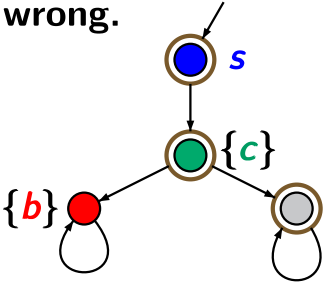
wrong.



$fair = \Box\Diamond gray$

$Sat_{fair}(\exists(c \,\mathsf{U}\, b)) = \varnothing$

$s \not\models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b)$

Correct or wrong?

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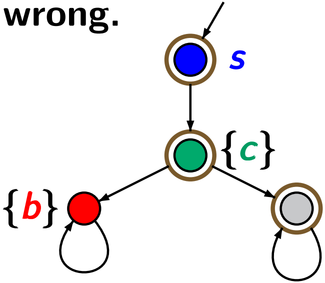

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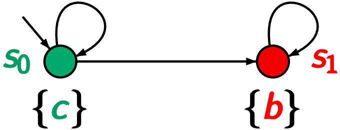
$s \models \exists\bigcirc(\exists(c \,\mathsf{U}\, b) \wedge a_{fair})$

Correct or wrong?

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

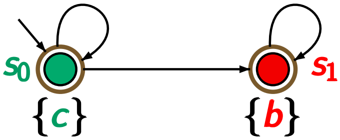
wrong.



$fair = \Box\Diamond b$

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

wrong.



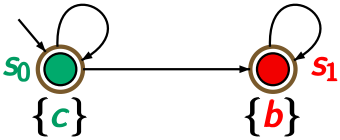
$fair = \Box\Diamond b$

$s_0 \models a_{fair}$

$s_1 \models a_{fair}$

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

wrong.



$fair = \Box\Diamond b$

$s_0 \models a_{fair}$

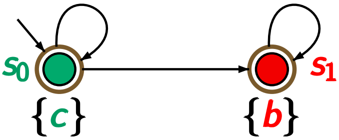
$s_1 \models a_{fair}$

regard state $s = s_0$:

Correct or wrong?

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

wrong.



$fair = \Box\Diamond b$

$s_0 \models a_{fair}$

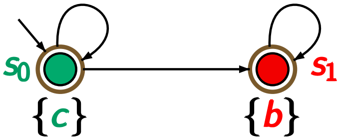
$s_1 \models a_{fair}$

regard state $s = s_0$:

$s \models \exists\Box(c \wedge a_{fair})$,

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

wrong.



$fair = \Box\Diamond b$

$s_0 \models a_{fair}$

$s_1 \models a_{fair}$

regard state $s = s_0$:

$s \models \exists\Box(c \wedge a_{fair})$,

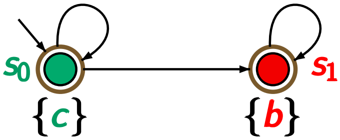
$\uparrow$

path $\pi = s_0\, s_0\, s_0\, s_0 \ldots \models \Box(c \wedge a_{fair})$

Correct or wrong?

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

wrong.



$fair = \Box\Diamond b$

$s_0 \models a_{fair}$

$s_1 \models a_{fair}$

regard state $s = s_0$:

$$s \models \exists\Box(c \wedge a_{fair}), \quad \text{but} \quad s \not\models_{fair} \exists\Box c$$

$\uparrow$

path $\pi = s_0\, s_0\, s_0\, s_0 \ldots \models \Box(c \wedge a_{fair})$

Idea of FairCTL model checking

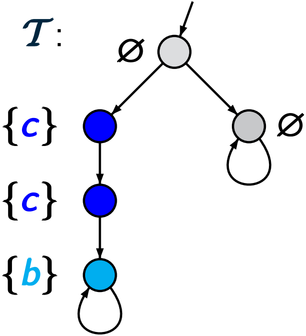
given: finite transition system $\mathcal{T}$
CTL formula Φ in $\exists$-normal form
CTL fairness assumption *fair*

question: does $\mathcal{T} \models_{fair} \Phi$ hold ?

1. ... preprocessing ...
2. Build the parse tree of Φ and process it in bottom-up-manner. Treatment of:
   - *true*, $a \in AP$, $\wedge$, $\neg$: as for standard CTL
   - $\exists\bigcirc$, $\exists U$: via standard CTL model checking
   - $\exists\Box$: via analysis of **SCCs**

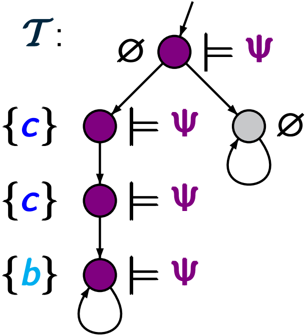
$\exists\Box\Psi$ under strong fairness

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ



$\mathcal{T} \models_{fair} \exists\Box\Psi$?

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ

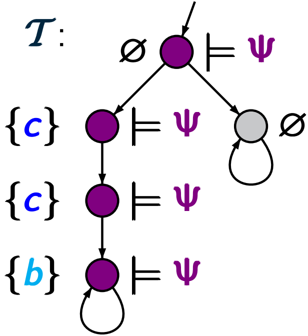


$\mathcal{T} \models_{fair} \exists\Box\Psi$?

1. calculate $Sat_{fair}(\Psi)$

$\exists\Box\Psi$ under strong fairness

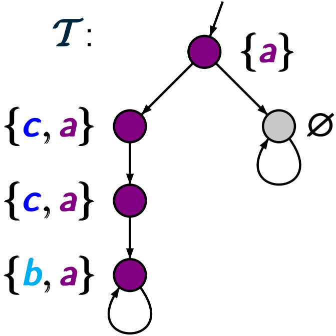
$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ



$\mathcal{T} \models_{fair} \exists\Box\Psi$?

1. calculate $Sat_{fair}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_\Psi$

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ



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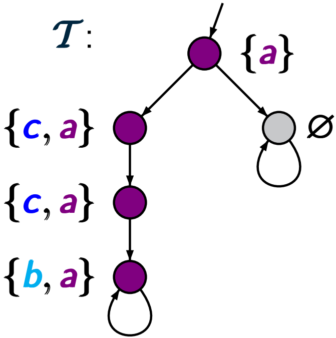
$\exists\Box\Psi$ under strong fairness

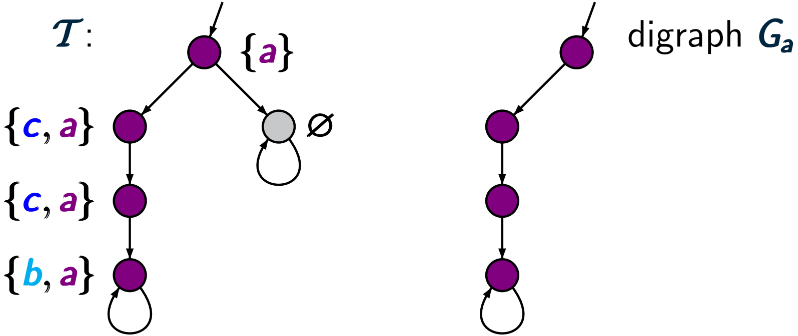
$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ



$\mathcal{T} \models_{fair} \exists\Box\Psi$?

