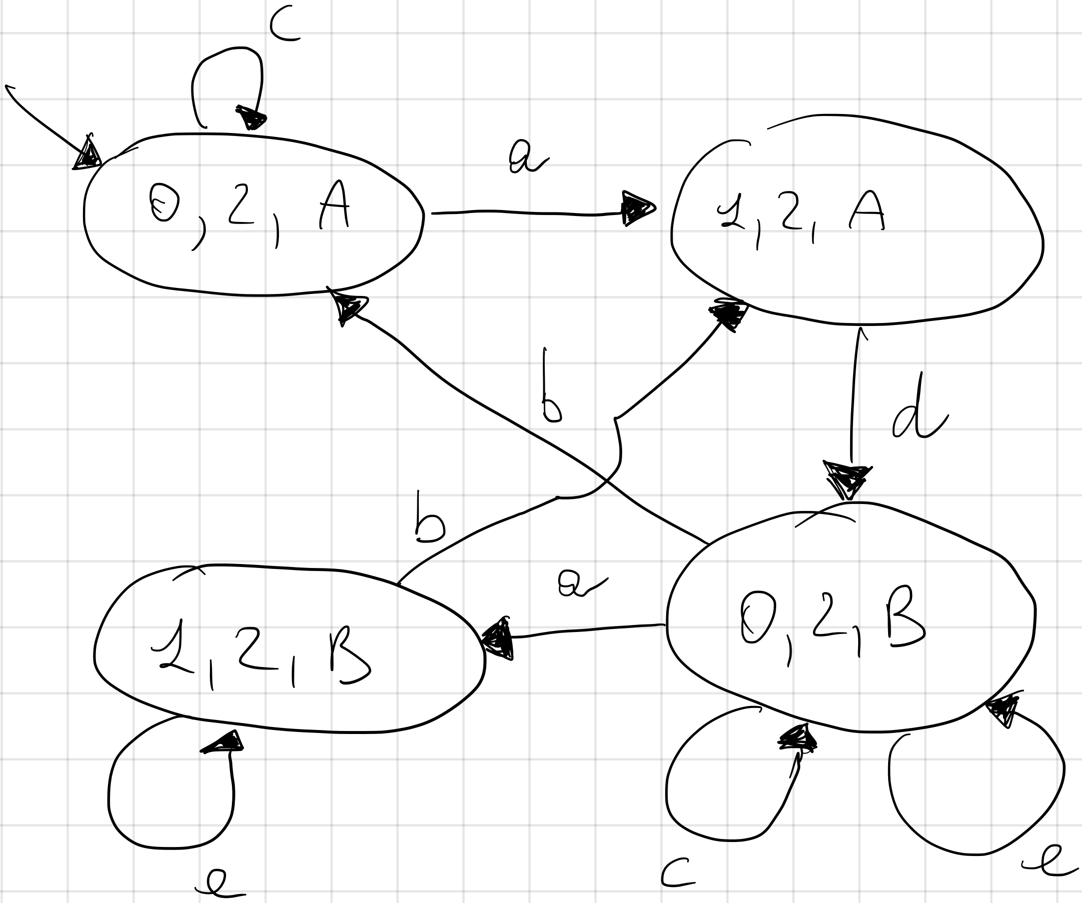
Ex 1

Let's construct the product transition system.
The states of this product contains the three components of the original TSs. Transitions belonging to $H = \{a, b, d\}$ must be done together by the involved components.

Initial state:



Transitions are derived with the rules:

$$\frac{s \xrightarrow{\alpha} s' \quad t \xrightarrow{\alpha} t' \quad \alpha \in H}{\langle s, t \rangle \xrightarrow{\alpha} \langle s', t' \rangle} \qquad \frac{s \xrightarrow{\alpha} s' \quad \alpha \notin H}{\langle s, t \rangle \xrightarrow{\alpha} \langle s', t \rangle}$$

