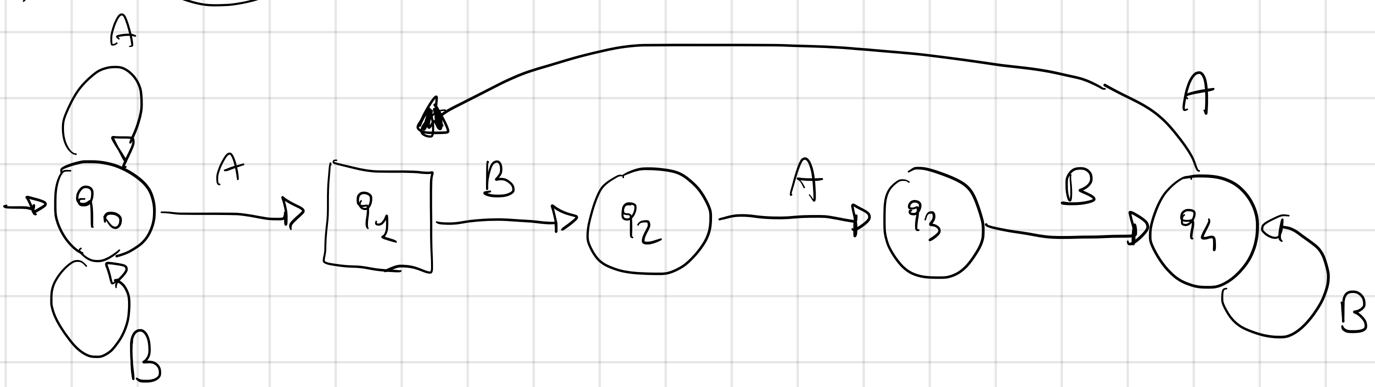
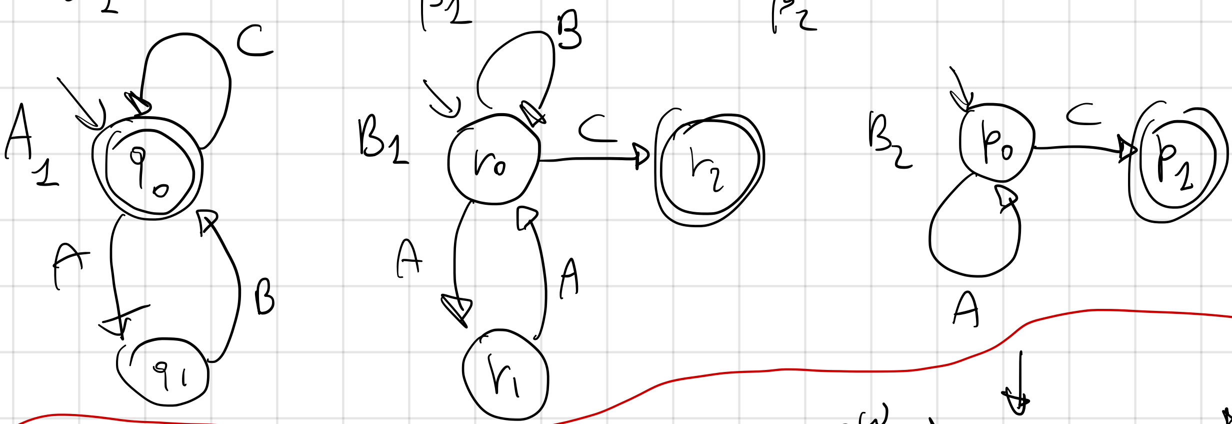
$L_1 = \{\sigma \in \{A,B\}^\omega \mid \sigma$ contains ABA infinitely often, but AA only finitely often$\}$