1. calculate $Sat_{fair}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_\Psi$
3. calculate $Sat_{fair}(\exists\Box a)$

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$,   CTL state formula Ψ



1. calculate $Sat_{fair}(\Psi)$
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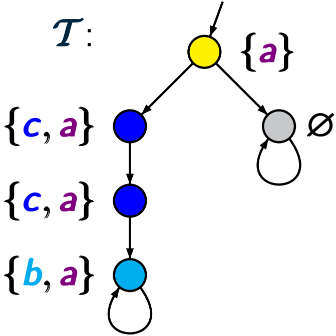
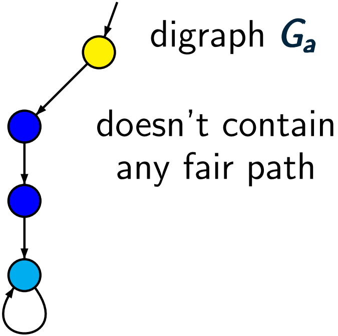
$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ





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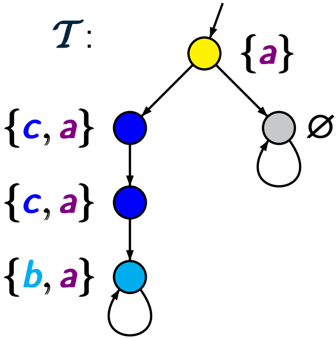
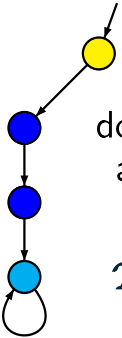
3. calculate $Sat_{fair}(\exists\Box a)$

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ

$\mathcal{T}$:





doesn't contain any fair path

$\Downarrow$

$\mathcal{T} \not\models_{fair} \exists\Box\Psi$

1. calculate $Sat_{fair}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_\Psi$
3. calculate $Sat_{fair}(\exists\Box a) = \varnothing$

Treatment of $\exists\Box a$ for FairCTL

given: finite TS $\mathcal{T}$, atomic proposition a
CTL fairness assumption *fair*

goal: compute $Sat_{fair}(\exists\Box a)$

Treatment of $\exists\Box a$ for FairCTL

given: finite TS $\mathcal{T}$, atomic proposition a
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if all states are labeled by a:

this technique yields a method
to compute $Sat_{fair}(\exists\Box true)$

Treatment of $\exists\Box a$ for FairCTL

given: finite TS $\mathcal{T}$, atomic proposition a
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goal: compute $Sat_{fair}(\exists\Box a)$

if all states are labeled by a:

this technique yields a method
to compute $Sat_{fair}(\exists\Box true)$

here: explanations only for strong fairness

$$fair \;=\; \bigwedge_{1\leq i\leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

Treatment of $\exists\Box$ under strong fairness

$$fair = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

Treatment of $\exists\Box$ under strong fairness

$$fair = \bigwedge_{1 \le i \le k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

$s \models_{fair} \exists\Box a$ iff there exists a path fragment

$$s_0\, s_1 \ldots s_n \ldots s_{n+r}$$

Treatment of $\exists\Box$ under strong fairness

$$fair = \bigwedge_{1\le i\le k} (\Box\Diamond b_i \to \Box\Diamond c_i)$$

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such that $r \geq 1$, $s = s_0$, $s_n = s_{n+r}$ and ...

Treatment of $\exists\Box$ under strong fairness

$$fair = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \to \Box\Diamond c_i)$$

$s \models_{fair} \exists\Box a$ iff there exists a path fragment

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such that $r \geq 1$, $s = s_0$, $s_n = s_{n+r}$ and

- $s_j \models a$ for all $0 \leq j \leq n + r$

$$fair = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

$s \models_{fair} \exists\Box a$ iff there exists a path fragment

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such that $r \geq 1$, $s = s_0$, $s_n = s_{n+r}$ and

- $s_j \models a$ for all $0 \leq j \leq n+r$
- the path $s_0\, s_1 \ldots s_n (s_{n+1} \ldots s_{n+r})^\omega$ is fair, i.e.,

Treatment of $\exists\Box$ under strong fairness

$$fair = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

$s \models_{fair} \exists\Box a$ iff there exists a path fragment

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such that $r \geq 1$, $s = s_0$, $s_n = s_{n+r}$ and

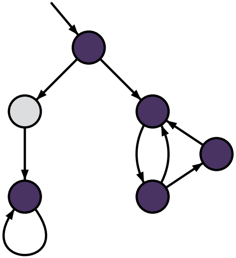
- $s_j \models a$ for all $0 \leq j \leq n + r$
- the path $s_0\, s_1 \ldots s_n (s_{n+1} \ldots s_{n+r})^\omega$ is fair, i.e., for all $1 \leq i \leq k$:

$$\{s_{n+1}, \ldots, s_{n+r}\} \cap Sat(b_i) = \varnothing$$

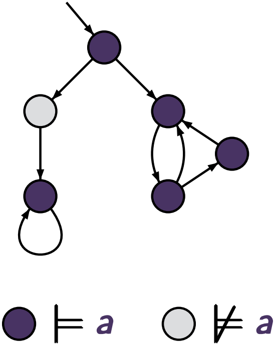
or $$\{s_{n+1}, \ldots, s_{n+r}\} \cap Sat(c_i) \neq \varnothing$$

$\exists\Box a$ under strong fairness

does $\mathcal{T} \models_{fair} \exists\Box a$ hold ?



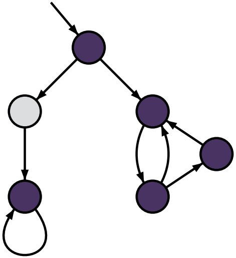
does $\mathcal{T} \models_{fair} \exists\Box a$ hold ?



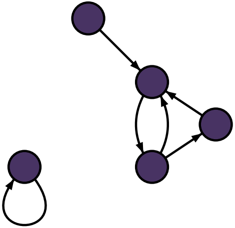
analyze the digraph G_a that results from $\mathcal{T}$ by removing all states s with $s \not\models a$

$\exists\Box a$ under strong fairness

does $\mathcal{T} \models_{fair} \exists\Box a$ hold ?

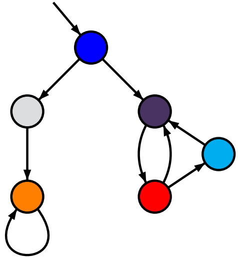


digraph G_a

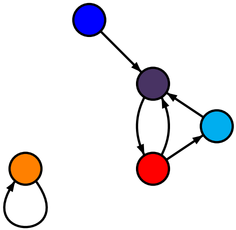


analyze the digraph G_a that results from $\mathcal{T}$ by removing all states s with $s \not\models a$

does $\mathcal{T} \models_{fair} \exists\Box a$ hold ?



digraph G_a



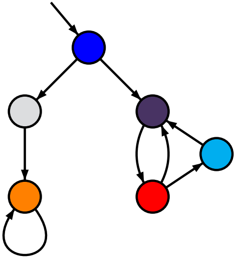
$\widehat{=} \{b_1\}$ $\widehat{=} \{c_1\}$

$\widehat{=} \{b_2\}$ $\widehat{=} \{c_2\}$

$$fair = (\Box\Diamond b_1 \rightarrow \Box\Diamond c_1) \wedge (\Box\Diamond b_2 \rightarrow \Box\Diamond c_2)$$

$\exists\Box a$ under strong fairness

does $\mathcal{T} \models_{fair} \exists\Box a$ hold ?



