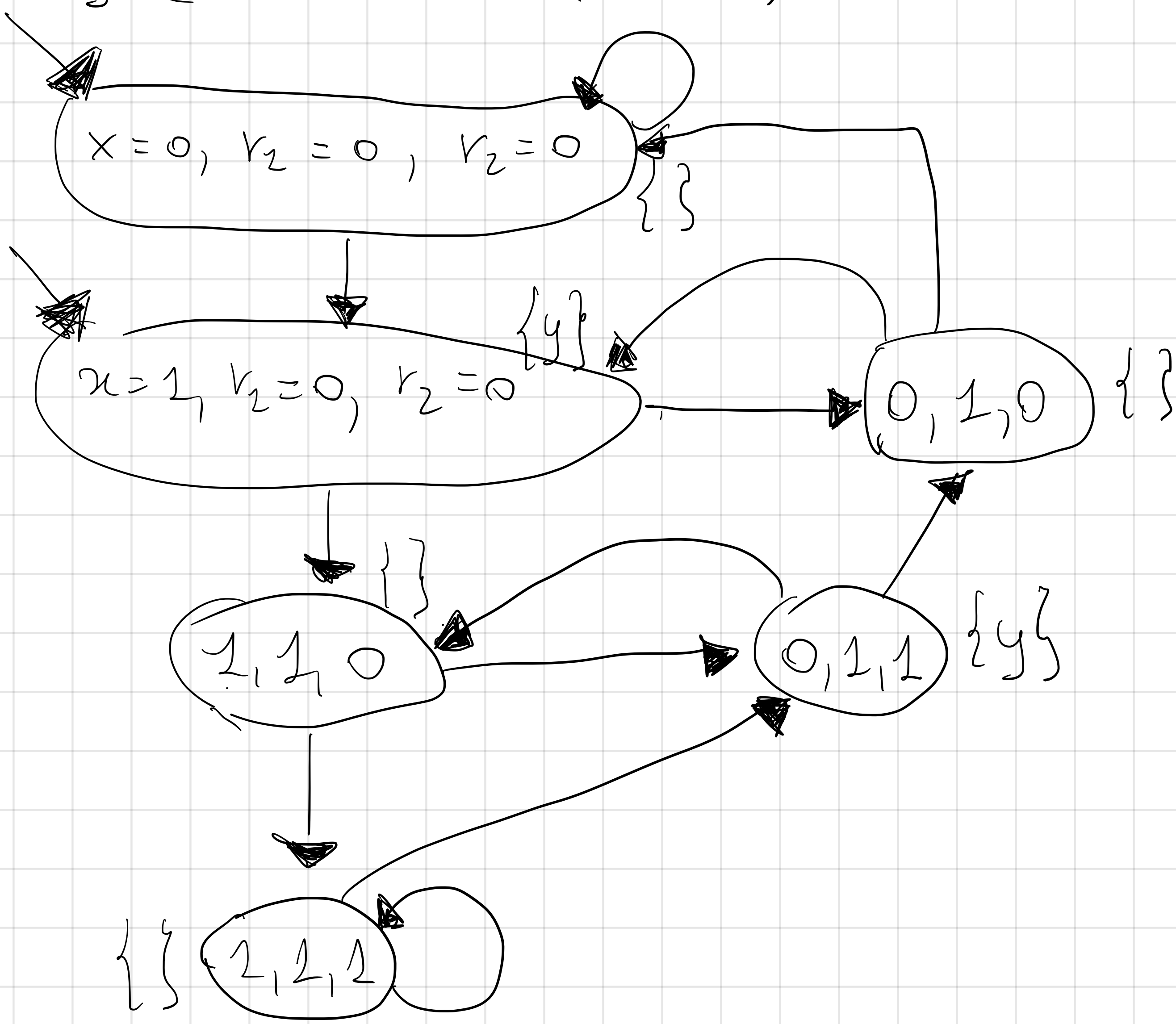
Ex1 $\delta_{r_1} = r_2 \vee x \qquad \delta_{r_2} = r_1 \wedge x$

