Master of Science in Computer Science - University of Camerino
Reactive Systems Verification A. Y. 2016/2017
Written Test of 29th June 2017 (Appello I)
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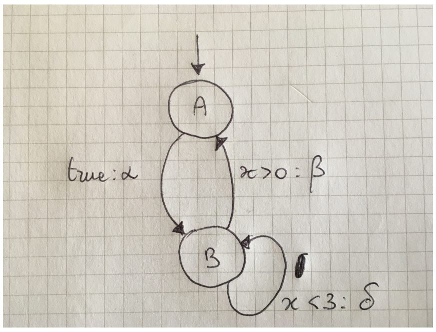
EXERCISE 1 (6 points)

Consider the following program graph



where x is a variable such that $\mathrm{dom}(x) = \{0, 1, 2\}$, $g_0 \equiv (x = 1)$, $\mathrm{Loc}_0 = \{A\}$, $\mathit{Effect}(\alpha, \eta) = \eta[x := 0]$, $\mathit{Effect}(\beta, \eta) = \eta[x := \eta(x) - 1]$ and $\mathit{Effect}(\delta, \eta) = \eta[x := \eta(x) + 1]$.

1. Draw the transition system that is the semantics of the given program graph.

EXERCISE 2 (8 points)

Consider the alphabet $AP = \{A, B, C, D\}$ and the following linear time properties:

(a) Whenever A holds then B holds and C does not hold

(b) A and D hold together at least twice

(c) C holds at least once and only finitely many times

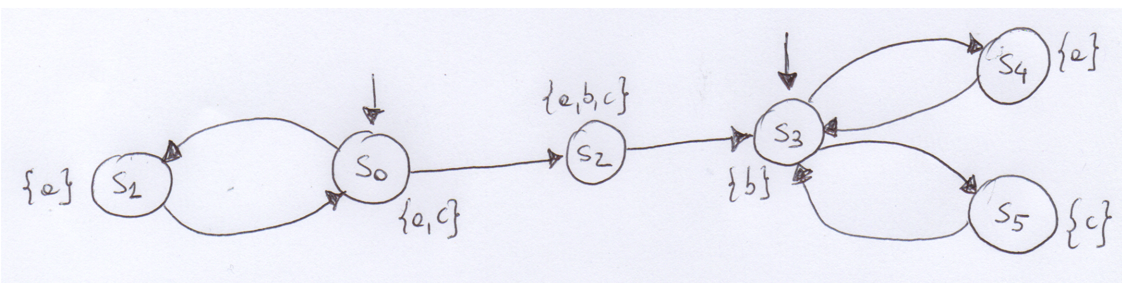
(d) Whenever B holds then after two steps C holds

For each property:

1. formalise it using set expressions and first order logic;
2. formalise it in LTL (you can use the operators next, until, box and diamond, together with all boolean connectives);
3. tell if it is a safety, liveness or mixed property; in case it is a safety property provide an NFA for the language of the **minimal** bad prefixes.

EXERCISE 3

Consider the following transition system TS on $AP = \{a, b, c\}$.



1. **(3 points)** Decide, for each LTL formula φ_i below, whether or not $TS \models \varphi_i$. Justify your answers! If $TS \not\models \varphi_i$ provide a path $\pi \in Paths(TS)$ such that $\pi \not\models \varphi_i$.

$$\varphi_1 = \Diamond b$$
$$\varphi_2 = \bigcirc \bigcirc (c \vee b)$$
$$\varphi_3 = \Box(b \longrightarrow (\bigcirc \Diamond c))$$

2. Consider the following fairness assumptions written as LTL formulas:

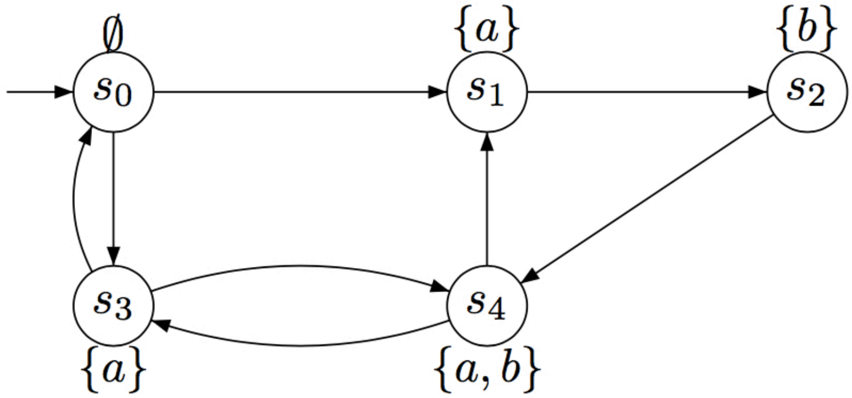
$$\psi_1^{\text{fair}} = \Box\Diamond c \longrightarrow \Box\Diamond b \qquad \psi_2^{\text{fair}} = \Box\Diamond a \qquad \psi_3^{\text{fair}} = \Box\Diamond b \longrightarrow ((\Box\Diamond a) \wedge (\Box\Diamond c))$$

   (a) **(3 points)** Decide whether or not $TS \models_{\text{fair}} \varphi_1$ under the three different fairness conditions ψ_{fair}^i, $i \in \{1, 2, 3\}$, **separately**. Whenever $TS \not\models_{\text{fair}} \varphi_1$ provide a path $\pi \in Paths(TS)$ such that $\pi \not\models \varphi_1$ and arguing that π is fair with respect to ψ_{fair}^i.

   (b) **(3 points)** Decide whether or not $TS \models_{\text{fair}} \varphi_3$ under the three different fairness conditions ψ_{fair}^i, $i \in \{1, 2, 3\}$, **separately**. Whenever $TS \not\models_{\text{fair}} \varphi_6$ provide a path $\pi \in Paths(TS)$ such that $\pi \not\models \varphi_6$ and arguing that π is fair with respect to ψ_{fair}^i.

EXERCISE 4 (8 points)

Consider the following transition system



1. Calculate $\text{Sat}(a \leftrightarrow b)$, $\text{Sat}(\exists(a \vee b)\mathcal{U}(a \wedge b))$ and $\text{Sat}(\exists\Box(a \vee \neg b))$. Justify your answers by showing the steps of the algorithm used for the CTL formulas.