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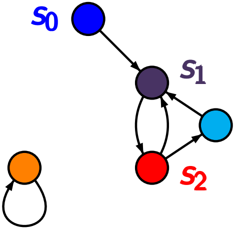

● $\widehat{=} \{b_1\}$ ● $\widehat{=} \{c_1\}$

● $\widehat{=} \{b_2\}$ ● $\widehat{=} \{c_2\}$

$s_0 \, (s_1 \, s_2)^\omega \models \neg\Box\Diamond b_2 \wedge \Box\Diamond c_1$

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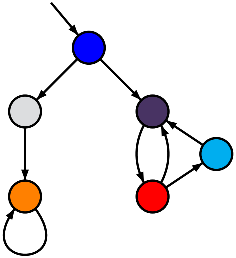

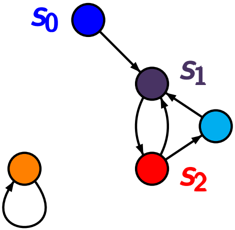

$\widehat{=}\{b_1\}$ $\widehat{=}\{c_1\}$

$\widehat{=}\{b_2\}$ $\widehat{=}\{c_2\}$

$s_0\,(s_1\,s_2)^\omega \models \neg\Box\Diamond b_2 \wedge \Box\Diamond c_1$

$s_0\,(s_1\,s_2)^\omega \models fair$

$$fair = (\Box\Diamond b_1 \rightarrow \Box\Diamond c_1) \wedge (\Box\Diamond b_2 \rightarrow \Box\Diamond c_2)$$

Treatment of $\exists\Box$ under strong fairness

$$fair = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \to \Box\Diamond c_i)$$

$s \models_{fair} \exists\Box a$ iff there exists a path fragment

$$s_0\, s_1 \ldots s_n \ldots s_{n+r}$$

such that $r \geq 1$, $s = s_0$, $s_n = s_{n+r}$ and

- $s_j \models a$ for all $0 \leq j \leq n+r$
- for all $1 \leq i \leq k$: $\{s_{n+1}, \ldots, s_{n+r}\} \cap Sat(b_i) = \varnothing$
  or $\{s_{n+1}, \ldots, s_{n+r}\} \cap Sat(c_i) \neq \varnothing$

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↑

Thus: $D = \{s_{n+1}, \ldots, s_{n+r}\}$ is a strongly connected node-set of the digraph G_a

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↑

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G_a: digraph that arises from $\mathcal{T}$ by removing all states s' with $s' \not\models a$

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(1) D is reachable from s

(2) for all $1 \leq i \leq k$:

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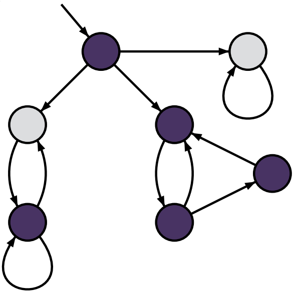
note: if $s \models_{fair} \exists\Box a$ then there might be **no SCC** D where (1) and (2) hold

Example: computation of $Sat_{fair}(\exists\Box a)$

$\mathcal{T}$



computation of $Sat_{fair}(\exists\Box a)$

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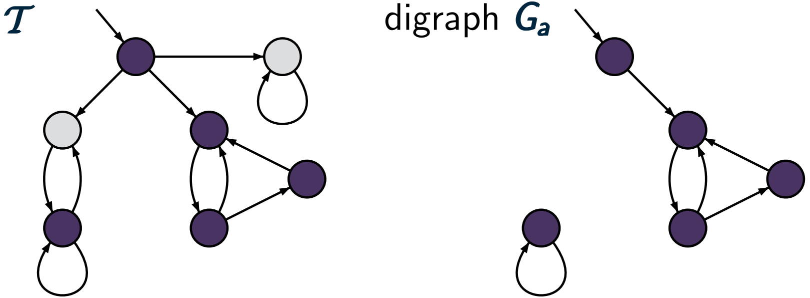
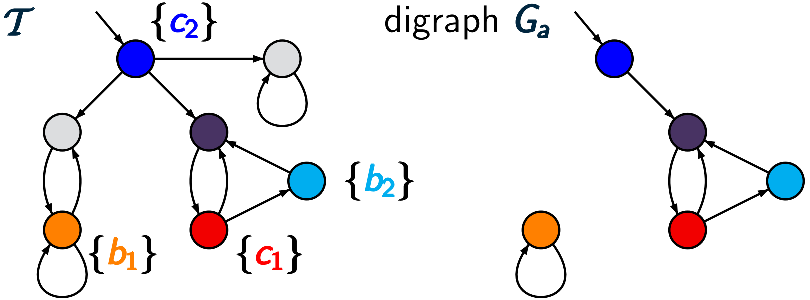


computation of $Sat_{fair}(\exists\Box a)$
by analyzing the digraph G_a

Example: computation of $Sat_{fair}(\exists\Box a)$



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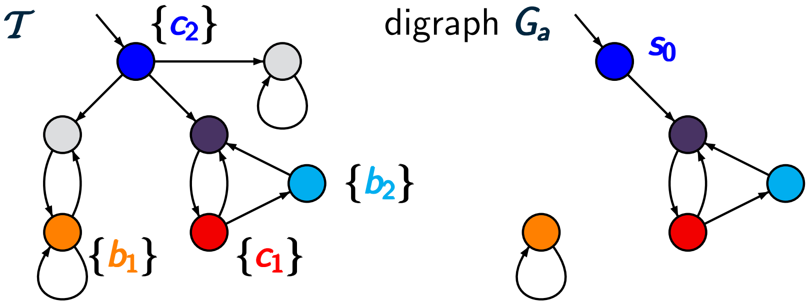

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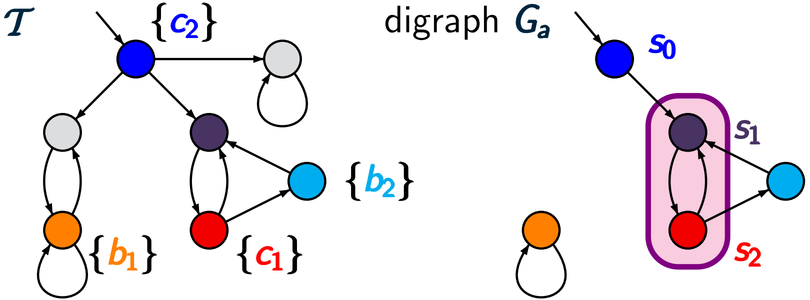
$s_0 \models_{fair} \exists\Box a$

Example: computation of $Sat_{fair}(\exists\Box a)$



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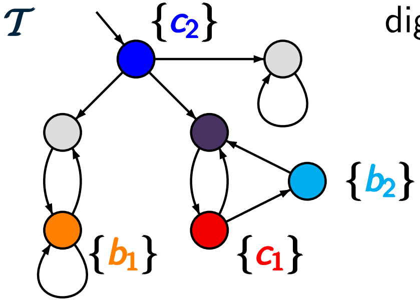

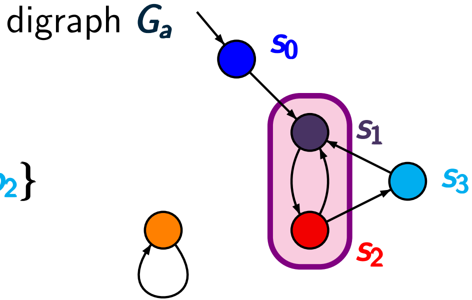

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$s_0 \models_{fair} \exists\Box a$ as $s_0\, s_1\, s_2\, s_1\, s_2 \ldots \models_{LTL} fair$

$Sat_{fair}(\exists\Box a) = \{s_0, s_1, s_2, s_3\}$

CTL model checking with fairness

treatment of $\exists\Box$ for **CTL** with fairness

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here: explanations only for strong fairness

weak fairness and combinations of weak/strong fairness can be treated in an analogous way

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case 1: unconditional fairness

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$s \models_{fair} \exists\Box a$ iff **?**