$y = (r_2 \vee x) \text{ XOR } (r_1 \wedge x)$




Ex2 a) $E_a = \{ A_0 A_1 \cdots \in (2^{AP})^\omega \mid$

$(\forall i \in \mathbb{N} : A \in A_i \Rightarrow (\exists j \in \mathbb{N} : j > i \wedge B \in A_j)) \wedge$

$(\forall k \in \mathbb{N} . B \in A_k \Rightarrow C \in A_k) \}$

In LTL: $\Box(A \rightarrow \Diamond B) \wedge \Box(B \rightarrow C)$

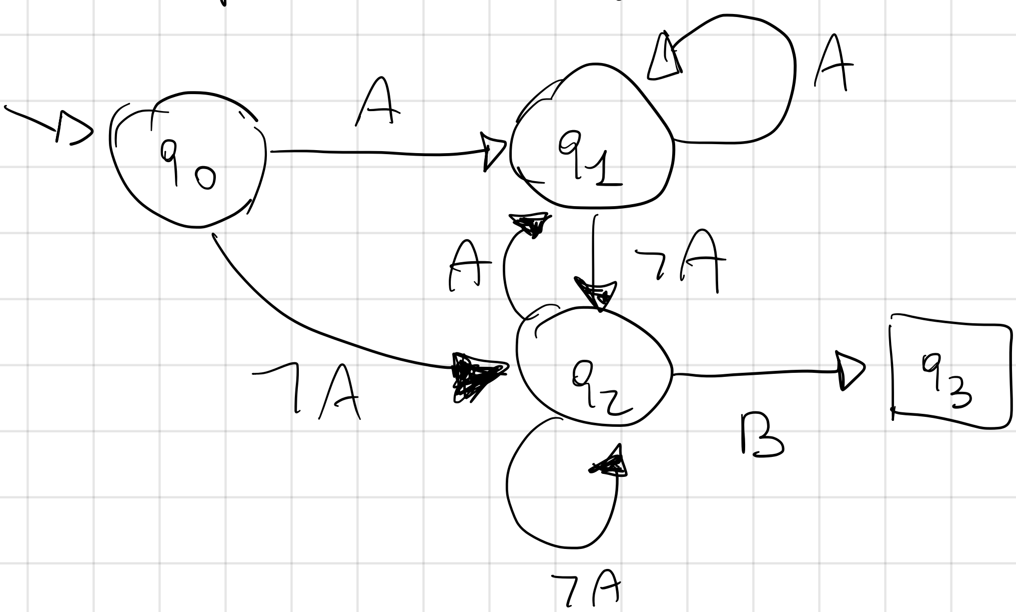
This is a mixed property.

b) $E_b = \{A_0 A_1 \cdots \in (2^{AP})^\omega \mid \forall i \in \mathbb{N}.\ i > 0 \Rightarrow (A \in A_i \Rightarrow B \in A_{i-1})\}$

In LTL: $\Box(\bigcirc A \rightarrow B)$

This is an invariant, so a safety property.

An NFA recognising the language of minimal Bad Prefixes is as follows:




The state $q_1$ records the information "A was seen in the previous step". The state $q_2$ records the information "A was not seen in the previous step".

c) $E_C = \{ A_0 A_1 \dots \in (2^{AP})^\omega \mid \forall i \in \mathbb{N}.\ B \in A_i \Rightarrow (\exists k \in \mathbb{N}: k \geq i \wedge C \in A_k) \}$

In LTL : $\Box (B \to \Diamond C)$

This is a pure liveness property.

