Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation-Tree Logic

Equivalences and Abstraction

- bisimulation
- CTL, CTL*-equivalence ⟵
- computing the bisimulation quotient
- abstraction stutter steps
- simulation relations

Recall: CTL*

CTL* state formulas

$$\Phi \ ::= \ \mathit{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \exists\varphi$$

CTL* path formulas

$$\varphi \ ::= \ \Phi \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

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derived operators:

- $\Diamond$, $\Box$, ... as in **LTL**

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derived operators:

- $\Diamond$, $\Box$, ... as in **LTL**
- universal quantification: $\forall\varphi \overset{\text{def}}{=} \neg\exists\neg\varphi$

CTL* state formulas

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- with path quantifiers $\exists$ and $\forall$

Recall: CTL* and CTL

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CTL: sublogic of **CTL***

- with path quantifiers $\exists$ and $\forall$
- restricted syntax of path formulas:
  - *no* boolean combinations of path formulas
  - arguments of temporal operators $\bigcirc$ and **U** are state formulas

CTL equivalence

CTLEQ5.2-1

CTL equivalence

Let s_1, s_2 be states of a TS $\mathcal{T}$ without terminal states

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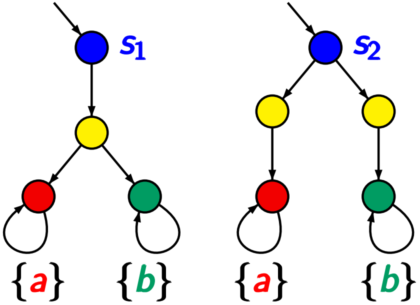

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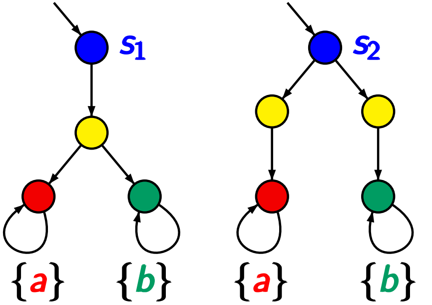

s_1, s_2 are not **CTL** equivalent

$s_1 \models \exists\bigcirc(\exists\bigcirc a \wedge \exists\bigcirc b)$

$s_2 \not\models \exists\bigcirc(\exists\bigcirc a \wedge \exists\bigcirc b)$

CTL, CTL* and LTL equivalence

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analogous definition for **CTL*** and **LTL**

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s_1, s_2 are **CTL*** equivalent if for all **CTL*** formulas Φ:

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---

s_1, s_2 are **LTL** equivalent if for all **LTL** formulas φ:

$$s_1 \models \varphi \quad \text{iff} \quad s_2 \models \varphi$$

CTL/CTL* and bisimulation

bisimulation equivalence

= **CTL** equivalence

= **CTL*** equivalence

CTL/CTL* and bisimulation

	bisimulation equivalence
=	**CTL** equivalence
=	**CTL*** equivalence

$\longleftarrow$ for finite TS

	bisimulation equivalence	
=	CTL equivalence	⟵ for finite TS
=	CTL* equivalence	

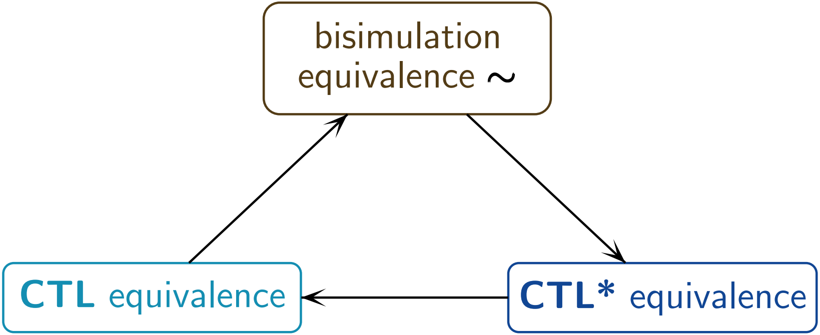
Let $\mathcal{T}$ be a finite TS without terminal states, and s_1, s_2 states in $\mathcal{T}$. Then:

$$s_1 \sim_{\mathcal{T}} s_2$$

iff s_1 and s_2 are CTL equivalent

iff s_1 and s_2 are CTL* equivalent

CTL/CTL* and bisimulation



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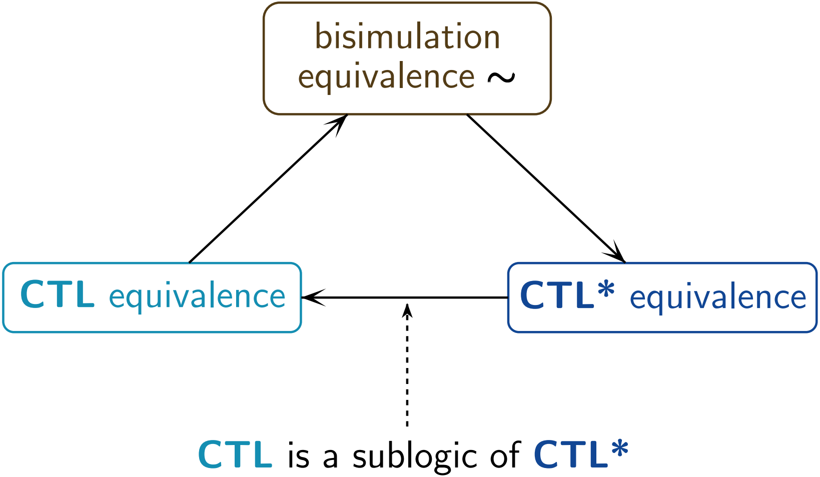