$\exists\Box a$ under unconditional fairness

$$fair = \bigwedge_{1 \le i \le k} \Box\Diamond c_i$$

$s \models_{fair} \exists\Box a$	iff	there exists a nontrivial **SCC** C in G_a that is reachable from s and $C \cap Sat(c_i) \neq \varnothing$ for $i = 1, \ldots, k$

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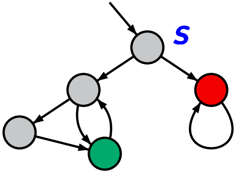
digraph G_a



fairness assumption:
$fair = \Box\Diamond c_1 \wedge \Box\Diamond c_2$

$\exists\Box a$ under unconditional fairness

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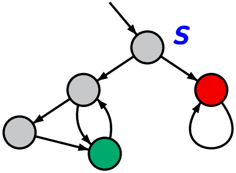

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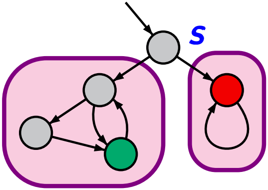

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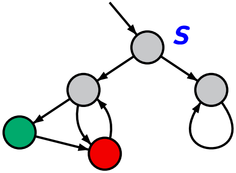

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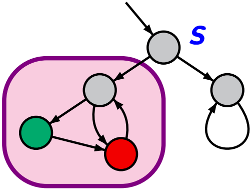

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$s \models_{fair} \exists\Box a$

treatment of $\exists\Box$ for CTL with fairness

here: explanations only for strong fairness

case 1: unconditional fairness $\surd$

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Strong fairness: 1 fairness requirement

$$fair = \Box\Diamond b \rightarrow \Box\Diamond c$$

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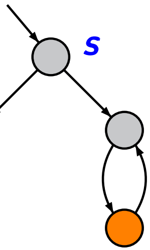


grey $\hat{=}\ \varnothing$

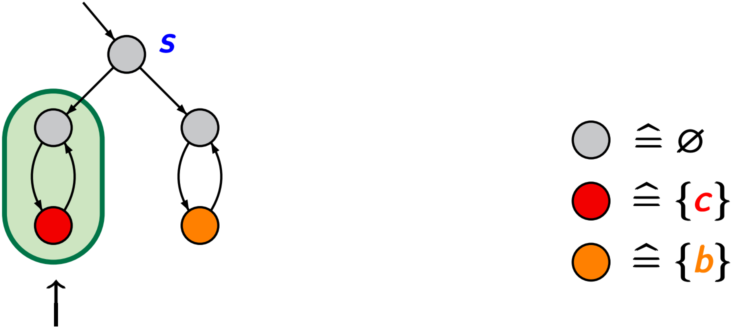
red $\hat{=}\ \{c\}$

orange $\hat{=}\ \{b\}$

Strong fairness: 1 fairness requirement

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$

digraph G_a



nontrivial SCC C of G_a with $C \cap Sat(c) \neq \varnothing$

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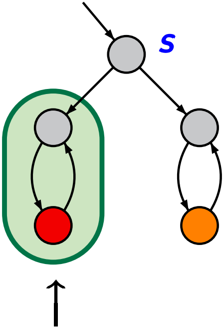
digraph G_a



$\longleftarrow$ $s \models_{fair} \exists\Box a$

$\bigcirc \mathrel{\widehat{=}} \varnothing$

$\bigcirc \mathrel{\widehat{=}} \{c\}$

$\bigcirc \mathrel{\widehat{=}} \{b\}$

nontrivial **SCC** C of G_a with $C \cap Sat(c) \neq \varnothing$

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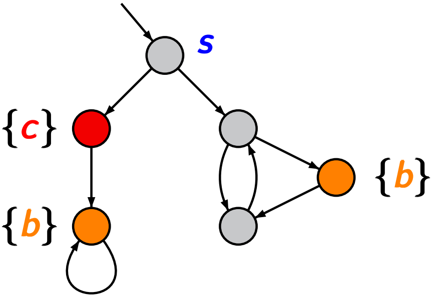

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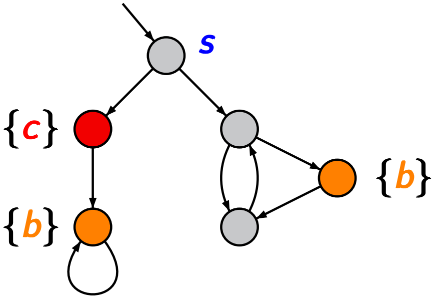

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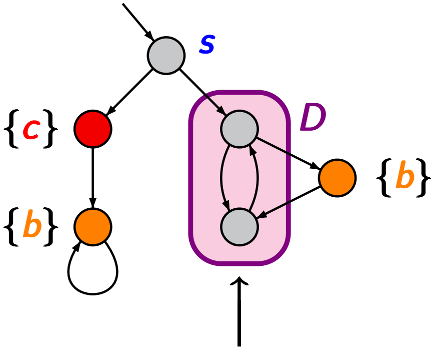


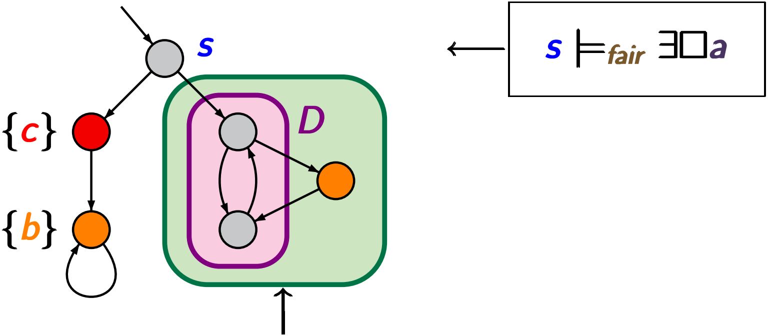
$s \models_{fair} \exists\Box a$

strongly connected node-set D of G_a with
$D \cap Sat(b) = \varnothing$

Strong fairness: 1 fairness requirement

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$

digraph G_a



nontrivial SCC C of G_a that contains a nontrivial SCC D of $G_a|_C \setminus Sat(b)$

CTL model checking with fairness

treatment of $\exists\Box$ for CTL with fairness

here: explanations only for strong fairness

case 1:	unconditional fairness	√
case 2:	$fair = \Box\Diamond b \rightarrow \Box\Diamond c$	√
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Example: 2 strong fairness conditions

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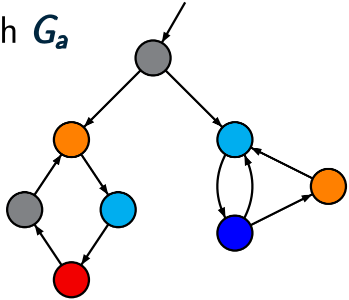

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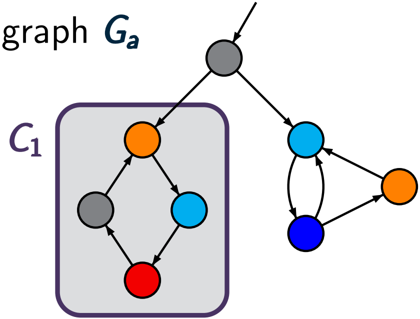

first SCC: $C_1 \cap Sat(c_2) = \varnothing$

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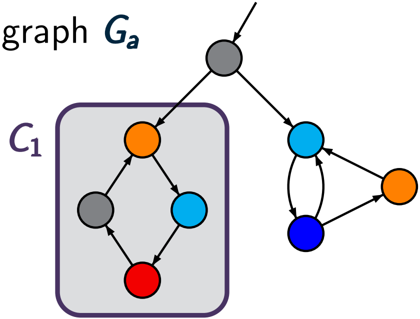

