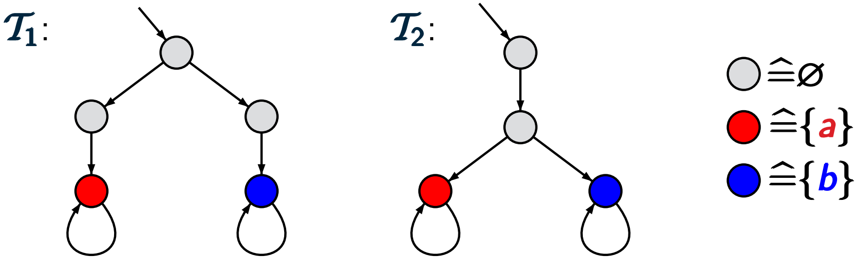
Overview

- Introduction
- Modelling parallel systems
- Linear Time Properties
- Regular Properties
- Linear Temporal Logic (LTL)
- Computation-Tree Logic
- **Equivalences and Abstraction**

Trace equivalence



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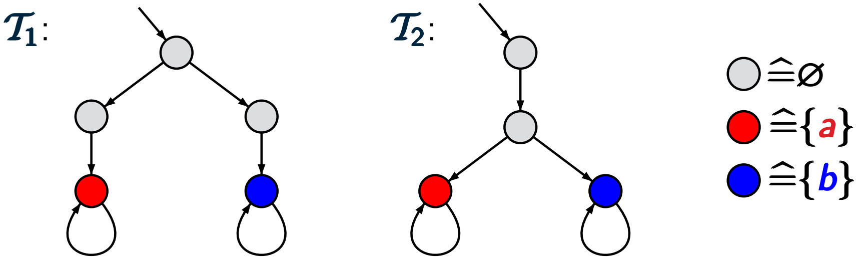

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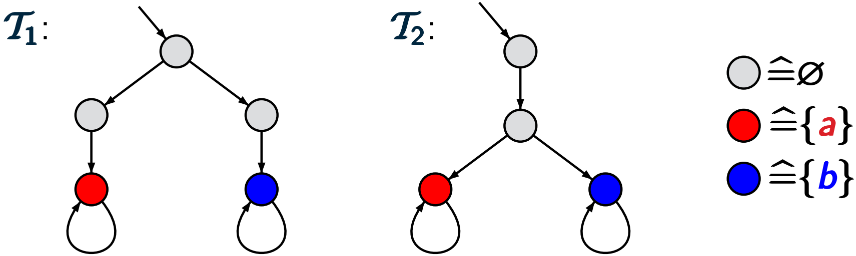

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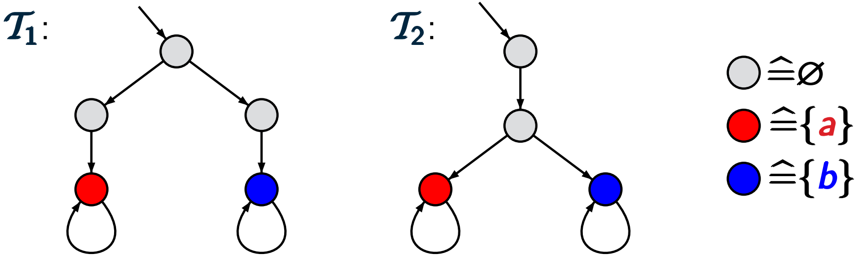

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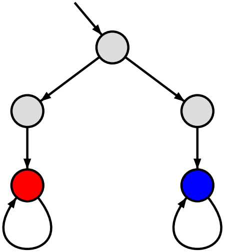
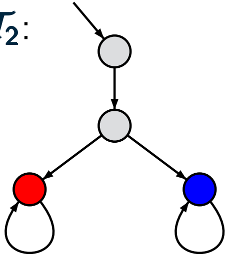
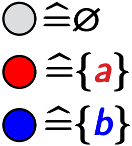
Trace equivalence is not compatible with CTL

$\mathcal{T}_1$:



$\mathcal{T}_2$:





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- strong vs. weak relations
  - * strong: reasoning about all transitions
  - * weak: abstraction from stutter steps

Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation-Tree Logic

Equivalences and Abstraction

- bisimulation ⟵
- CTL, CTL*-equivalence
- computing the bisimulation quotient
- abstraction stutter steps
- simulation relations

Bisimulation for two transition systems

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let $\mathcal{T}_1 = (S_1, Act_1, \rightarrow_1, S_{0,1}, AP, L_1)$,

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Bisimulation equivalence of $\mathcal{T}_1$ and $\mathcal{T}_2$ requires that $\mathcal{T}_1$ and $\mathcal{T}_2$ can simulate each other in a stepwise manner.