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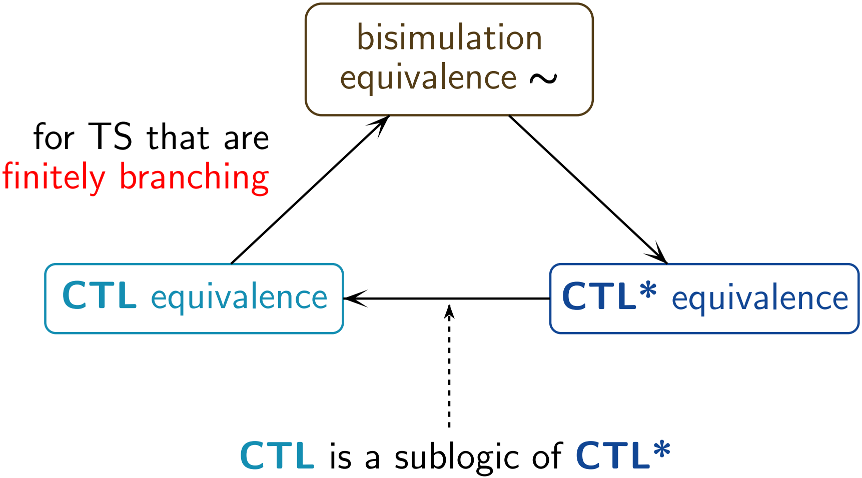
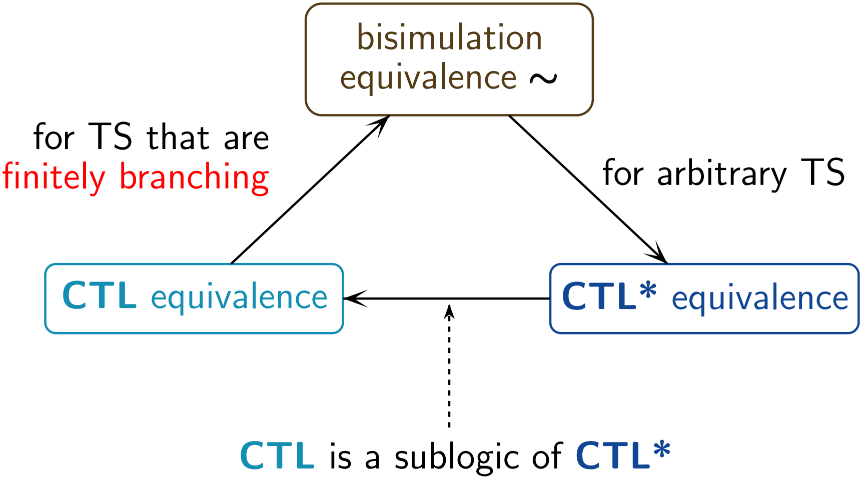


CTL/CTL* and bisimulation



Bisimulation equivalence ⇒ CTL* equivalence

For arbitrary (possibly infinite) transition systems without terminal states:

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

For arbitrary (possibly infinite) transition systems without terminal states:

If s_1, s_2 are states with $s_1 \sim_{\mathcal{T}} s_2$ then for all **CTL*** formulas Φ:

$$s_1 \models \Phi \quad \text{iff} \quad s_2 \models \Phi$$

Bisimulation equivalence ⇒ CTL* equivalence

show by structural induction on **CTL*** formulas:

(a) if s_1, s_2 are states with $s_1 \sim_{\mathcal{T}} s_2$ then for all **CTL*** state formulas Φ:

$$s_1 \models \Phi \quad \text{iff} \quad s_2 \models \Phi$$

(b) if π_1, π_2 are paths with $\pi_1 \sim_{\mathcal{T}} \pi_2$ then for all **CTL*** path formulas φ:

$$\pi_1 \models \varphi \quad \text{iff} \quad \pi_2 \models \varphi$$

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

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$\pi_1 \sim_{\mathcal{T}} \pi_2 \quad \overset{\text{def}}{\Longleftrightarrow} \quad$ π_1 and π_2 are statewise bisimulation equivalent

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

statewise bisimulation equivalent paths:

$$
\begin{array}{ccc}
s_1 & \sim_{\mathcal{T}} & s_2 \\
\downarrow & & \downarrow \\
s_{11} & \sim_{\mathcal{T}} & s_{12} \\
\downarrow & & \downarrow \\
s_{21} & \sim_{\mathcal{T}} & s_{22} \\
\downarrow & & \downarrow \\
s_{31} & \sim_{\mathcal{T}} & s_{32} \\
\downarrow & & \downarrow \\
\ldots & & \ldots \\
\uparrow & & \uparrow \\
\boxed{\text{path } \pi_1} & & \boxed{\text{path } \pi_2}
\end{array}
$$

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

For all CTL* state formulas Φ and path formulas φ:

(a) if $s_1 \sim_{\mathcal{T}} s_2$ then: $s_1 \models \Phi$ iff $s_2 \models \Phi$

(b) if $\pi_1 \sim_{\mathcal{T}} \pi_2$ then: $\pi_1 \models \varphi$ iff $\pi_2 \models \varphi$

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

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Proof by structural induction

base of induction:

(a) $\Phi = \mathit{true}$ or $\Phi = a \in AP$

(b) $\varphi = \Phi$ for some state formula Φ
s.t. statement (a) holds for Φ

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

For all CTL* state formulas Φ and path formulas φ:

(a) if $s_1 \sim_{\mathcal{T}} s_2$ then: $s_1 \models \Phi$ iff $s_2 \models \Phi$

(b) if $\pi_1 \sim_{\mathcal{T}} \pi_2$ then: $\pi_1 \models \varphi$ iff $\pi_2 \models \varphi$

Proof by structural induction

step of induction:

(a) consider $\Phi = \Phi_1 \wedge \Phi_2$, $\neg\Psi$ or $\exists\varphi$ s.t.

- (a) holds for Φ_1, Φ_2, Ψ
- (b) holds for φ

(b) consider $\varphi = \varphi_1 \wedge \varphi_2$, $\neg\varphi'$, $\bigcirc\varphi'$, $\varphi_1 \, \mathsf{U} \, \varphi_2$ s.t.