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analyze $C_1 \setminus Sat(b_2)$ w.r.t. $\Box\Diamond b_1 \rightarrow \Box\Diamond c_1$

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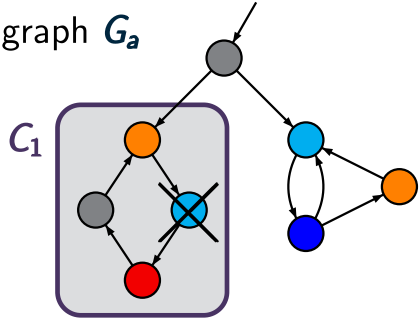

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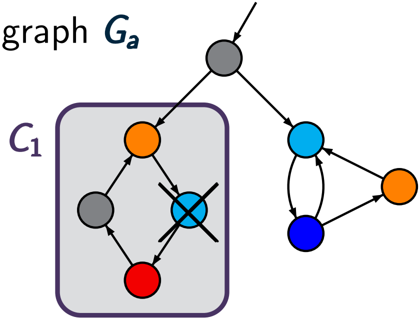

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$\rightsquigarrow$ there is no cycle

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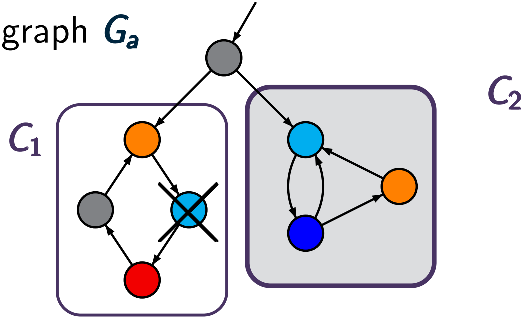

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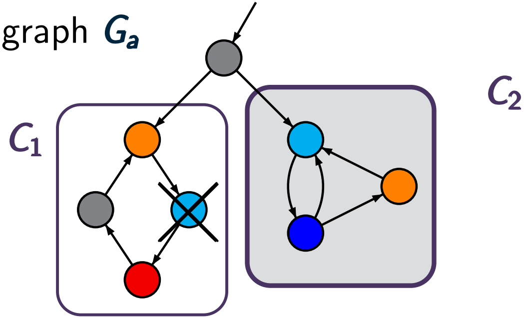

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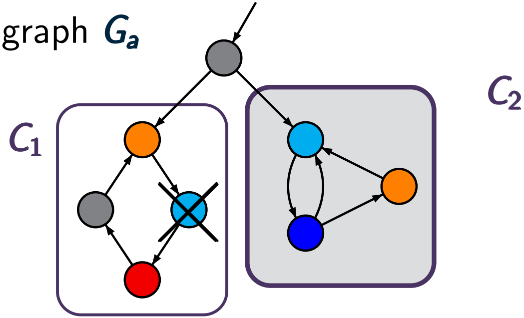

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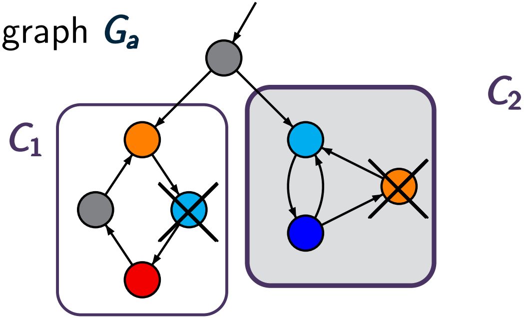

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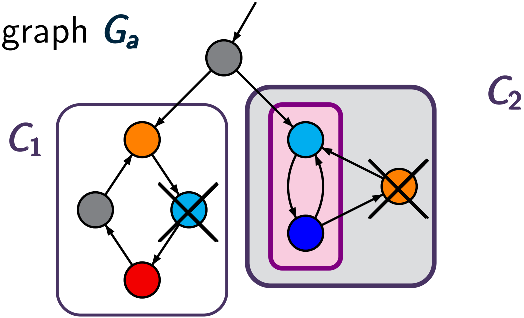

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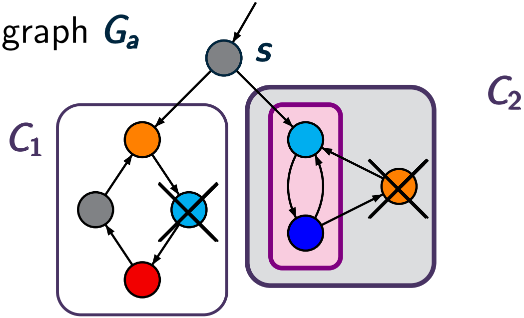

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hence: $s \models_{fair} \exists\Box a$

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Recursive algorithm *CheckFair*(...)

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algorithm $\mathit{CheckFair}\big(C, k, \bigwedge_{1 \le i \le k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)\big)$

Recursive algorithm *CheckFair*(...)

algorithm *CheckFair*$\left(C, k, \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \to \Box\Diamond c_i)\right)$ returns

"true" if there exists a cyclic path fragment $s_0\, s_1 \ldots s_n$ in C such that

$$(s_0\, s_1 \ldots s_{n-1})^\omega \models \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \to \Box\Diamond c_i)$$

"false" otherwise

Recursive algorithm $CheckFair(\ldots)$

pseudo code for $CheckFair(C, k, \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i))$

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recurrence for the time complexity:

$T(n, k) = \ \ldots$ where $n = size(C)$

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CTL model checking with fairness

input:	finite transition system $\mathcal{T}$
	CTL fairness assumption fair
	CTL formula Φ
output:	"**yes**", if $\mathcal{T} \models_{\mathit{fair}} \Phi$. "**no**" otherwise.

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i.e., with the basic modalities $\exists\bigcirc$, $\exists\mathbf{U}$ and $\exists\Box$

Model checking algorithm for FairCTL

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calculate $Sat_{fair}(\exists\Box true)$;

label all states in $Sat_{fair}(\exists\Box true)$ with a_{fair}

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OD
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CASE Ψ is:

$\vdots$

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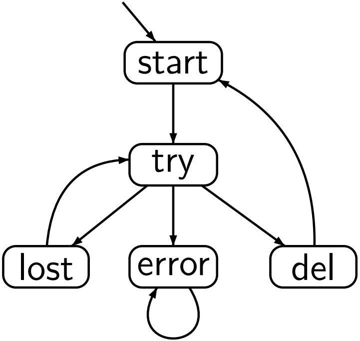
OD

IF $S_0 \subseteq Sat_{fair}(\Phi)$ THEN return "**yes**"

ELSE return "**no**"

FI

Example: CTL model checking with fairness



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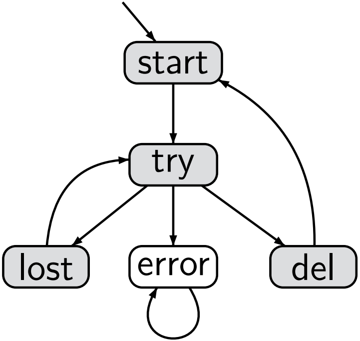

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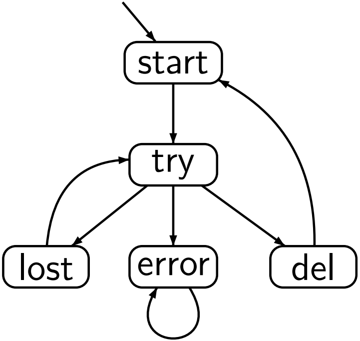