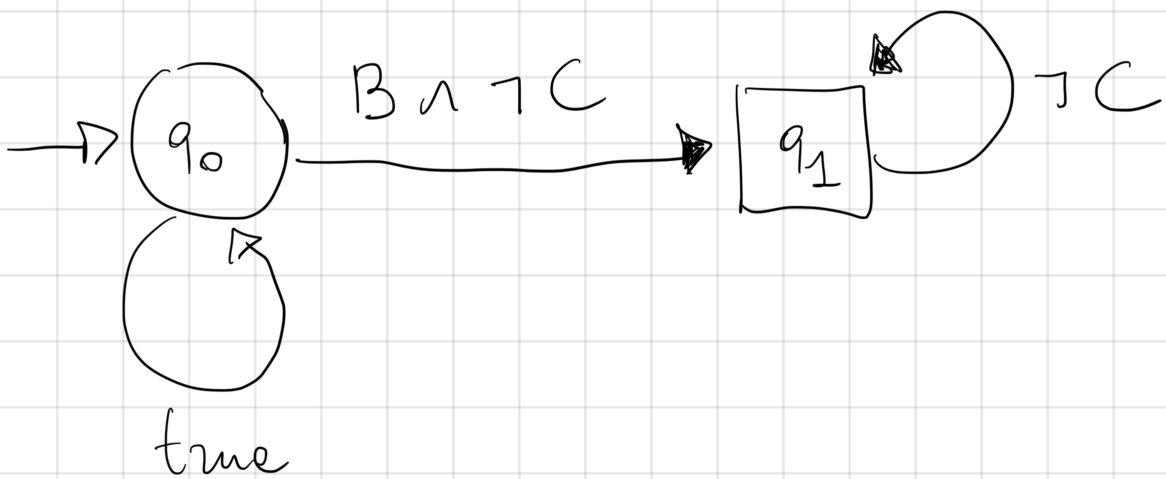
Bad Behaviours are those satisfying

$\neg \Box (B \to \Diamond C) \equiv \Diamond \neg (B \to \Diamond C)$

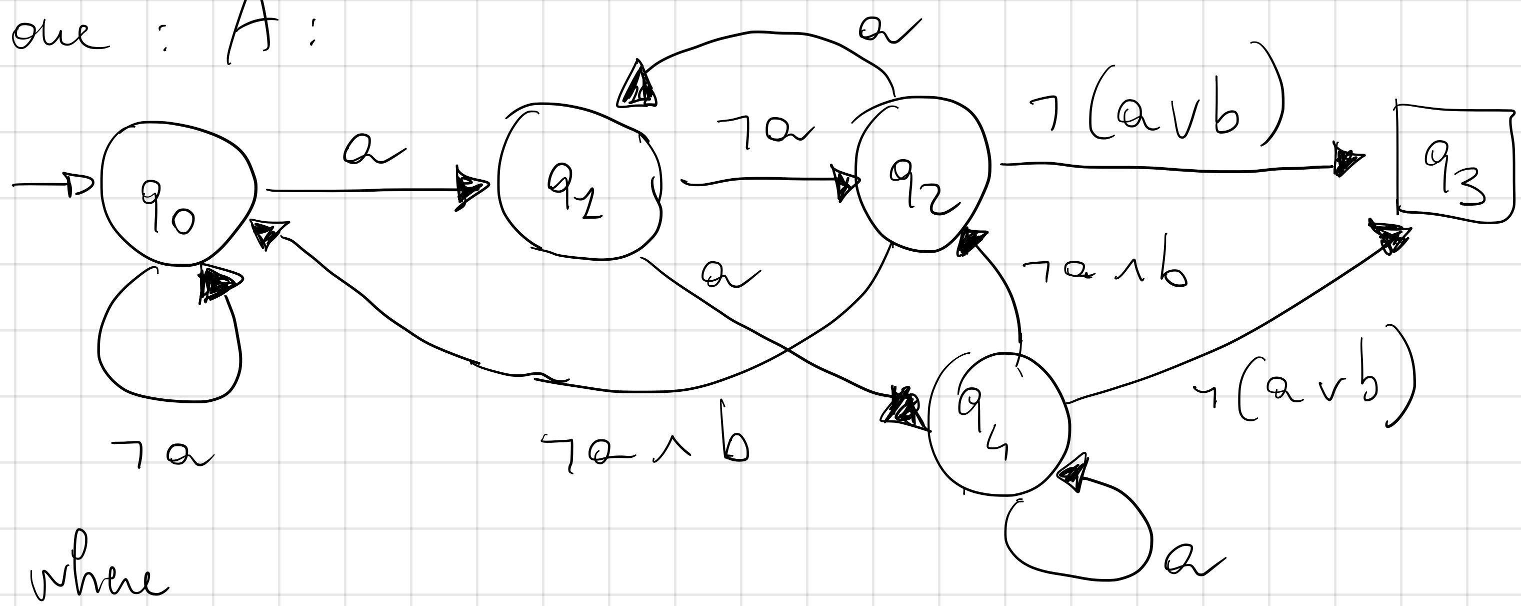
$\equiv \Diamond \neg (\neg B \vee \Diamond C) \equiv \Diamond (\neg\neg B \wedge \neg \Diamond C)$