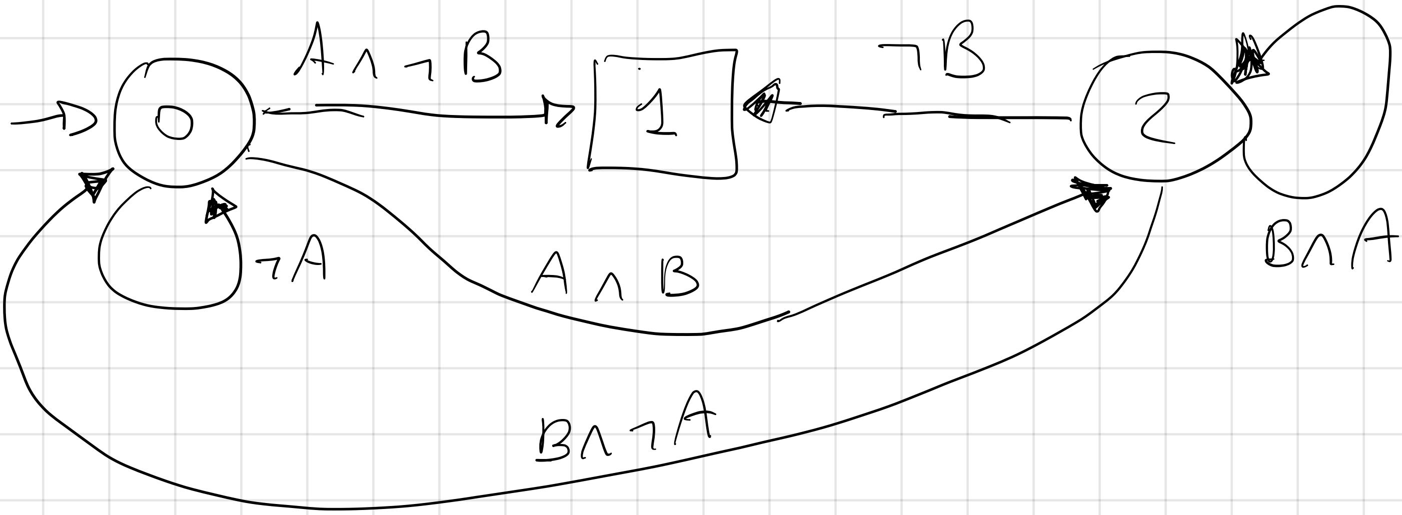
$$\frac{t \xrightarrow{\alpha} t' \quad \alpha \notin H}{\langle s, t \rangle \xrightarrow{\alpha} \langle s, t' \rangle}$$

Ex 2

a) $E_a = \{ A_0 A_1 \dots \in (2^{AP})^\omega \mid \forall i \in \mathbb{N}$
$A \in A_i \Rightarrow (B \in A_i \wedge B \in A_{i+1}) \}$

In LTL: $\square(A \rightarrow (B \wedge \bigcirc B))$

This is a pure safety property. The NFA accepting the language of minimal bad prefixes is:



where 1 is the accepting state.

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b) $E_b = \{ A_0 A_1 \dots \in (2^{AP})^\omega \mid \overset{\infty}{\exists} i \in \mathbb{N} : B \in A_i \wedge$
$\overset{\infty}{\forall} j \in \mathbb{N}. \ C \notin A_i \}$

