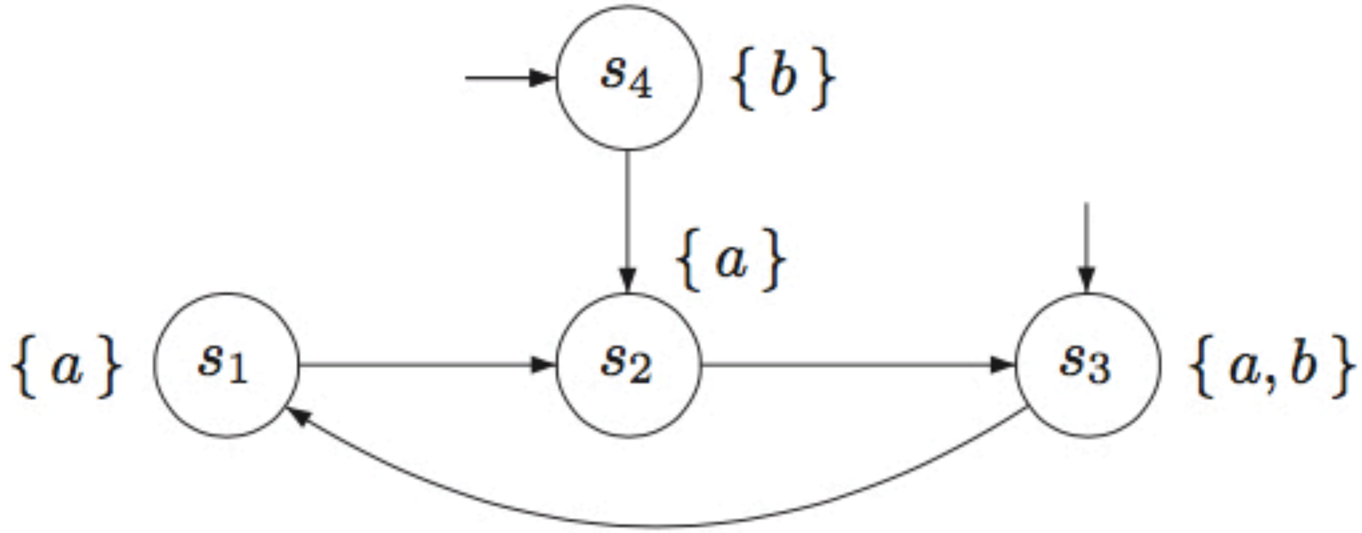
EXERCISE 5.1. Consider the following transition system over the set of atomic propositions $\{a, b\}$:




Indicate for each of the following LTL formulae the set of states for which these formulae are fulfilled:

(a) $\bigcirc\, a$

(b) $\bigcirc \bigcirc \bigcirc\, a$

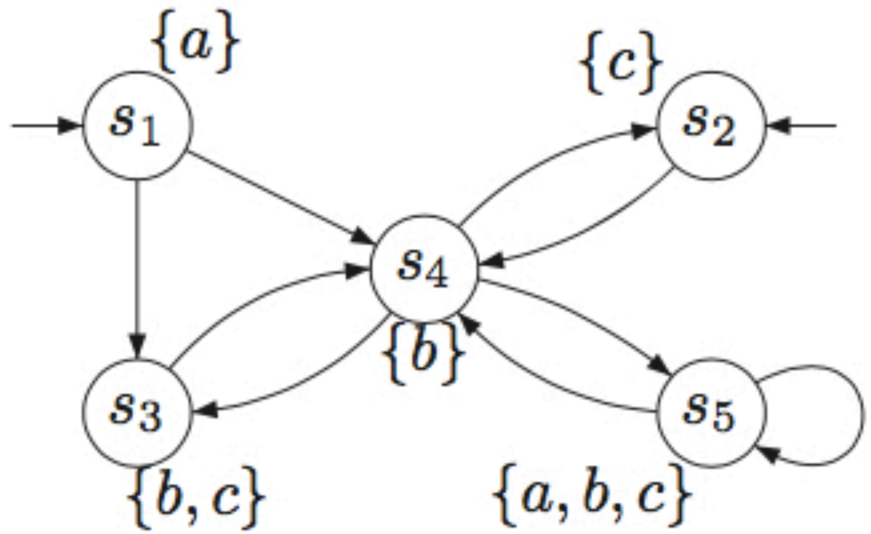
(c) $\square\, b$

(d) $\square \Diamond a$

(e) $\square (b \,\mathsf{U}\, a)$

(f) $\Diamond (a \,\mathsf{U}\, b)$

**Exercise 5.2.** Consider the transition system $TS$ over the set of atomic propositions $AP = \{a, b, c\}$:




Decide for each of the LTL formulae $\varphi_i$ below, whether $TS \models \varphi_i$ holds. Justify your answers! If $TS \not\models \varphi_i$, provide a path $\pi \in Paths(TS)$ such that $\pi \not\models \varphi_i$.

$$
\begin{aligned}
\varphi_1 &= \Diamond \Box c \\
\varphi_2 &= \Box \Diamond c \\
\varphi_3 &= \bigcirc \neg c \rightarrow \bigcirc \bigcirc c \\
\varphi_4 &= \Box a \\
\varphi_5 &= a \,\mathsf{U}\, \Box (b \vee c) \\
\varphi_6 &= (\bigcirc \bigcirc b) \,\mathsf{U}\, (b \vee c)
\end{aligned}
$$

EXERCISE 5.4. Suppose we have two users, *Peter* and *Betsy*, and a single printer device *Printer*. Both users perform several tasks, and every now and then they want to print their results on the *Printer*. Since there is only a single printer, only one user can print a job at a time. Suppose we have the following atomic propositions for *Peter* at our disposal:

- *Peter.request* ::= indicates that *Peter* requests usage of the printer;
- *Peter.use* ::= indicates that *Peter* uses the printer;
- *Peter.release* ::= indicates that *Peter* releases the printer.

For *Betsy*, similar predicates are defined. Specify in LTL the following properties:

(a) Mutual exclusion, i.e., only one user at a time can use the printer.

(b) Finite time of usage, i.e., a user can print only for a finite amount of time.

(c) Absence of individual starvation, i.e., if a user wants to print something, he/she eventually is able to do so.

(d) Absence of blocking, i.e., a user can always request to use the printer

(e) Alternating access, i.e., users must strictly alternate in printing.

EXERCISE 5.6. Which of the following equivalences are correct? Prove the equivalence or provide a counterexample that illustrates that the formula on the left and the formula on the right are not equivalent.

(a) $\Box\varphi \rightarrow \Diamond\psi \quad \equiv \quad \varphi \, \mathsf{U} \, (\psi \vee \neg\varphi)$

(b) $\Diamond\Box\varphi \rightarrow \Box\Diamond\psi \quad \equiv \quad \Box(\varphi \, \mathsf{U} \, (\psi \vee \neg\varphi))$

(c) $\Box\Box(\varphi \vee \neg\psi) \equiv \neg\Diamond(\neg\varphi \wedge \psi)$

(d) $\Diamond(\varphi \wedge \psi) \equiv \Diamond\varphi \ \wedge \ \Diamond\psi$

(e) $\Box\varphi \ \wedge \ \bigcirc\Diamond\varphi \equiv \Box\varphi$

(f) $\Diamond\varphi \ \wedge \ \bigcirc\Box\varphi \equiv \Diamond\varphi$

(g) $\Box\Diamond\varphi \rightarrow \Box\Diamond\psi \equiv \Box(\varphi \rightarrow \Diamond\psi)$

(h) $\neg(\varphi_1 \, \mathsf{U} \, \varphi_2) \equiv \neg\varphi_2 \, \mathsf{W} \, (\neg\varphi_1 \wedge \neg\varphi_2)$

(i) $\bigcirc\Diamond\varphi_1 \equiv \Diamond\bigcirc\varphi_2$

(j) $(\Diamond\Box\varphi_1) \wedge (\Diamond\Box\varphi_2) \equiv \Diamond(\Box\varphi_1 \wedge \Box\varphi_2)$

(k) $(\varphi_1 \, \mathsf{U} \, \varphi_2) \, \mathsf{U} \, \varphi_2 \equiv \varphi_1 \, \mathsf{U} \, \varphi_2$