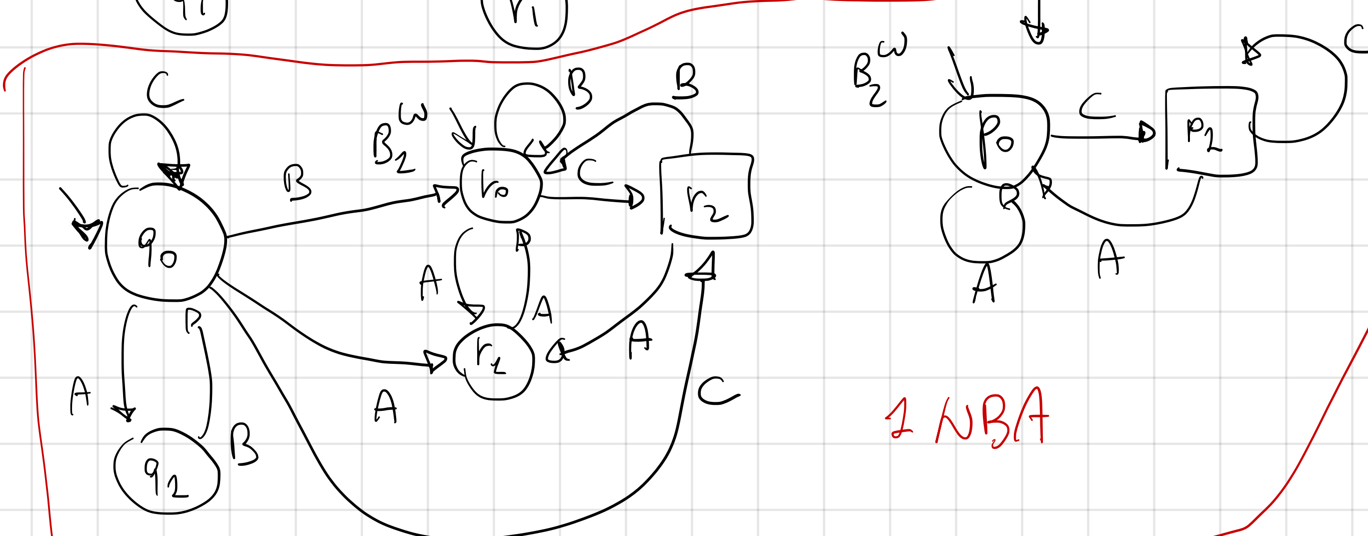
BBBAA BABAAA   ABAABA

ABABAB   BABA



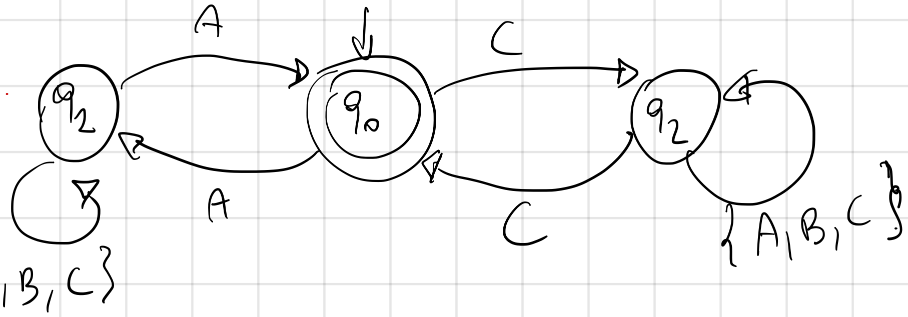
$$\underbrace{(AB + C)^*}_{\alpha_1}\underbrace{((AA+B)C)^\omega}_{\beta_1} + \underbrace{(A^*C)^\omega}_{\beta_2}$$





1 NBA

A_1

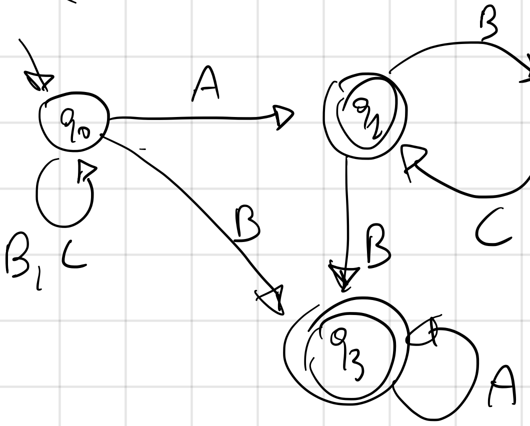


As NFA the language accepted is

$$\mathcal{L}(A_{1\,q_0 q_0}) = \left(\left(A(A+B+C)^*A\right) + C(A+B+C)^*C\right)^*$$

$$= \left(A(A+B+C)^*A + C(A+B+C)^*C\right)^\omega$$

A_2



$$\mathcal{L}(A_{q_0 q_1}) = (B+C)^* A (BC)^*$$

$$\mathcal{L}(A_{q_0 q_3}) = (B+C)^* A (BC)^* B A^* + (B+C)^* B A^* = (B+C)^* \left(A(BC)^* + \varepsilon\right) B A^*$$

$$\mathcal{L}(A_{q_1 q_1}) = (BC)^* \rightsquigarrow (BC)^+$$

$$\mathcal{L}(A_{q_3 q_3}) = A^* \rightsquigarrow (A^+)$$

$$\mathcal{L}_\omega(A) = (B+C)^* A (BC)^* \left((BC)^+\right)^\omega + (B+C)^* \left(A(BC)^* + \varepsilon\right) B A^* (A^+)^\omega$$