- (b) holds for $\varphi_1, \varphi_2, \varphi'$

Path lifting for $\sim_{\mathcal{T}}$

CTLEQ5.2-4

$$\begin{array}{ccc} s_1 & \sim_{\mathcal{T}} & s_2 \\ \downarrow & & \\ s_{11} & & \\ \downarrow & & \\ s_{21} & & \\ \downarrow & & \\ s_{31} & & \\ \downarrow & & \\ \vdots & & \end{array}$$

can be completed to

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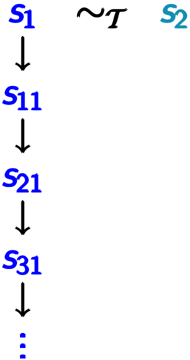
Path lifting for $\sim_{\mathcal{T}}$

CTLEQ5.2-4



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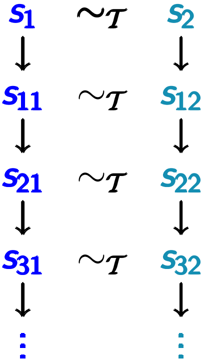

If $s_1 \sim_{\mathcal{T}} s_2$ then for all $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ with $\pi_1 \sim_{\mathcal{T}} \pi_2$

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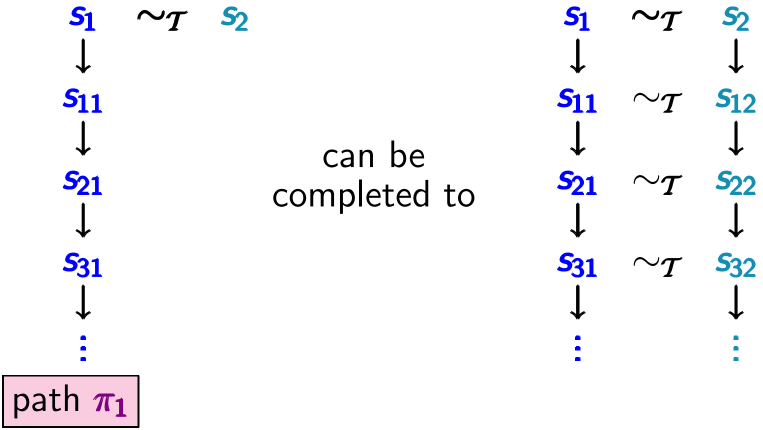

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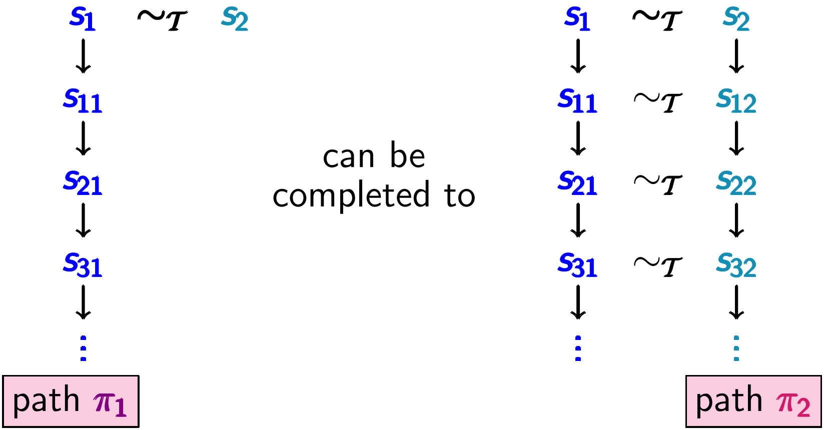

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Correct or wrong?

If s_1, s_2 are not **CTL** equivalent then there exists a **CTL** formula Φ with $s_1 \models \Phi$ and $s_2 \not\models \Phi$

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$$s_1 \models \Phi \wedge s_2 \not\models \Phi$$

or $$s_1 \not\models \Phi \wedge s_2 \models \Phi$$

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$$s_1 \models \Phi \wedge s_2 \not\models \Phi$$

$$\text{or} \quad s_1 \not\models \Phi \wedge s_2 \models \Phi \implies s_1 \models \neg\Phi \wedge s_2 \not\models \neg\Phi$$

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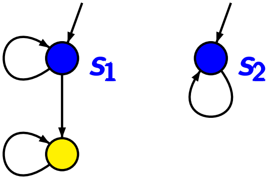

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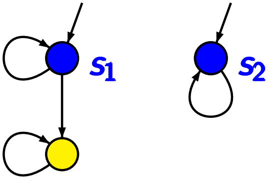
$Traces(s_2) \subset Traces(s_1)$



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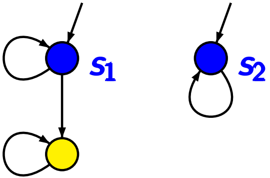
wrong.

$Traces(s_2) \subset Traces(s_1)$

hence: $s_1 \models \varphi$ implies $s_2 \models \varphi$



CTL equivalence $\Longrightarrow$ bisimulation equivalence

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If $\mathcal{T}$ is a finite TS then, for all states s_1, s_2 in $\mathcal{T}$:

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Proof: show that

$$\mathcal{R} \stackrel{\text{def}}{=} \{ (s_1, s_2) : s_1, s_2 \text{ satisfy the same } \textbf{CTL} \text{ formulas} \}$$

is a bisimulation

CTL equivalence $\Longrightarrow$ bisimulation equivalence

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Proof: show that

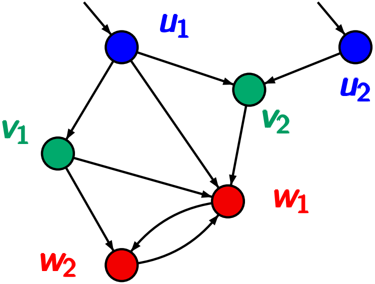
$$\mathcal{R} \stackrel{\text{def}}{=} \{ (s_1, s_2) : s_1, s_2 \text{ satisfy the same } \textbf{CTL} \text{ formulas} \}$$

is a bisimulation, i.e., for all $(s_1, s_2) \in \mathcal{R}$:

(1) $L(s_1) = L(s_2)$

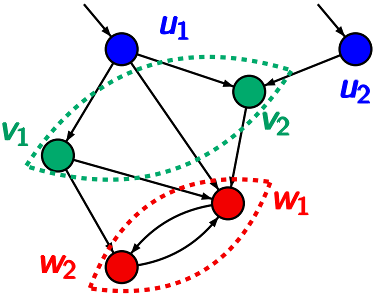
(2) if $s_1 \rightarrow t_1$ then there exists a transition $s_2 \rightarrow t_2$ s.t. $(t_1, t_2) \in \mathcal{R}$

Example: CTL master formulas



- (blue) $\widehat{=}$ $\{a\}$
- (red) $\widehat{=}$ $\{b\}$
- (green) $\widehat{=}$ $\varnothing$

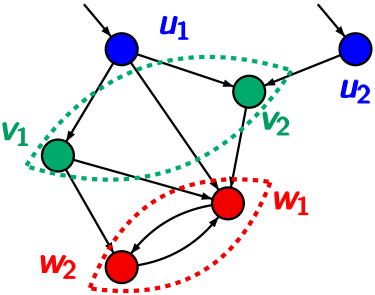
Example: CTL master formulas



bisimulation equivalence $\sim_{\mathcal{T}}$
$= \{(v_1, v_2), (w_1, w_2), ...\}$

- blue $\widehat{=} \{a\}$
- red $\widehat{=} \{b\}$
- green $\widehat{=} \varnothing$

Example: CTL master formulas



bisimulation equivalence $\sim_{\mathcal{T}}$
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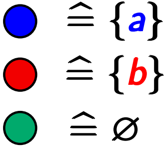
but $u_1 \not\sim_{\mathcal{T}} u_2$



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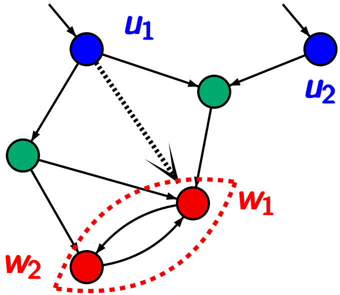
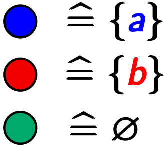




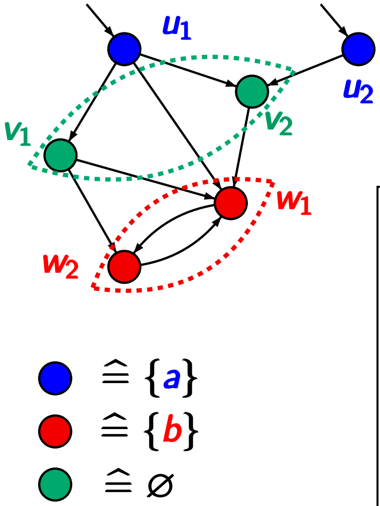
bisimulation equivalence $\sim_{\mathcal{T}}$
$= \{(v_1, v_2), (w_1, w_2), \ldots\}$

but $u_1 \not\sim_{\mathcal{T}} u_2$

as $u_1 \rightarrow \{w_1, w_2\}$

$u_2 \not\rightarrow \{w_1, w_2\}$

Example: CTL master formulas



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CTL master formulas:

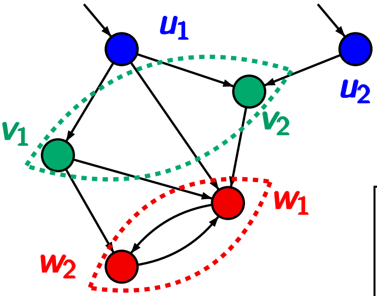
$w_1, w_2 \models$?

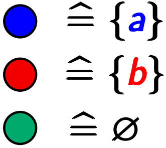
$v_1, v_2 \models$?

$u_1 \models$?

$u_2 \models$?

Example: CTL master formulas





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CTL master formulas:

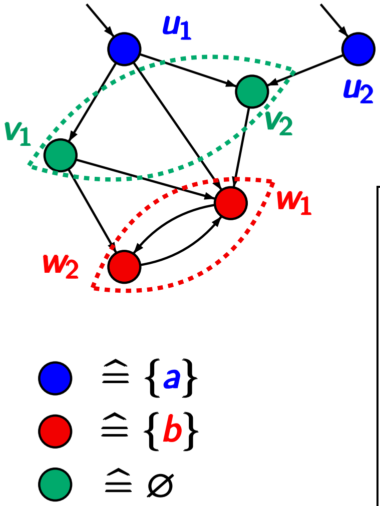
$w_1, w_2 \models b$

$v_1, v_2 \models ?$

$u_1 \models ?$

$u_2 \models ?$

Example: CTL master formulas



bisimulation equivalence $\sim_{\mathcal{T}}$
$= \{(v_1, v_2), (w_1, w_2), \ldots\}$

CTL master formulas:

$w_1, w_2 \models b$

$v_1, v_2 \models \neg a \wedge \neg b$

$u_1 \models$?

$u_2 \models$?