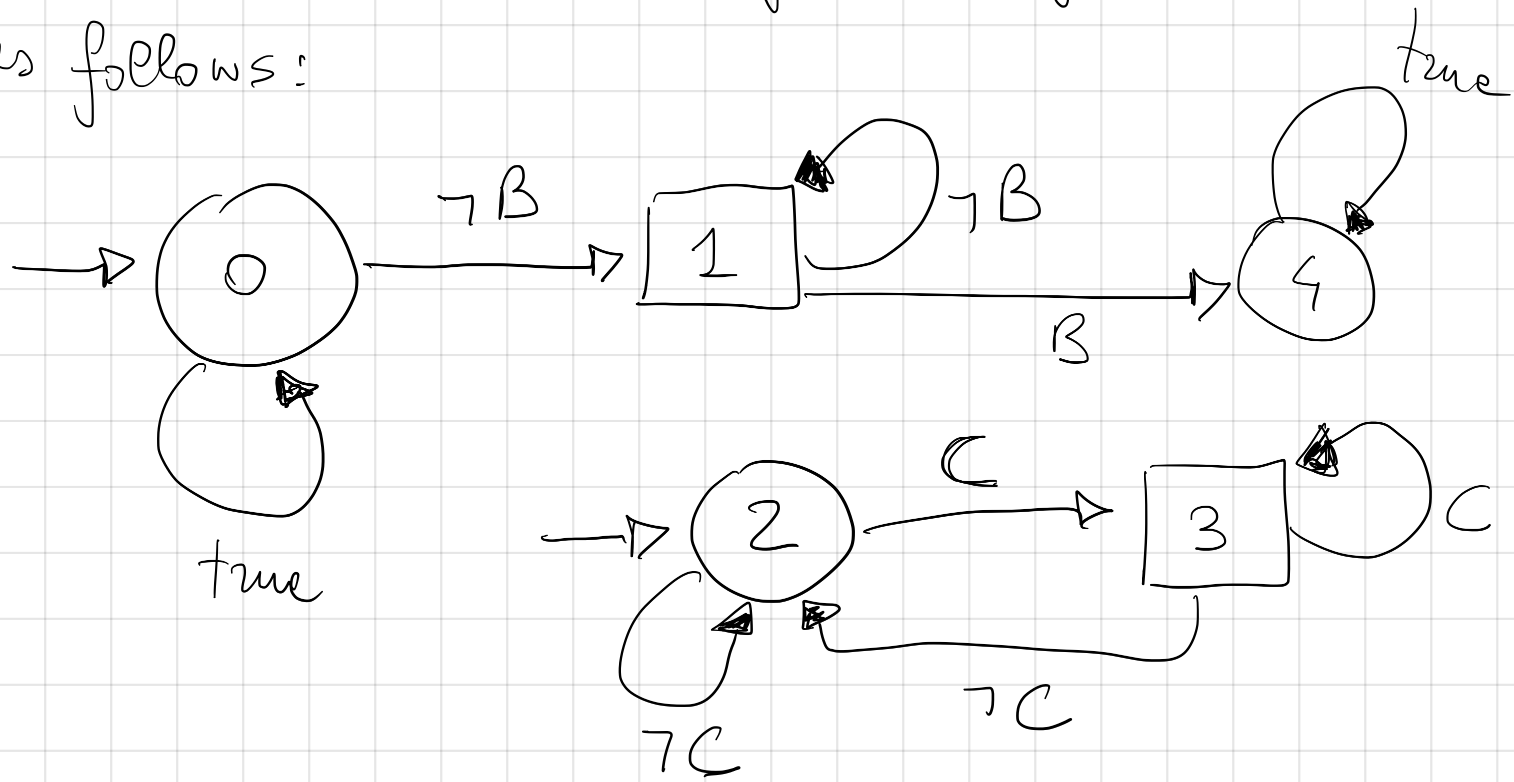
In LTL: $\square \lozenge B \wedge \lozenge \square \neg C$

This is a pure liveness property. To derive an NBA accepting the bad behaviours

We first negate the formula:

$$\neg(\Box\Diamond B \wedge \Diamond\Box\neg C) \equiv \Diamond\Box\neg B \vee \Box\Diamond C$$

A simple NFA accepting this language is as follows:



where the set of initial states is $\{0, 2\}$ and the set of accepting states is $\{1, 3\}$.

c) $E_C = \{ A_0 A_1 \cdots \in (2^{AP})^\omega \mid (\exists i, j \in \mathbb{N} : i \neq j \wedge A \in A_i \wedge A \in A_j) \wedge (\forall k \in \mathbb{N} : B \in A_k \Rightarrow C \in A_k) \}$

In LTL: $\Diamond(A \wedge \bigcirc(\Diamond A)) \wedge \Box(B \to C)$

This is a mixed property.

Ex3 The paths of the TS are the following:

$\pi_0 : ((04)^+ 4^*)^\omega$ $\quad\quad$ $\pi_3 : (213)^\omega$

$\pi_1 : ((04)^+ 4^*)^+ (321)^\omega$

$\pi_2 : (0(4^+))^+ (132)^\omega$

The formula $\varphi = \Diamond(b \, U \, c)$ is satisfied only by the paths of type π_1, π_2 and π_3, where the strongly connected component $\{1,2,3\}$ is reached.

Let's consider φ_1^{fair}: the strong fairness of γ can exclude the paths of the form π_1, but not all of them. In particular the path 04^ω is still fair, so this path is a counter example to show $TS \not\models_{\varphi_1^{fair}} \varphi$.