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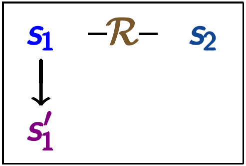
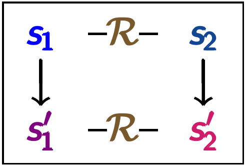


can be completed to



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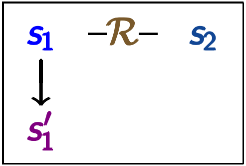

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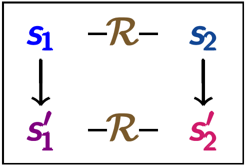

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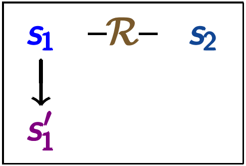

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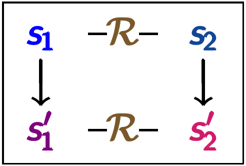

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(2), (3) } mutual stepwise simulation

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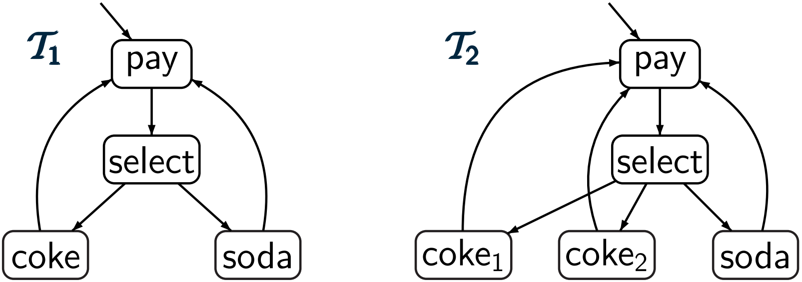
for state s_1 of $\mathcal{T}_1$ and state s_2 of $\mathcal{T}_2$:

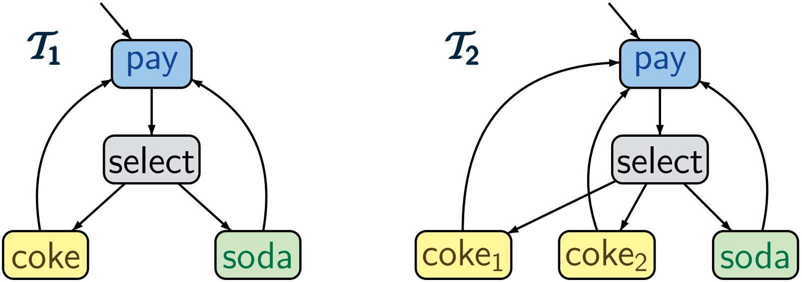
$s_1 \sim s_2$ iff there exists a bisimulation $\mathcal{R}$ for $(\mathcal{T}_1, \mathcal{T}_2)$ such that $(s_1, s_2) \in \mathcal{R}$

Two beverage machines



$AP = \{pay, coke, soda\}$

Two beverage machines



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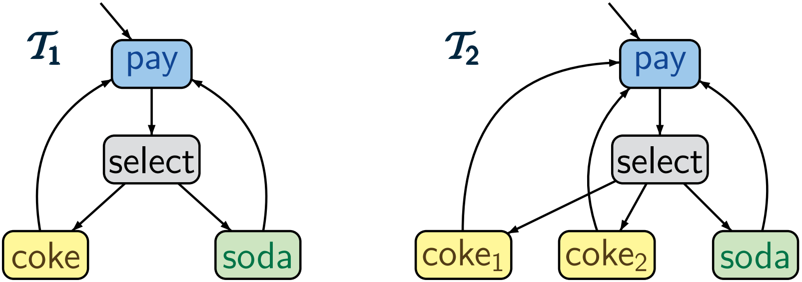