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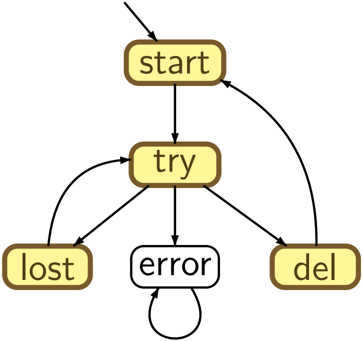

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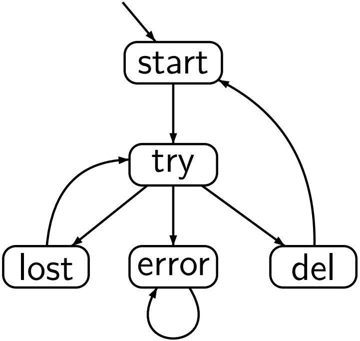

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existential normal form

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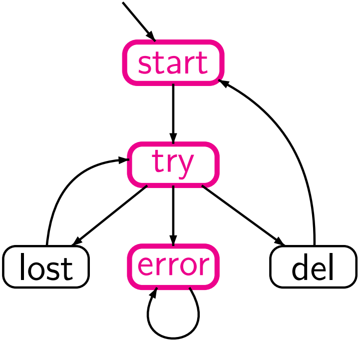

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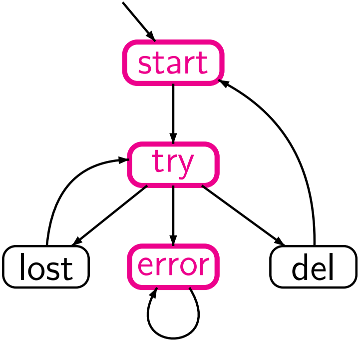

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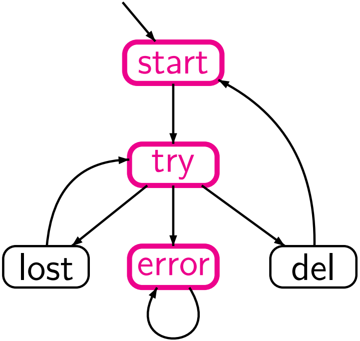

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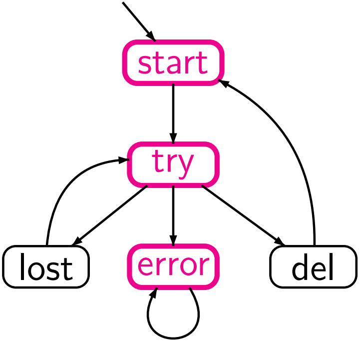

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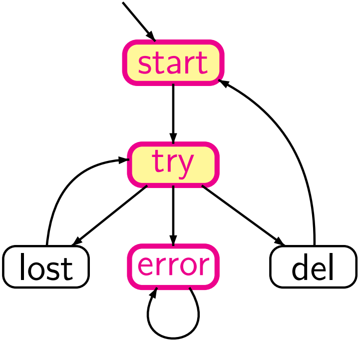

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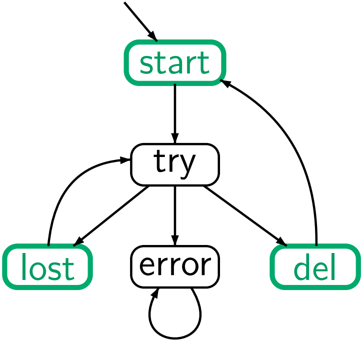

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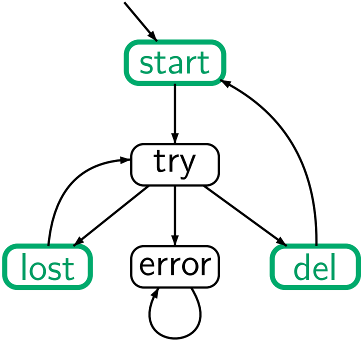

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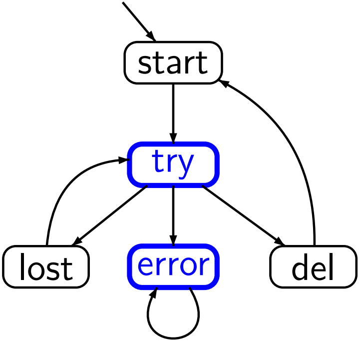

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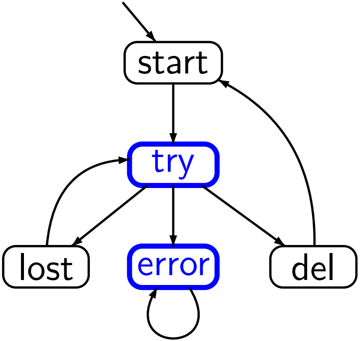
$$Sat_{fair}(\exists\bigcirc a) = Sat(\exists\bigcirc(\ a \wedge a_{fair}\ )) = \{start, lost, del\}$$

$$Sat_{fair}(\neg\exists\bigcirc a) = \{try, error\}$$

Example: CTL model checking with fairness



$$\Phi = \exists\Diamond\ \forall\bigcirc(lost \vee del)$$

$$\equiv \exists\Diamond\neg\exists\bigcirc\ (\neg lost \wedge \neg del)$$

$$\rightsquigarrow \exists\Diamond\ \boxed{\neg\exists\bigcirc a}$$

$$\rightsquigarrow \exists\Diamond b$$

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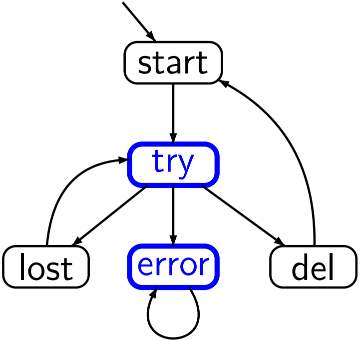

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$Sat_{fair}(\exists\Diamond b)$

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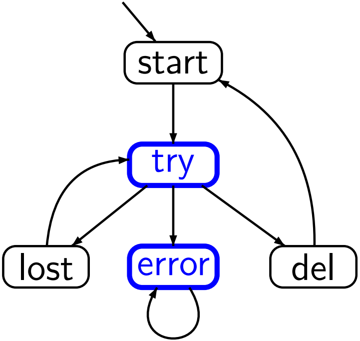

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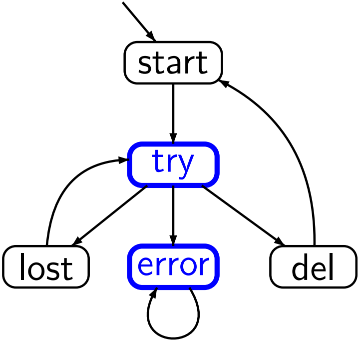

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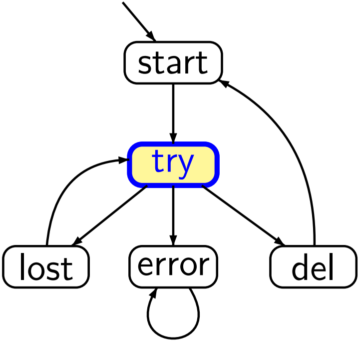
Example: CTL model checking with fairness



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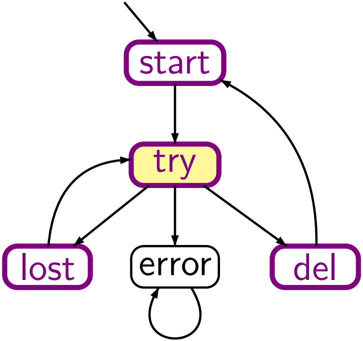

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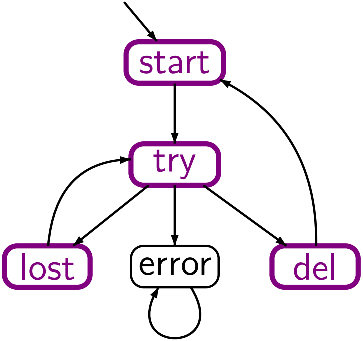

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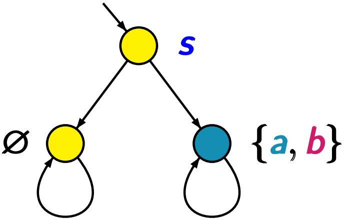
Correct or wrong?

$$s \models_{fair} \forall \bigcirc a \quad \text{iff} \quad s \models \forall \bigcirc (a \wedge a_{fair})$$

$$s \models_{fair} \forall \bigcirc a \quad \text{iff} \quad s \models \forall \bigcirc (a \wedge a_{fair})$$

wrong.