Let's consider φ_2^{fair}: weak fairness on α is trivially satisfied by the paths of the form π_1, so again

$$TS \nvDash_{\psi_2^{fair}} \varphi$$

where $(04)^\omega$ is a counterexample path.
Lets consider ψ_3^{fair}. In this case, strong fairness of both α and β requires that from the paths of the form π_1, which satisfy the enabling condition "infinitely many times α or β enabled", α or β must be taken infinitely many times. This is clearly impossible, so these paths are all unfair. Thus:

$$TS \vDash_{\psi_3^{fair}} \varphi.$$

Finally, ψ_4^{fair} has strong fairness of α and weak fairness of γ. While a lot of paths of the form π_1 will be excluded by the strong fairness of α, still some paths, e.g. 04^ω, are fair so $TS \nvDash_{\psi_4^{fair}} \varphi$.

Ex 4 | First of all we have to put the formula in EXISTENTIAL NORMAL FORM:

$\forall(a \, \mathcal{U} \, b) \vee \exists \bigcirc (\forall \Box b)$

$\equiv \{ \forall \Box \varphi \equiv \neg \exists \Diamond \neg \varphi \}$

$\forall(a \, \mathcal{U} \, b) \vee \exists \bigcirc (\neg \exists \Diamond \neg b)$

$\equiv \{ \forall \varphi_1 \mathcal{U} \varphi_2 \equiv \neg \exists (\neg \varphi_2 \, \mathcal{U} \, (\neg \varphi_1 \wedge \neg \varphi_2)) \wedge \neg \exists \Box \neg \varphi_2 \}$

$\left(\neg \exists (\neg b \, \mathcal{U} \, (\neg a \wedge \neg b)) \wedge \neg \exists \Box \neg b\right) \vee$
$\left(\exists \bigcirc (\neg \exists \, true \, \mathcal{U} \neg b)\right)$

$Sat(a) = \{s_1, s_2\}$ $\quad Sat(\neg a) = \{s_0, s_3, s_4\}$

$Sat(b) = \{s_1, s_2, s_3, s_4\}$ $\quad Sat(\neg b) = \{s_0, s_1\}$

$Sat(\neg a \wedge \neg b) = \{s_0, s_3, s_4\} \cap \{s_0, s_1\} = \{s_0\}$

$Sat(\exists \neg b \, \mathcal{U} \, (\neg a \wedge \neg b))$: least fixpoint computation

$T_0 = \{s_0\}$, $T_1 = \{s_0\} \leftarrow$ Solution.

$Sat(\neg \exists \neg b \, \mathcal{U} (\neg a \wedge \neg b)) = S - \{s_0\} = \{s_1, s_2, s_3, s_4\}$

$Sat(\exists\Box\neg b)$; greatest fixpoint computation:

$T_0 = \{s_0, s_1\}$, $T_1 = \{s_0\}$, $T_2 = \{\}$, $T_3 = \{\}$

Solution: $\{\}$.

$Sat(\neg\exists\Box\neg b) = S - \{\} = \{s_0, s_1, s_2, s_3, s_4\}$

$Sat(\neg\exists\neg b\,\mathcal{U}(\neg a \wedge \neg b) \wedge \neg\exists\Box\neg b) =$

$\{s_1, s_2, s_3, s_4\} \cap \{s_0, s_1, s_2, s_3, s_4\} = \{s_1, s_2, s_3, s_4\}$

$Sat(\exists\, true\, \mathcal{U} \neg b)$; least fixpoint computation:

$T_0 = \{s_0, s_1\}$, $T_1 = \{s_0, s_1, s_3\}$, $T_2 = \{s_0, s_1, s_3, s_2\}$, $T_3 = \{s_0, s_1, s_3, s_2\}$ ← Solution.

$Sat(\neg\exists\, true\, \mathcal{U} \neg b) = S - \{s_0, s_1, s_3, s_2\} = \{s_4\}$

$Sat(\exists\bigcirc(\neg\exists\, true\, \mathcal{U} \neg b)) = \{s_0, s_4\} =$

$\{s \in S \mid Succ(s) \cap \{s_4\} \neq \emptyset\}$.

$Sat(\varphi) = \{s_1, s_2, s_3, s_4\} \cup \{s_0, s_4\} = S$