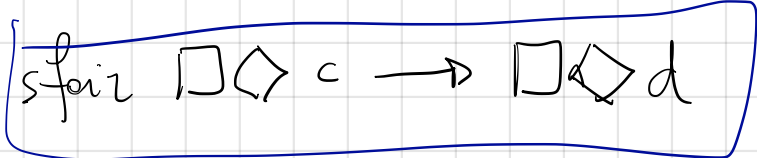


$$\text{Sat}(a \wedge \neg b) = \{s_0, s_5\} \cap \{s_1, s_2, s_5\} = \{s_5\}$$

$$\text{Sat}(b) = \{s_0, s_2, s_3\}$$

$$\text{Sat}(b \wedge \neg a) = \text{Sat}(b) \cap \text{Sat}(\neg a) = \{s_0, s_2, s_3\} \cap \{s_1, s_2, s_3, s_4\} = \{s_2, s_3\} \xrightarrow{\psi_2} c$$

$$\text{Sat}(\exists(b \cup (a \wedge \neg b))) = \{s_5, s_2, s_0\} \rightarrow d \quad T := \emptyset$$

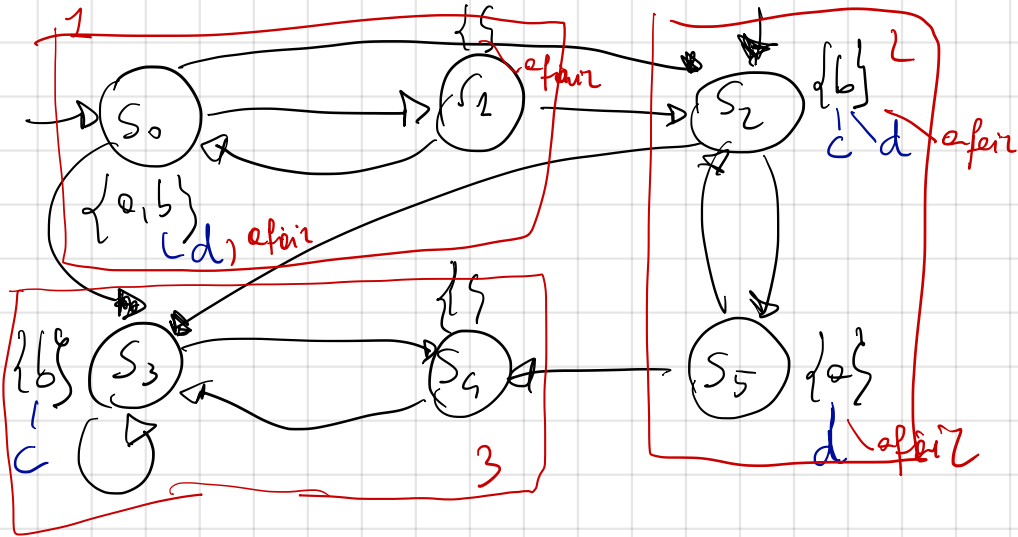


- SCC1:  $\checkmark$  fair is satisfied  $\leftarrow +$
- SCC2:  $\checkmark$  ...  $\leftarrow +$
- SCC3: NO  $\leftarrow \times$

$$T = \text{SCC1} \cup \text{SCC2} = \{s_0, s_1, s_2, s_5\}$$

T can be reached by the states  $\{s_0, s_1, s_2, s_5\}$

$$\text{Thus } \text{Sat}_{\text{fair}}(\exists \Diamond \text{true}) = \{s_0, s_1, s_2, s_5\} \leftarrow \text{fair}$$



$$\begin{aligned} \Phi &= \forall \square \forall \diamond a \equiv \neg \exists \diamond \neg \forall \diamond a \equiv \neg \exists \diamond \exists \square \neg a \equiv \\ &\equiv \neg \exists (\text{true} \wedge \exists \square \neg a) \end{aligned}$$

Let's find  $\text{Set}_{\text{fair}}(\neg \exists (\text{true} \wedge \exists \square \neg a))$

traversing the parse tree of the formula bottom-up.

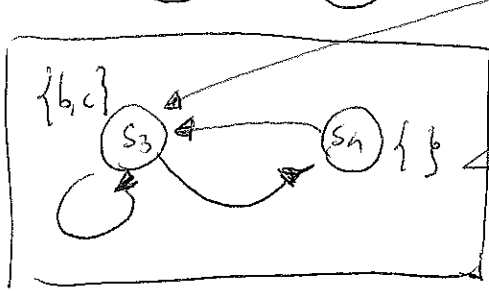
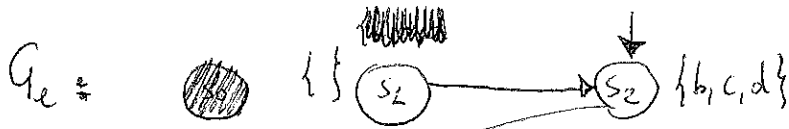
$$\text{Set}_{\text{ofair}}(\neg \exists (\text{true} \cup \exists \Diamond \neg a))$$

$$a_{\text{fair}} \equiv \Box \leftrightarrow c \longrightarrow \Box \leftrightarrow d$$

$$\text{Set}_{\text{ofair}}(a) = \{s_0, s_5\}$$

$$\text{Set}_{\text{ofair}}(\neg a) = S \setminus \{s_0, s_5\} = \{s_1, s_2, s_3, s_4\} \leftarrow \text{new AP "e" labels these states}$$

$$\text{Set}_{\text{ofair}}(\exists \Diamond e) \equiv \{\} \text{ Explanation: } \leftarrow \text{new AP "f" labels no states}$$



SCC

there is no cycle such that

$$D \cap \text{Set}(d) \neq \emptyset$$

or

$$D \cap \text{Set}(c) = \emptyset$$

$$\text{Set}_{\text{ofair}}(\exists \text{true} \cup \neg f) = \text{Set}(\exists (\text{true} \cup (f \wedge a_{\text{fair}}))) = \{\} \rightarrow \text{new AP "g" labels no states}$$

↑  
since  $\text{Set}(f \wedge a_{\text{fair}}) = \{\}$

$$\text{Set}_{\text{ofair}}(\neg g) = S = \{s_0, s_1, s_2, s_3, s_4, s_5\}$$

NOTE:  $s_3, s_4 \models \forall \Box \forall \langle a \rangle a$  because there is NO fair path starting from them, so the CTL formulas are  $\forall$  are trivially true. Hence.