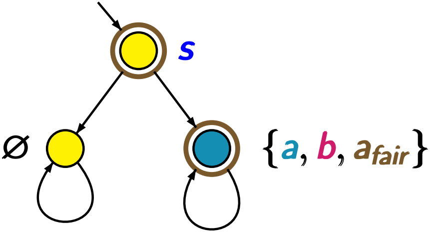


$fair = \Box \Diamond b$

Correct or wrong?

$$s \models_{fair} \forall \bigcirc a \quad \text{iff} \quad s \models \forall \bigcirc (a \wedge a_{fair})$$

wrong.



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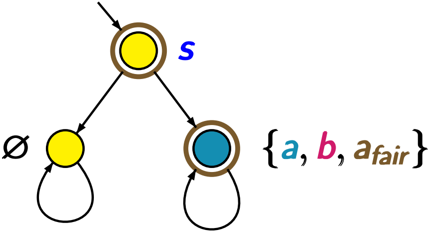

$fair = \Box\Diamond b$

$s \not\models \forall\bigcirc(a \wedge a_{fair})$

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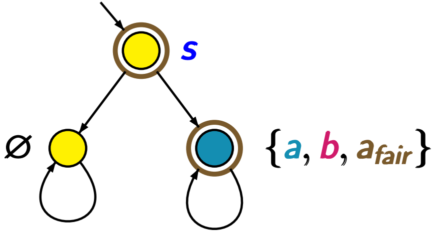

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Correct or wrong?

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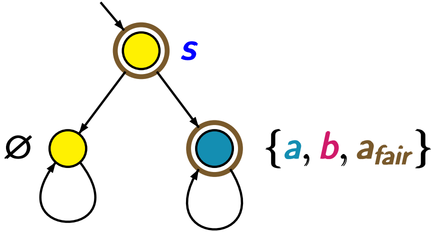


$fair = \Box\Diamond b$

$s \not\models \forall \bigcirc (a \wedge a_{fair})$

$s \models_{fair} \forall \bigcirc a$

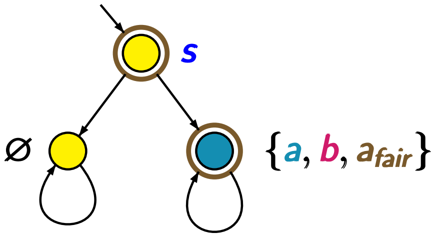
but correct is:

$$s \models_{fair} \forall \bigcirc a \quad \text{iff} \quad ?$$

Correct or wrong?

$s \models_{fair} \forall \bigcirc a$ iff $s \models \forall \bigcirc (a \wedge a_{fair})$

wrong.



$fair = \Box \Diamond b$

$s \not\models \forall \bigcirc (a \wedge a_{fair})$

$s \models_{fair} \forall \bigcirc a$

but correct is:

$s \models_{fair} \forall \bigcirc a$ iff $s \models \forall \bigcirc (a_{fair} \rightarrow a)$

Correct or wrong?

$$s \models_{fair} \forall\Box a \quad \text{iff} \quad s \models \forall\Box(a_{fair} \rightarrow a)$$

Correct or wrong?

$$s \models_{fair} \forall\Box a \quad \text{iff} \quad s \models \forall\Box(a_{fair} \rightarrow a)$$

iff there is no state s' reachable from s with $s' \models \neg a \wedge a_{fair}$

Correct or wrong?

$s \models_{fair} \forall\Box a$ iff $s \models \forall\Box(a_{fair} \rightarrow a)$

iff there is no state s' reachable from s with $s' \models \neg a \wedge a_{fair}$

correct

$s \models_{fair} \forall\Box a$ iff $s \models \forall\Box(a_{fair} \rightarrow a)$

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correct

$s \models_{fair} \forall\Box a$

Correct or wrong?

$s \models_{fair} \forall\Box a$ iff $s \models \forall\Box(a_{fair} \rightarrow a)$

iff there is no state s' reachable from s with $s' \models \neg a \wedge a_{fair}$

correct

$s \models_{fair} \forall\Box a$ iff $s \models_{fair} \neg\exists\Diamond\neg a$

Correct or wrong?

$s \models_{fair} \forall\Box a$ iff $s \models \forall\Box(a_{fair} \rightarrow a)$

iff there is no state s' reachable from s with $s' \models \neg a \wedge a_{fair}$

correct

$s \models_{fair} \forall\Box a$ iff $s \models_{fair} \neg\exists\Diamond\neg a$

iff $s \not\models_{fair} \exists\Diamond\neg a$

$$s \models_{fair} \forall\Box a \quad \text{iff} \quad s \models \forall\Box(a_{fair} \rightarrow a)$$

iff there is no state s' reachable from s with $s' \models \neg a \wedge a_{fair}$

correct

$$s \models_{fair} \forall\Box a \quad \text{iff} \quad s \models_{fair} \neg\exists\Diamond\neg a$$

$$\text{iff} \quad s \not\models_{fair} \exists\Diamond\neg a$$

$$\text{iff} \quad s \not\models \exists\Diamond(\neg a \wedge a_{fair})$$

$$s \models_{fair} \forall\Box a \quad \text{iff} \quad s \models \forall\Box(a_{fair} \rightarrow a)$$

iff there is no state s' reachable from s with $s' \models \neg a \wedge a_{fair}$

correct

$$\begin{aligned} s \models_{fair} \forall\Box a \quad &\text{iff} \quad s \models_{fair} \neg\exists\Diamond\neg a \\ &\text{iff} \quad s \not\models_{fair} \exists\Diamond\neg a \\ &\text{iff} \quad s \not\models \exists\Diamond(\neg a \wedge a_{fair}) \\ &\text{iff} \quad s \models \neg\exists\Diamond(\neg a \wedge a_{fair}) \end{aligned}$$

Correct or wrong?

$s \models_{fair} \forall\Box a$ iff $s \models \forall\Box(a_{fair} \rightarrow a)$

iff there is no state s' reachable from s with $s' \models \neg a \wedge a_{fair}$

correct

$s \models_{fair} \forall\Box a$ iff $s \models_{fair} \neg\exists\Diamond\neg a$

iff $s \not\models_{fair} \exists\Diamond\neg a$

iff $s \not\models \exists\Diamond(\neg a \wedge a_{fair})$

iff $s \models \neg\exists\Diamond(\neg a \wedge a_{fair}) \equiv \forall\Box(a_{fair} \rightarrow a)$

Summary

We just saw:

$$s \models_{fair} \forall \bigcirc a \quad \text{iff} \quad s \models \forall \bigcirc (a_{fair} \rightarrow a)$$

$$s \models_{fair} \forall \Box a \quad \text{iff} \quad s \models \forall \Box (a_{fair} \rightarrow a)$$

Correct or wrong?