$\equiv \Diamond (B \wedge \Box \neg C)$

An NBA accepting the words that satisfy $\Diamond (B \wedge \Box \neg C)$ is as follows :



Ex3 An NFA (non-blocking) for the minimal bad prefixes of $\varphi$ is the following one : $\mathcal{A}$ :




where

$\neg a \equiv \{\{\}, \{b\}, \{c\}, \{b,c\}\}$

$a \equiv \{\{a\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$

$\neg(a \vee b) \equiv \{\{\}, \{c\}\}$

$\neg a \wedge b \equiv \{\{b\}, \{b,c\}\}$

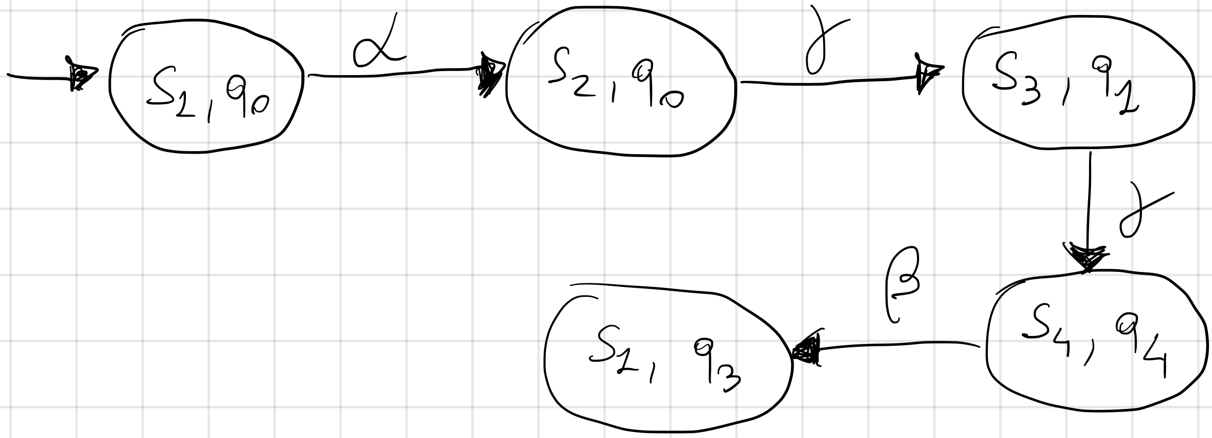
Let us construct the product $TS \otimes \mathcal{A}$

The initial states are given by

$$\delta(q_0, L(s_1)) = \delta(q_0, \{\}) = \{q_0\}$$

There is only one initial state $(s_1, q_0)$

The following is a fragment of $TS \otimes A$




The transitions where derived as follows:

$s_1 \xrightarrow{\alpha} s_2$ $\quad \delta(q_0, L(s_2)) = \delta(q_0, \{b, c\}) = \{q_1\}$

$s_2 \xrightarrow{\gamma} s_3$ $\quad \delta(q_1, L(s_3)) = \delta(q_1, \{a\}) = \{q_4\}$

$s_4 \xrightarrow{\beta} s_1$ $\quad \delta(q_4, L(s_1)) = \delta(q_4, \{\}) = \{q_3\}$

Since $(s_1, q_3)$ is reachable from the initial state we can conclude that

$$TS \not\models \varphi$$

The counterexample is given by the path $s_1\, s_2\, s_3\, s_4\, s_1$ corresponding to the finite trace $\{\}\ \{b, c\}\ \{a\}\ \{b\}\ \{\}$.

