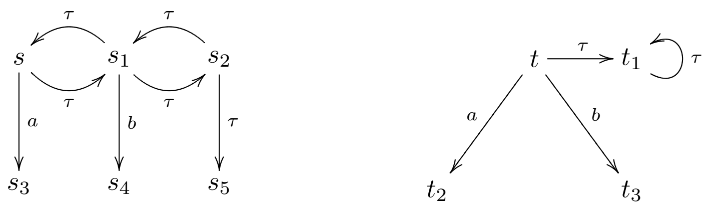
Exercise 2*

Consider the following labelled transition system.



Show that $s \approx t$ by finding a weak bisimulation R containing the pair (s,t).

Exercise 3*

Decide whether the following claims are true or false. Support your claims either by using bisimulation games or directly the definition of strong/weak bisimilarity.

- $a.\tau.Nil \overset{?}{\sim} \tau.a.Nil$
- $\tau.a.A + b.B \overset{?}{\sim} \tau.(a.A + b.B)$
- $\tau.Nil + (a.Nil \,|\, \overline{a}.Nil) \smallsetminus \{a,b\} \overset{?}{\sim} \tau.Nil$
- $a.(\tau.Nil + b.B) \overset{?}{\sim} a.Nil + a.b.B$

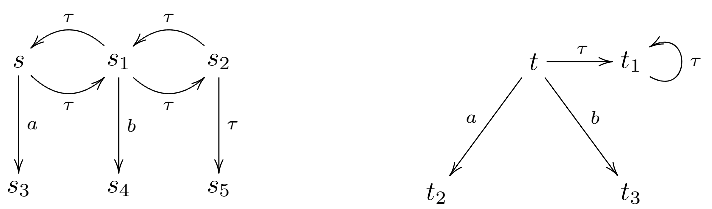
The same processes but weak bisimilarity instead of the strong one.

- $a.\tau.Nil \overset{?}{\approx} \tau.a.Nil$
- $\tau.a.A + b.B \overset{?}{\approx} \tau.(a.A + b.B)$
- $\tau.Nil + (a.Nil \,|\, \overline{a}.Nil) \smallsetminus \{a,b\} \overset{?}{\approx} \tau.Nil$
- $a.(\tau.Nil + b.B) \overset{?}{\approx} a.Nil + a.b.B$

Hint: draw first the LTS generated by the CCS processes.
Home exercise: try to verify your claims by using the tool CWB.

Exercise 2*

Consider the following labelled transition system.



Show that $s \approx t$ by finding a weak bisimulation R containing the pair (s, t).

Solution

Let $R = \{(s,t), (s_1,t), (s_2,t), (s_3,t_2), (s_4,t_3), (s_5,t_1)\}$. Now one can argue that R is a weak bisimulation as follows.

- Transitions from the pair (s,t): if $s \xrightarrow{a} s_3$ then $t \overset{a}{\Longrightarrow} t_2$ and $(s_3,t_2) \in R$. If $s \xrightarrow{\tau} s_1$ then $t \overset{\tau}{\Longrightarrow} t$ and $(s_1,t) \in R$. If $t \xrightarrow{a} t_2$ then $s \overset{a}{\Longrightarrow} s_3$ and $(s_3,t_2) \in R$. If $t \xrightarrow{b} t_3$ then $s \overset{b}{\Longrightarrow} s_4$ and $(s_4,t_3) \in R$. If $t \xrightarrow{\tau} t_1$ then $s \overset{\tau}{\Longrightarrow} s_5$ and $(s_5,t_1) \in R$.
- The transitions from the remaining pairs can be checked in a similar way.

Exercise 3*

Decide whether the following claims are true or false. Support your claims either by using bisimulation games or directly the definition of strong/weak bisimilarity.

- $a.\tau.Nil \not\sim \tau.a.Nil$
  - The attacker plays the action a in the left process and the defender does not have any a-move available in the right process and looses.

- $\tau.a.A + b.B \not\sim \tau.(a.A + b.B)$
  - The attacker plays the action b from the left process, there is no action b available in the right process in the first round. The attacker clearly wins.
- $\tau.Nil + (a.Nil \,|\, \overline{a}.Nil) \smallsetminus \{a, b\} \sim \tau.Nil$
  - $R = \{(\tau.Nil + (a.Nil \,|\, \overline{a}.Nil) \smallsetminus \{a, b\}, \tau.Nil), (Nil, Nil), ((Nil \,|\, Nil) \smallsetminus \{a, b\}, Nil)\}$ is a strong bisimulation.
- $a.(\tau.Nil + b.B) \not\sim a.Nil + a.b.B$
  - In the first round the attacker plays from the left the action a and in the second round he plays again from left the action τ. The defender looses as he can never play the same sequence of a followed by τ from the right process.

The same processes but weak bisimilarity instead of the strong one.

- $a.\tau.Nil \approx \tau.a.Nil$
  - $R = \{(a.\tau.Nil, \tau.a.Nil), (\tau.Nil, Nil), (Nil, Nil), (a.\tau.Nil, a.Nil)\}$ is a weak bisimulation.
- $\tau.a.A + b.B \not\approx \tau.(a.A + b.B)$
  - The attacker plays the action τ from the left and reaches the process $a.A$. The defender can either answer by (i) doing nothing on the right and staying in the process $\tau.(a.A + b.B)$ or (ii) by playing the action τ and reaching $a.A + b.B$. In case (i) the attacker will play in second round on the right the action τ, the defender can only stay in $a.A$ and in the next round the attacker wins by making the b-move on the right. In case (ii) the attacker wins already in the second round by playing b from the right process.
- $\tau.Nil + (a.Nil \,|\, \overline{a}.Nil) \smallsetminus \{a, b\} \approx \tau.Nil$
  - These two processes are even strongly bisimilar so they must be also weakly bisimilar.
- $a.(\tau.Nil + b.B) \not\approx a.Nil + a.b.B$
  - The attacker plays $a.Nil + a.b.B \stackrel{a}{\longrightarrow} b.B$ on the right, the defender can answer either by $a.(\tau.Nil + b.B) \stackrel{a}{\Longrightarrow} \tau.Nil + b.B$ or by $a.(\tau.Nil + b.B) \stackrel{a}{\Longrightarrow} Nil$. In the first case the attacker plays $\tau.Nil + b.B \stackrel{\tau}{\longrightarrow} Nil$ and the defender can only do nothing and will loose in the next round. In the second case, the attacker plays the action b from the left and the defender looses.

Home exercise: try to verify your claims by using the tool CWB.