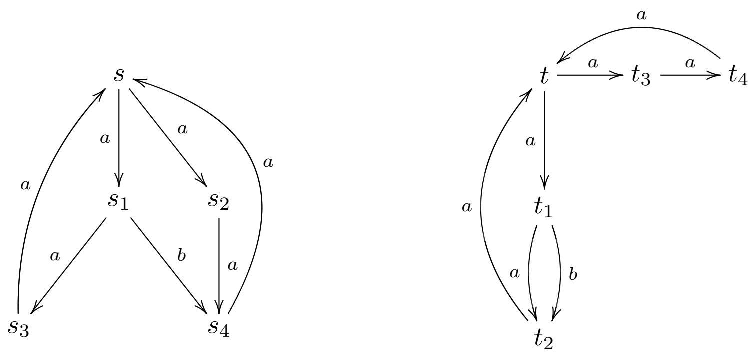
Tutorial 3

Exercise 1*

Consider the following labelled transition system.



Show that $s \sim t$ by finding a strong bisimulation R containing the pair (s, t).

Exercise 2*

Consider the CCS processes P and Q defined by:

$$\begin{aligned} P &\stackrel{\text{def}}{=} a.P_1 \\ P_1 &\stackrel{\text{def}}{=} b.P + c.P \end{aligned}$$

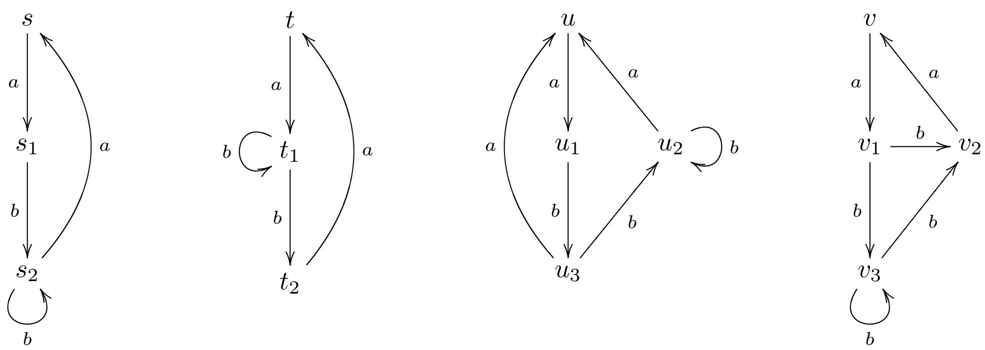
and

$$\begin{aligned} Q &\stackrel{\text{def}}{=} a.Q_1 \\ Q_1 &\stackrel{\text{def}}{=} b.Q_2 + c.Q \\ Q_2 &\stackrel{\text{def}}{=} a.Q_3 \\ Q_3 &\stackrel{\text{def}}{=} b.Q + c.Q_2 \ . \end{aligned}$$

Show that $P \sim Q$ holds by finding an appropriate strong bisimulation.

Exercise 3*

Consider the following labelled transition system.



Decide whether $s \stackrel{?}{\sim} t$, $s \stackrel{?}{\sim} u$, and $s \stackrel{?}{\sim} v$. Support your claims by giving a universal winning strategy either for the attacker (in the negative case) or the defender (in the positive case). In the positive case you can also define a strong bisimulation relating the pair in question.

Tutorial 3 - Solutions

Exercise 1*

If we can show that $R = \{(s,t),(s_1,t_1),(s_3,t_2),(s_4,t_2),(s_2,t_3),(s_4,t_4)\}$ is a strong bisimulation, then $s \sim t$. Indeed R is a strong bisimulation since:

- Consider $(s,t) \in R$. Transitions from s:
  - If $s \xrightarrow{a} s_1$, match by doing $t \xrightarrow{a} t_1$, and $(s_1,t_1) \in R$.
  - If $s \xrightarrow{a} s_2$, match by doing $t \xrightarrow{a} t_3$, and $(s_2,t_3) \in R$.
  - These are all transitions from s.

  Transitions from t:
  - If $t \xrightarrow{a} t_1$, match by doing $s \xrightarrow{a} s_1$, and $(s_1,t_1) \in R$.
  - If $t \xrightarrow{a} t_3$, match by doing $s \xrightarrow{a} s_2$, and $(s_2,t_3) \in R$.
  - These are all transitions from t.
- Consider $(s_1,t_1) \in R$. Transitions from s_1:
  - If $s_1 \xrightarrow{a} s_3$, match by doing $t_1 \xrightarrow{a} t_2$ and $(s_3,t_2) \in R$.
  - If $s_1 \xrightarrow{b} s_4$, match by doing $t_1 \xrightarrow{b} t_2$ and $(s_4,t_2) \in R$.

  Transitions from t_1:
  - If $t_1 \xrightarrow{a} t_2$, match by doing $s_1 \xrightarrow{a} s_3$ and $(s_3,t_2) \in R$.
  - If $t_1 \xrightarrow{b} t_2$, match by doing $s_1 \xrightarrow{b} s_4$ and $(s_4,t_2) \in R$.
- Consider $(s_3,t_2) \in R$. Transitions from s_3:
  - If $s_3 \xrightarrow{a} s$, match by doing $t_2 \xrightarrow{a} t$ and $(s,t) \in R$.

  Transitions from t_2:
  - If $t_2 \xrightarrow{a} t$, match by doing $s_3 \xrightarrow{a} s$ and $(s,t) \in R$.
- Consider $(s_4,t_2) \in R$. Transitions from s_4:
  - If $s_4 \xrightarrow{a} s$, match by doing $t_2 \xrightarrow{a} t$ and $(s,t) \in R$.