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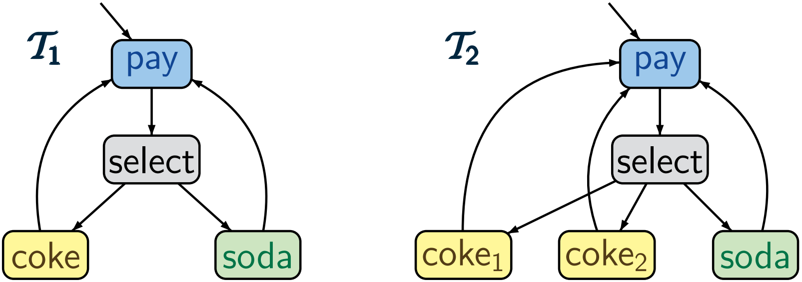

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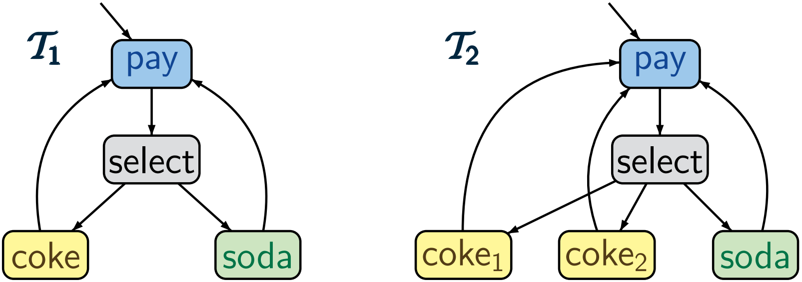

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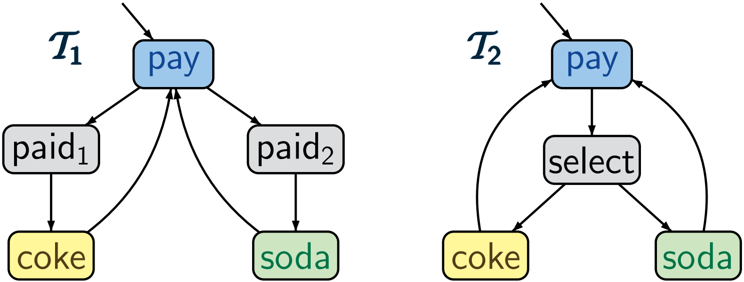
$\mathcal{T}_1 \sim \mathcal{T}_2$ as there is a bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$:

$\{$ (pay,pay), (select,select), (soda,soda)
(coke,coke$_1$), (coke,coke$_2$) $\}$

Two beverage machines



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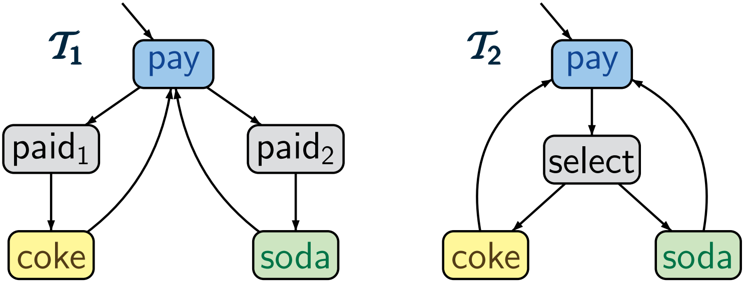

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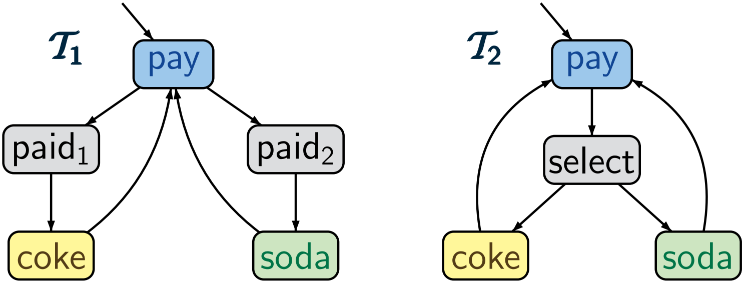