We just saw:

$$s \models_{fair} \forall \bigcirc a \quad \text{iff} \quad s \models \forall \bigcirc (a_{fair} \rightarrow a)$$

$$s \models_{fair} \forall \Box a \quad \text{iff} \quad s \models \forall \Box (a_{fair} \rightarrow a)$$

Is the following statement correct **?**

$$s \models_{fair} \forall (b \,\mathsf{U}\, a) \quad \text{iff} \quad s \models \forall \big(b \,\mathsf{U} (a_{fair} \rightarrow a)\big)$$

We just saw:

$$s \models_{fair} \forall\bigcirc a \quad \text{iff} \quad s \models \forall\bigcirc(a_{fair} \rightarrow a)$$

$$s \models_{fair} \forall\Box a \quad \text{iff} \quad s \models \forall\Box(a_{fair} \rightarrow a)$$

Is the following statement correct **?**

$$s \models_{fair} \forall(b \,\mathsf{U}\, a) \quad \text{iff} \quad s \models \forall\big(b \,\mathsf{U}(a_{fair} \rightarrow a)\big)$$

wrong.

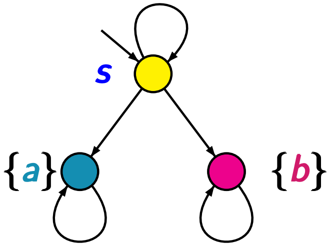
Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc\big((\exists\Diamond a) \wedge a_{fair}\big)$$

$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc\big((\exists\Diamond a) \wedge a_{fair}\big)$$

wrong.

$fair = \Box\Diamond b$



$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc\big((\exists\Diamond a) \wedge a_{fair}\big)$$

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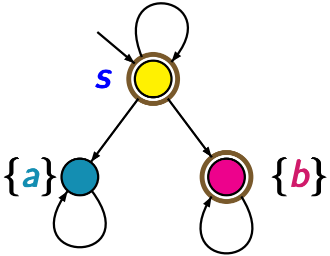

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc\big((\exists\Diamond a) \wedge a_{fair}\big)$$

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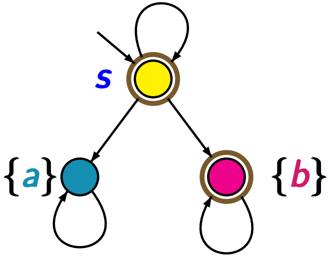
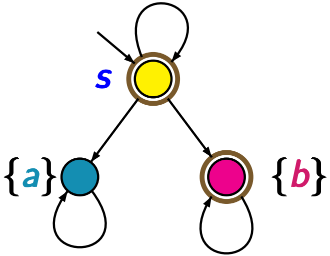


$fair = \Box\Diamond b$

$s \models \exists\bigcirc\big((\exists\Diamond a) \wedge a_{fair}\big)$

$$s \models_{fair} \exists\bigcirc\exists\lozenge a \quad \text{iff} \quad s \models \exists\bigcirc\big((\exists\lozenge a) \wedge a_{fair}\big)$$

wrong.



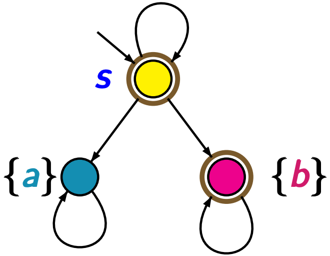
$fair = \Box\lozenge b$

$s \models \exists\bigcirc\big((\exists\lozenge a) \wedge a_{fair}\big)$

regard $s \rightarrow s$

$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc\big((\exists\Diamond a) \wedge a_{fair}\big)$$

wrong.



$fair = \Box\Diamond b$

$s \models \exists\bigcirc\big((\exists\Diamond a) \wedge a_{fair}\big)$

regard $s \rightarrow s$

$s \not\models_{fair} \exists\bigcirc\exists\Diamond a$

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc\big((\exists\Diamond a) \wedge a_{fair}\big)$$

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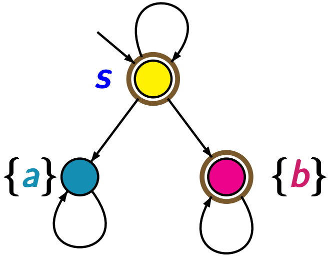

$fair = \Box\Diamond b$

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regard $s \rightarrow s$

$s \not\models_{fair} \exists\bigcirc\exists\Diamond a$

(note $Sat_{fair}(\exists\Diamond a) = \varnothing$)

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc((\exists\Diamond a) \wedge a_{fair})$$

wrong.

$$s \models_{fair} \exists(a \, \mathsf{W} \, c) \quad \text{iff} \quad s \models \exists(a \, \mathsf{W}(c \wedge a_{fair}))$$

remind: **W** = weak until

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc\big((\exists\Diamond a) \wedge a_{fair}\big)$$

wrong.

$$s \models_{fair} \exists(a \,\mathsf{W}\, c) \quad \text{iff} \quad s \models \exists(a \,\mathsf{W}(c \wedge a_{fair}))$$

remind: **W** = weak until

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$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc(\,(\exists\Diamond a) \wedge a_{fair}\,)$$

wrong.

$$s \models_{fair} \exists(a\,\mathsf{W}\,c) \quad \text{iff} \quad s \models \exists(a\,\mathsf{W}(c \wedge a_{fair}))$$

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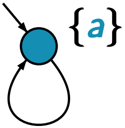

$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc((\exists\Diamond a) \wedge a_{fair})$$

wrong.

$$s \models_{fair} \exists(a \mathsf{W} c) \quad \text{iff} \quad s \models \exists(a \mathsf{W}(c \wedge a_{fair}))$$

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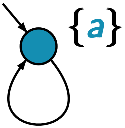

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Correct or wrong?

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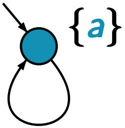
wrong.

$$s \models_{fair} \exists(a \,\mathsf{W}\, c) \quad \text{iff} \quad s \models \exists(a \,\mathsf{W}(c \wedge a_{fair}))$$

remind: $\mathbf{W}$ = weak until

wrong.



$fair = \Box\Diamond b$

$s \models \exists(a \,\mathsf{W}(c \wedge a_{fair}))$

$s \not\models_{fair} \exists(a \,\mathsf{W}\, c)$

Summary: fairness in CTL

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CTL fairness assumptions: formulas similar to **LTL**

e.g., $fair = \bigwedge_{1 \leq i \leq k} (\Box\Diamond\Psi_i \rightarrow \Box\Diamond\Phi_i)$

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CTL satisfaction relation with fairness:

$s \models_{fair} \exists\varphi$ iff there exists $\pi \in Paths(s)$ with $\pi \models fair$ and $\pi \models_{fair} \varphi$

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model checking for **CTL** with fairness:

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model checking for **CTL** with fairness:

- $\exists\bigcirc$, $\exists\mathbf{U}$, $\forall\bigcirc$, $\forall\Box$ via **CTL** model checker

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- analysis of **SCCs** for $\exists\Box$, $\forall$**U**

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model checking for **CTL** with fairness:

- $\exists\bigcirc$, $\exists$**U**, $\forall\bigcirc$, $\forall\Box$ via **CTL** model checker
- analysis of **SCCs** for $\exists\Box$, $\forall$**U**
- complexity: $\mathcal{O}(size(\mathcal{T}) \cdot |\Phi| \cdot |fair|)$