  Transitions from t_2:
  - If $t_2 \xrightarrow{a} t$, match by doing $s_4 \xrightarrow{a} s$ and $(s,t) \in R$.
- Consider $(s_2,t_3) \in R$. Transitions from s_2:
  - If $s_2 \xrightarrow{a} s_4$, match by doing $t_3 \xrightarrow{a} t_4$ and $(s_4,t_4) \in R$.

  Transitions from t_3:
  - If $t_3 \xrightarrow{a} t_4$, match by doing $s_2 \xrightarrow{a} s_4$ and $(s_4,t_4) \in R$.
- Consider $(s_4,t_4) \in R$. Transitions from s_4:
  - If $s_4 \xrightarrow{a} s$, match by $t_4 \xrightarrow{a} t$ and $(s,t) \in R$.

  Transitions from t_4:
  - If $t_4 \xrightarrow{a} t$, match by $s_4 \xrightarrow{a} s$ and $(s,t) \in R$.

Exercise 2*

Let $R = \{(P, Q), (P_1, Q_1), (P, Q_2), (P_1, Q_3)\}$. We only outline the proof; it follows along the lines as the proof in Exercise 1. You should complete the details.

- From $(P, Q) \in R$ either P or Q can do an a transition.
  - In either case the response is to match by making an a transition from the remaining state, so we end up in $(P_1, Q_1) \in R$.
- From $(P_1, Q_1) \in R$ we end up in either $(P, Q) \in R$ or $(P, Q_2) \in R$.
- From $(P, Q_2) \in R$ we can only end up in $(P_1, Q_3) \in R$.
- From $(P_1, Q_3) \in R$ we end up in either $(P, Q) \in R$ or $(P, Q_2) \in R$.

Exercise 3*

In this exercise you are asked to train yourself in the use of the game characterization for strong bisimulation. We therefore give universal winning strategy for the attacker or the defender in order to prove strong nonbisimilarity or bisimilarity. Let A denote the attacker and D the defender.

- Claim: $s \not\sim t$. The universal winning strategy for A is as follows.
  - In configuration (s, t), A chooses s and makes the move $s \xrightarrow{a} s_1$.
    - D's only possible response is to choose t and make the move $t \xrightarrow{a} t_1$. The current configuration is now (s_1, t_1)
  - In configuration (s_1, t_1), A chooses s_1 and makes the move $s_1 \xrightarrow{b} s_2$.

  Now the winning strategy depends on D's next move and is as follows. D can only choose the state t_1, but has two possible moves. Suppose D chooses $t_1 \xrightarrow{b} t_1$. Then the current configuration becomes (s_2, t_1). Now A choose s_2 and makes the move $s_2 \xrightarrow{a} s$. Then D looses since there are no a-transitions from t_1. If D uses the other possible move, namely $t_1 \xrightarrow{b} t_2$, the current configuration becomes (s_2, t_2). But then A chooses s_2 and makes the move $s_2 \xrightarrow{b} s_2$. Again D looses since there are no b-transitions from t_2.

  Remark: there is another winning strategy for the attacker which is easier to describe; try to find it.

- Claim: $s \sim u$: The universal winning strategy for D is as follows.
  - Starting in (s, u), A has two possible moves. Either (a) $s \xrightarrow{a} s_1$ or (b) $u \xrightarrow{a} u_1$.
    - If A chooses (a), then D takes the move $u \xrightarrow{a} u_1$, and the current configuration becomes (s_1, u_1).
    - If A chooses (b), then D takes the move $s \xrightarrow{a} s_1$, and the current configuration again becomes (s_1, u_1).
  - In configuration (s_1, u_1), A can choose either (a) $s_1 \xrightarrow{b} s_2$, or (b) $u_1 \xrightarrow{b} u_3$.
    - If A chooses (a), then D takes the move $u_1 \xrightarrow{b} u_3$, and the current configuration becomes (s_2, u_3).
    - If A chooses (b), then D takes the move $s_1 \xrightarrow{b} s_2$, and the current configuration again becomes (s_2, u_3).
  - In configuration (s_2, u_3), A can choose either (a) $s_2 \xrightarrow{b} s_2$ or (b) $s_2 \xrightarrow{a} s$ or (c) $u_3 \xrightarrow{a} u$ or (d) $u_3 \xrightarrow{b} u_2$.
    - If A chooses (a), then D takes the move $u_3 \xrightarrow{b} u_2$ and the current configuration becomes (s_2, u_2).

  - $*$ If A chooses (b), then D takes the move $u_3 \xrightarrow{a} u$ and the current configuration becomes (s, u) which is exactly the start configuration.
  - $*$ If A chooses (c), then D takes the move $s_2 \xrightarrow{a} s$ and the current configuration becomes (s, u) which is the start configuration.
  - $*$ If A chooses (d), then D takes the move $s_2 \xrightarrow{b} s_2$ and the current configuration becomes (s_2, u_2) as when the attacker played (a). Hence from now we only need to consider games form the state (s_2, u_2).

  Now we can argue that D has a winning strategy. From (s_2, u_2), D's response to any move from A will be to take the same transition. This means that the next configuration is either (s_2, u_2) or (s, u). The game will be infinite, and hence D is the winner.

- Claim: $s \not\sim v$: The universal winning strategy for A is as follows.
  - In configuration (s, v), A makes the move $s \xrightarrow{a} s_1$.
    - $*$ Now D must make the move $v \xrightarrow{a} v_1$ and the current configuration becomes (s_1, v_1).
  - In configuration (s_1, v_1), A chooses $v_1 \xrightarrow{b} v_2$.
    - $*$ D must make the move $s_1 \xrightarrow{b} s_2$. The current configuration is (s_2, v_2).

  Now A wins since from (s_2, v_2) as he can choose to make the move $s_2 \xrightarrow{b} s_2$. Since there are no b-transitions from v_2, D looses.