Exercise 5.11. Consider the transition system $TS$ in Figure 5.25 with the set $AP = \{a, b, c\}$ of atomic propositions. Note that this is a single transition system with two initial states. Consider the LTL fairness assumption

$$fair = (\Box\Diamond(a \wedge b) \rightarrow \Box\Diamond\neg c) \wedge (\Diamond\Box(a \wedge b) \rightarrow \Box\Diamond\neg b).$$

Questions:

(a) Determine the fair paths in $TS$, i.e., the initial, infinite paths satisfying *fair*

(b) For each of the following LTL formulae:

$$\begin{aligned}
\varphi_1 &= \Diamond\Box a \\
\varphi_2 &= \bigcirc\neg a \longrightarrow \Diamond\Box a \\
\varphi_3 &= \Box a \\
\varphi_4 &= b \,\mathsf{U}\, \Box\neg b \\
\varphi_5 &= b \,\mathsf{W}\, \Box\neg b \\
\varphi_6 &= \bigcirc\bigcirc b \,\mathsf{U}\, \Box\neg b
\end{aligned}$$

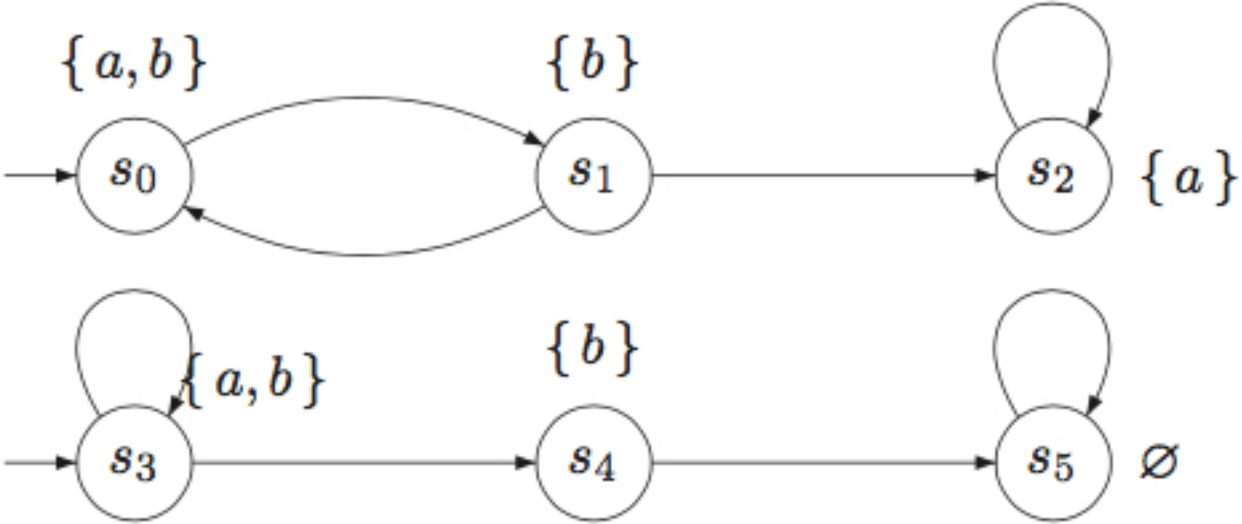



Figure 5.25: Transition system for Exercise 5.11.

determine whether $TS \models_{fair} \varphi_i$. In case $TS \not\models_{fair} \varphi_i$, indicate a path $\pi \in Paths(TS)$ for which $\pi \not\models \varphi_i$.

EXERCISE 5.13. Provide an NBA for each of the following LTL formulae:

$$\Box(a \vee \neg \bigcirc b) \quad \text{and} \quad \Diamond a \ \vee \ \Box\Diamond(a \leftrightarrow b) \quad \text{and} \quad \bigcirc \bigcirc (a \vee \Diamond\Box b).$$

EXERCISE 5.17. Let $\psi = \Box\,(a \leftrightarrow \bigcirc \neg a)$ and $AP = \{\, a \,\}$.

(a) Show that $\psi$ can be transformed into the following equivalent basic LTL formula

$$\varphi = \neg \left[ \text{true}\, \mathsf{U} \left( \neg\,(a \wedge \bigcirc \neg a) \wedge \neg\,(\neg a \wedge \neg \bigcirc \neg a) \right) \right].$$