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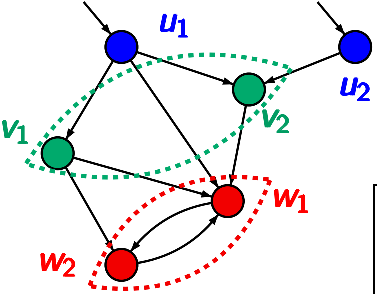

● $\widehat{=} \{a\}$
● $\widehat{=} \{b\}$
● $\widehat{=} \varnothing$

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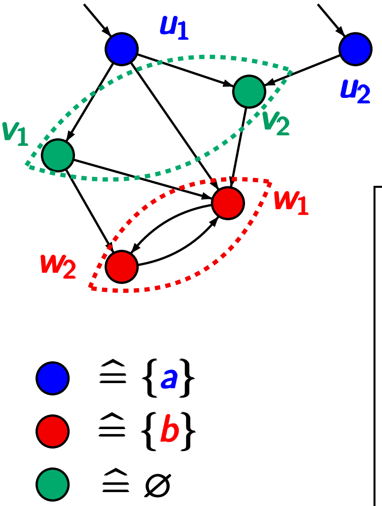

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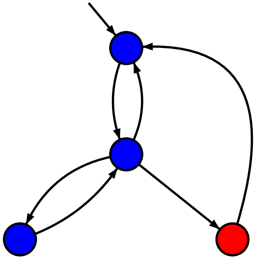
...master formulas for $\sim_{\mathcal{T}}$-classes?



$AP = \{blue, red\}$

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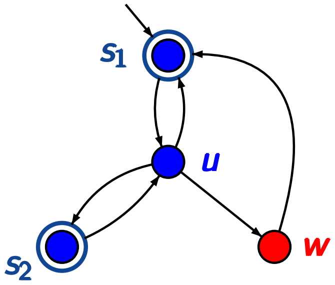

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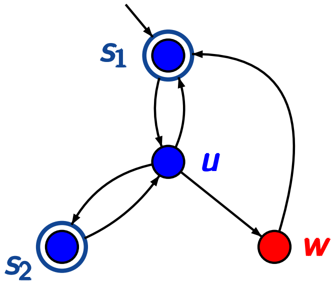

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$\Phi_w = ?$

$\Phi_C = ?$ where $C = \{s_1, s_2\}$

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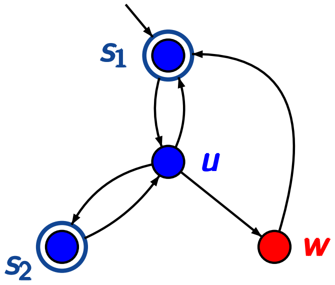

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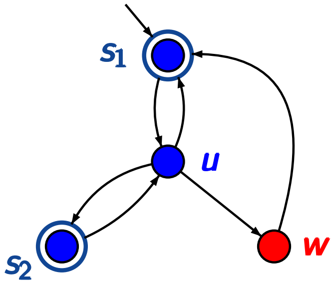

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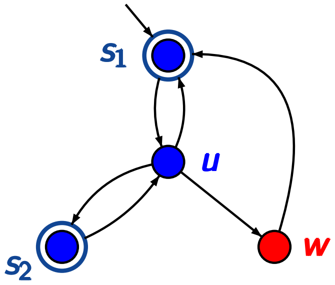

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CTL equivalence $\Longrightarrow$ bisimulation equivalence

If $\mathcal{T}$ is a finite TS then, for all states s_1, s_2 in $\mathcal{T}$:

if s_1, s_2 are **CTL** equivalent then $s_1 \sim_{\mathcal{T}} s_2$

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- wrong for infinite TS

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- but also holds for finitely branching TS

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- but also holds for finitely branching TS

↑

possibly infinite-state TS such that

- ∗ the number of initial states is finite
- ∗ for each state the number of successors is finite

CTL equivalence $\Longrightarrow$ bisimulation equivalence

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be finitely branching.

CTL equivalence $\Longrightarrow$ bisimulation equivalence

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* $Post(s)$ is finite for all $s \in S$

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Proof: as for finite TS. Amounts showing that

$$\mathcal{R} \overset{\text{def}}{=} \{ (s_1, s_2) : s_1, s_2 \text{ satisfy the same } \textbf{CTL} \text{ formulas} \}$$

is a bisimulation.

CTL equivalence $\Longrightarrow$ bisimulation equivalence

If $\mathcal{T}$ is a finitely branching TS then for all states s_1, s_2:

if s_1, s_2 are **CTL** equivalent then $s_1 \sim_{\mathcal{T}} s_2$

Proof: show that

$$\mathcal{R} \overset{\text{def}}{=} \{ (s_1, s_2) : s_1, s_2 \text{ satisfy the same } \textbf{CTL} \text{ formulas} \}$$

is a bisimulation, i.e., for $(s_1, s_2) \in \mathcal{R}$:

(1) $L(s_1) = L(s_2)$

(2) if $s_1 \rightarrow t_1$ then there exists a transition $s_2 \rightarrow t_2$ s.t. $(t_1, t_2) \in \mathcal{R}$

Summary: CTL/CTL* and bisimulation

Let $\mathcal{T}$ be a finite TS without terminal states, and s_1, s_2 states in $\mathcal{T}$. Then:

$$s_1 \sim_{\mathcal{T}} s_2$$

iff s_1 and s_2 are **CTL** equivalent

iff s_1 and s_2 are **CTL*** equivalent

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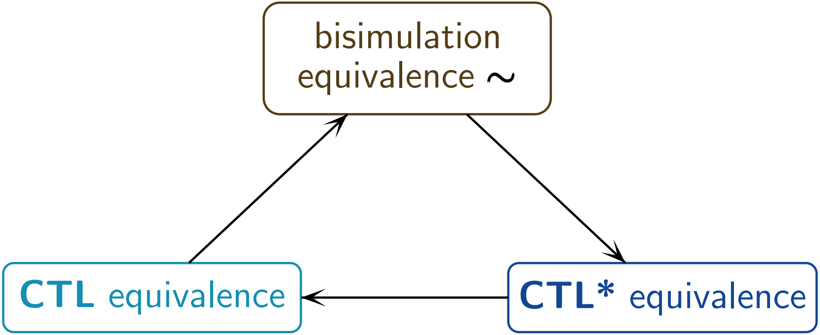
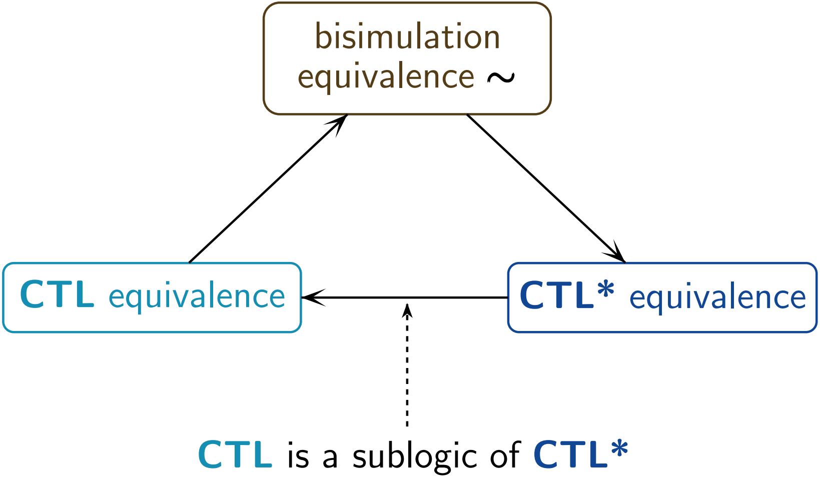


Summary: CTL/CTL* and bisimulation



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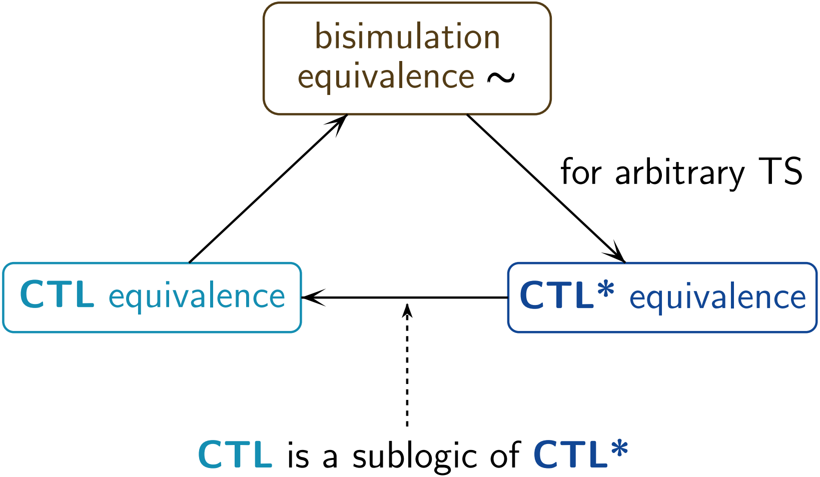

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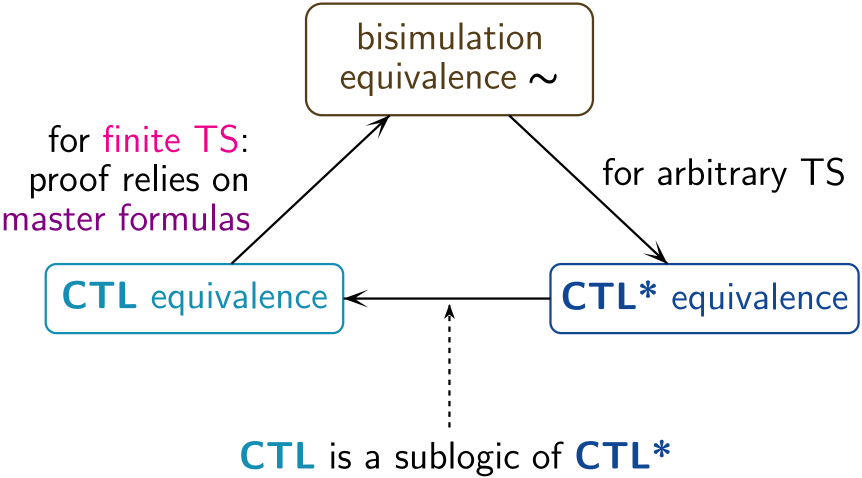

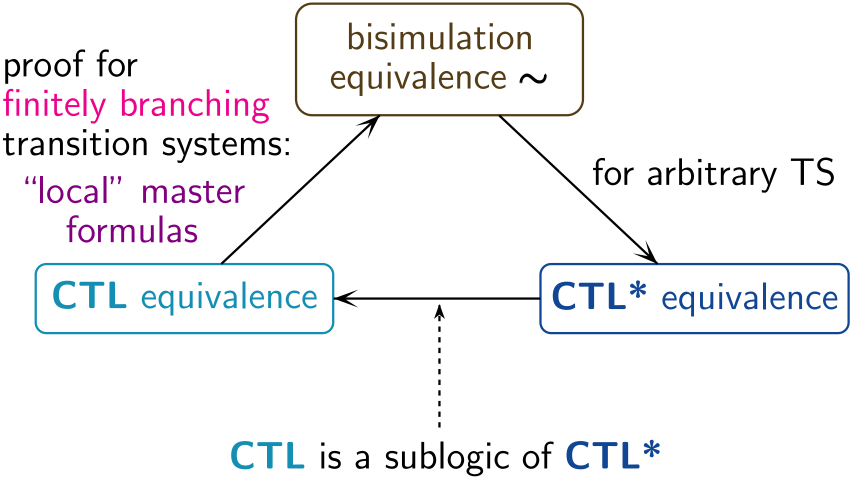
bisimulation equivalence ∼
proof for finitely branching transition systems: "local" master formulas
for arbitrary TS
CTL equivalence
CTL* equivalence
CTL is a sublogic of CTL*

CTL/CTL* and bisimulation for TS

CTL/CTL* and bisimulation for TS

so far: we considered

- **CTL**/**CTL*** equivalence
- bisimulation equivalence $\sim_{\mathcal{T}}$

for the states of a single transition system $\mathcal{T}$

If $\mathcal{T}_1$, $\mathcal{T}_2$ are finitely branching TS over AP without terminal states then:

$\mathcal{T}_1 \sim \mathcal{T}_2$

iff $\mathcal{T}_1$ and $\mathcal{T}_2$ satisfy the same **CTL** formulas

iff $\mathcal{T}_1$ and $\mathcal{T}_2$ satisfy the same **CTL*** formulas

Correct or wrong?