Ex4 First let us put the formula $\forall(a \, \mathcal{U} \, \forall\Diamond c)$ in existential normal form

$$\forall(a \, \mathcal{U} \, \forall\Diamond c)$$

$$\equiv \left\{ \forall \varphi_1 \mathcal{U} \varphi_2 \equiv \neg\exists(\neg\varphi_2 \, \mathcal{U} \, (\neg\varphi_1 \wedge \neg\varphi_2)) \wedge \neg\exists\Box\neg\varphi_2 \right\}$$

$$\neg\exists(\neg\forall\Diamond c \, \mathcal{U} \, (\neg a \wedge \neg\forall\Diamond c)) \wedge \neg\exists\Box\neg\forall\Diamond c$$

$$\equiv \left\{ \neg\forall\Diamond\varphi \equiv \exists\Box\neg\varphi \right\}$$

$$\neg\exists(\exists\Box\neg c \, \mathcal{U} \, (\neg a \wedge \exists\Box\neg c)) \wedge \neg\exists\Box\exists\Box\neg c \quad \leftarrow \text{In E.N.F.}$$

$$Sat(\neg c) = S - Sat(c) = S - \{s_2, s_3, s_4\} = \{s_0, s_2\}$$

$$Sat(\neg a) = S - Sat(a) = S - \{s_0, s_2\} = \{s_2, s_3, s_4\}$$

$Sat(\exists\Box\neg c)$ Calculation of a greatest fixpoint

$T_0 = Sat(\neg c) = \{s_0, s_2\}$

$T_1 = T_0 - \{s \in S \mid succ(s) \cap T_0 = \emptyset\} =$

$= T_0 - \{s_2\} = \{s_0\}$

$T_2 = T_1 - \{s \in S \mid succ(s) \cap T_1 = \emptyset\} =$

$T_1 - \{s_0\} = \{\}$ $\qquad T_3 = T_2$

Thus, $Sat(\exists\Box\neg c) = \{\}$

$Sat(\exists\Box(\exists\Box\neg c))$. Since $Sat(\exists\Box\neg c) = \{\}$

then $Sat(\exists\Box\{\}) = \{\}$

$Sat(\neg\exists\Box(\exists\Box\neg c)) = S - Sat(\exists\Box(\exists\Box\neg c)) =$

$S = \{s_0, s_1, s_2, s_3, s_4\}$

$Sat(\neg a \wedge \exists\Box\neg c) = Sat(\neg a) \cap Sat(\exists\Box\neg c)$

$= \{s_2, s_3, s_4\} \cap \{\} = \{\}$

$Sat(\exists(\exists\Box\neg c \; \mathcal{U} \; (\neg a \wedge \exists\Box\neg c)))$.

Since $Sat(\exists\Box\neg c) = \{\}$ and $Sat(\neg a \wedge \exists\Box\neg c) = \emptyset$ then the $\exists$-$\mathcal{U}$ cannot be satisfied by any state. So the result is the empty set.

Thus

$$Sat(\neg\exists(\exists\Box\neg c \; \mathcal{U} \; (\neg a \wedge \exists\Box\neg c))) =$$

$$S - \{\} = S = \{s_0, s_1, s_2, s_3, s_4\}$$

To conclude

$$Sat(\neg\exists(\exists\Box\neg c \; \mathcal{U} \; (\neg a \wedge \exists\Box\neg c)) \wedge \neg\exists\Box\exists\Box\neg c) = S \cap S = S$$