Does the following statements hold for finite TS without terminal states **?**

Correct or wrong?

CTL equivalence is finer than **LTL** equivalence

Correct or wrong?

CTL equivalence is finer than **LTL** equivalence

correct.

CTL equivalence is finer than **LTL** equivalence

correct.

↑

CTL equivalence = **CTL*** equivalence

LTL is sublogic of **CTL***

Correct or wrong?

CTL equivalence is finer than **LTL** equivalence

correct.

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Correct or wrong?

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correct.

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wrong.

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correct.

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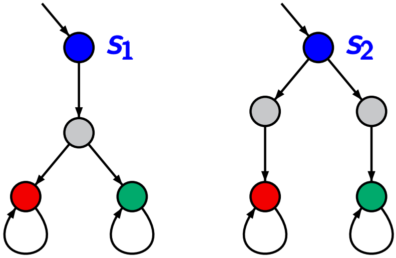
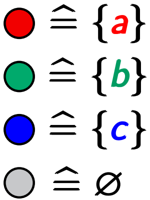
wrong.





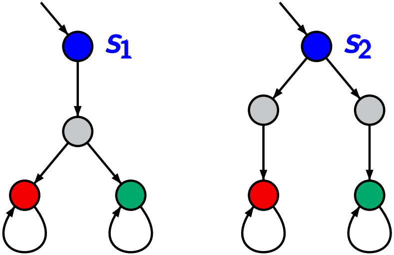
Correct or wrong?

CTL equivalence is finer than **LTL** equivalence

correct.

LTL equivalence is finer than **CTL** equivalence

wrong.



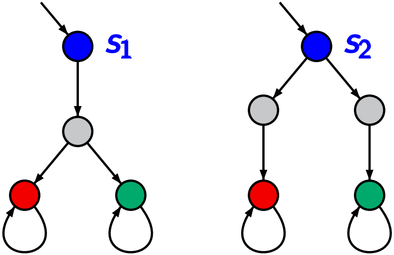
s_1, s_2 are trace equivalent

CTL equivalence is finer than **LTL** equivalence

correct.

LTL equivalence is finer than **CTL** equivalence

wrong.



s_1, s_2 are trace equivalent and **LTL** equivalent

Correct or wrong?

CTL equivalence is finer than **LTL** equivalence

correct.

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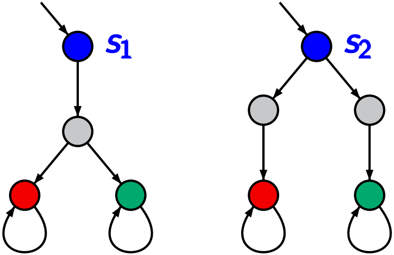
wrong.

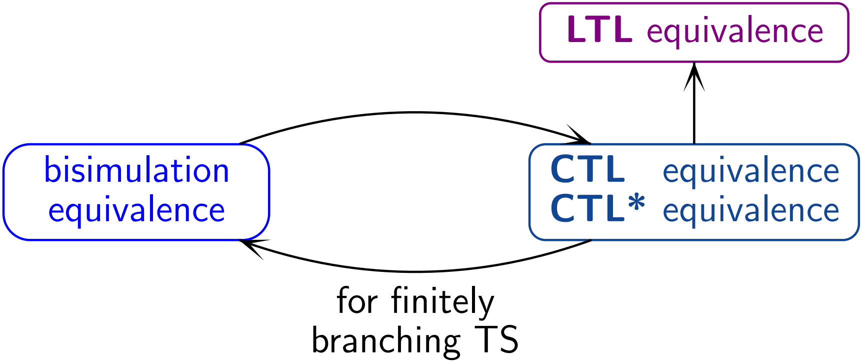


s_1, s_2 are trace equivalent

and **LTL** equivalent

$$s_1 \models \exists\bigcirc(\exists\bigcirc a \wedge \exists\bigcirc b)$$
$$s_2 \not\models \exists\bigcirc(\exists\bigcirc a \wedge \exists\bigcirc b)$$

Summary: equivalences



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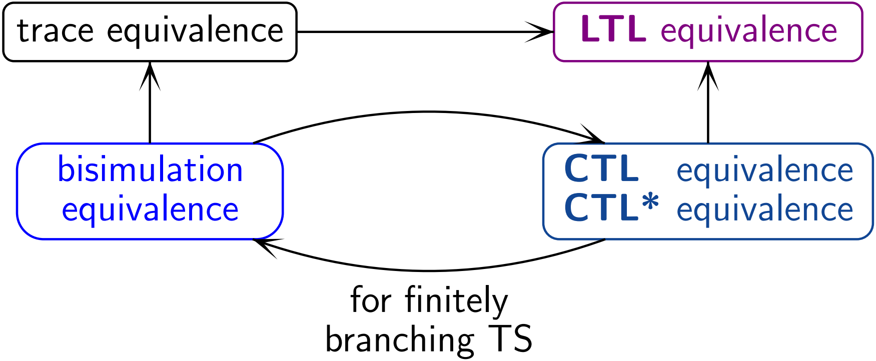

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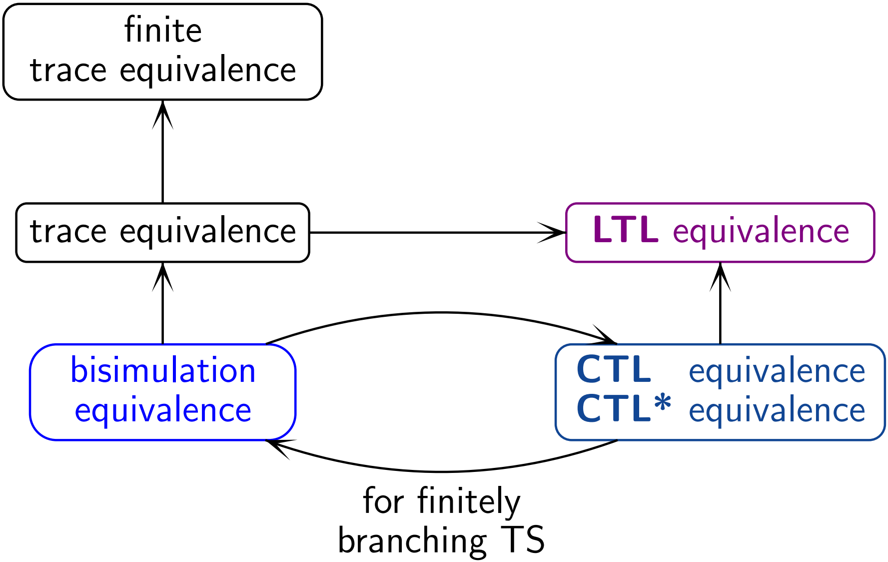
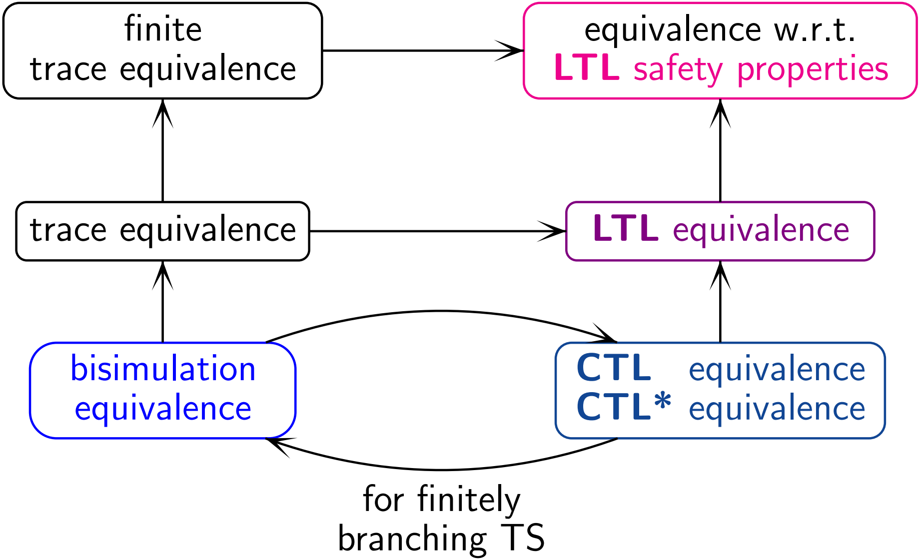


Summary: equivalences



Correct or wrong?

Let $\mathcal{T}$ be a finite TS without terminal states and s_1, s_2 states of $\mathcal{T}$.

> If s_1, s_2 satisfy the same $\mathrm{CTL}_{\setminus \mathrm{U}}$ formulas then
> $s_1 \sim_{\mathcal{T}} s_2$.

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correct.

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correct. see the proof

"**CTL** equivalence $\Longrightarrow$ bisimulation equivalence"

$\text{CTL}_{\setminus U}$-equivalence ⇒ bisimulation equivalence

Let $\mathcal{T}$ be a finite TS without terminal states and s_1, s_2 states of $\mathcal{T}$.

> If s_1, s_2 satisfy the same $\text{CTL}_{\setminus U}$ formulas then
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Proof. Show that $\text{CTL}_{\setminus U}$ equivalence is a bisimulation

$\text{CTL}_{\setminus U}$-equivalence $\Rightarrow$ bisimulation equivalence

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If s_1, s_2 satisfy the same $\text{CTL}_{\setminus U}$ formulas then
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- labeling condition only uses atomic propositions

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Let $\mathcal{T}$ be a finite TS without terminal states and s_1, s_2 states of $\mathcal{T}$.

> If s_1, s_2 satisfy the same $CTL_{\setminus U}$ formulas then
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- labeling condition only uses atomic propositions
- simulation condition can be established by $CTL_{\setminus U}$ master formulas of the form:

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$$\exists \bigcirc \Phi_C \quad \text{where} \quad \Phi_C = \bigwedge_D \Phi_{C,D}$$

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$$\text{and} \quad Sat(\Phi_{C,D}) \subseteq C \setminus D$$

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Let $\mathcal{T}$ be a finite TS without terminal states.

> $\mathcal{T}$ and its bisimulation quotient $\mathcal{T}/\sim$ satisfy the same **CTL*** formulas.

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correct.

Let $\mathcal{T}$ be a finite TS without terminal states.

$\mathcal{T}$ and its bisimulation quotient $\mathcal{T}/\!\sim$ satisfy the same **CTL*** formulas.

correct. Recall that $\mathcal{T} \sim \mathcal{T}/\!\sim$

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$\mathcal{T}$ and its bisimulation quotient $\mathcal{T}/\!\sim$ satisfy the same **CTL*** formulas.

correct. Recall that $\mathcal{T} \sim \mathcal{T}/\!\sim$ as

$$\mathcal{R} = \big\{(s, [s]) : s \in S\big\}$$

is a bisimulation for $(\mathcal{T}, \mathcal{T}/\!\sim)$

here: $[s] = \sim_{\mathcal{T}}$-equivalence class of state s