$AP = \{pay, coke, soda\}$

$\mathcal{T}_1 \not\sim \mathcal{T}_2$

because there is no state in $\mathcal{T}_1$ that has both

- a successor labeled with coke and
- a successor labeled with soda

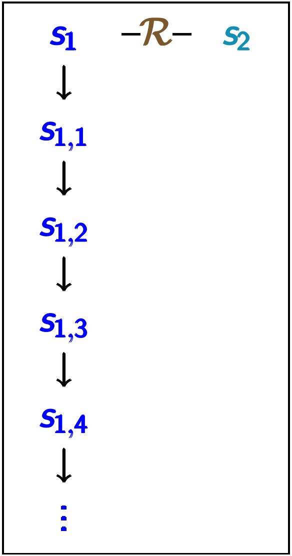
Simulation condition of bisimulations

$$\begin{array}{ccc} s_1 & -\mathcal{R}- & s_2 \\ \downarrow & & \\ s_1' & & \end{array}$$

can be completed to

$$\begin{array}{ccc} s_1 & -\mathcal{R}- & s_2 \\ \downarrow & & \downarrow \\ s_1' & -\mathcal{R}- & s_2' \end{array}$$

Path lifting for bisimulation $\mathcal{R}$



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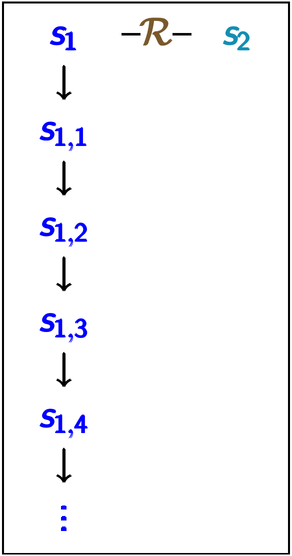

can be
completed to

Path lifting for bisimulation $\mathcal{R}$

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \\
s_{1,1} & & \\
\downarrow & & \\
s_{1,2} & & \\
\downarrow & & \\
s_{1,3} & & \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

can be completed to

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \downarrow \\
s_{1,1} & -\mathcal{R}- & s_{2,1} \\
\downarrow & & \\
s_{1,2} & & \\
\downarrow & & \\
s_{1,3} & & \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

Path lifting for bisimulation $\mathcal{R}$

BSEQOR5.1-9-BIS

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \\
s_{1,1} & & \\
\downarrow & & \\
s_{1,2} & & \\
\downarrow & & \\
s_{1,3} & & \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

can be completed to

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \downarrow \\
s_{1,1} & -\mathcal{R}- & s_{2,1} \\
\downarrow & & \downarrow \\
s_{1,2} & -\mathcal{R}- & s_{2,2} \\
\downarrow & & \\
s_{1,3} & & \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \\
s_{1,1} & & \\
\downarrow & & \\
s_{1,2} & & \\
\downarrow & & \\
s_{1,3} & & \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
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can be completed to

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
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s_{1,1} & -\mathcal{R}- & s_{2,1} \\
\downarrow & & \downarrow \\
s_{1,2} & -\mathcal{R}- & s_{2,2} \\
\downarrow & & \downarrow \\
s_{1,3} & -\mathcal{R}- & s_{2,3} \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

Path lifting for bisimulation $\mathcal{R}$

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \\
s_{1,1} & & \\
\downarrow & & \\
s_{1,2} & & \\
\downarrow & & \\
s_{1,3} & & \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

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$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \downarrow \\
s_{1,1} & -\mathcal{R}- & s_{2,1} \\
\downarrow & & \downarrow \\
s_{1,2} & -\mathcal{R}- & s_{2,2} \\
\downarrow & & \downarrow \\
s_{1,3} & -\mathcal{R}- & s_{2,3} \\
\downarrow & & \downarrow \\
s_{1,4} & -\mathcal{R}- & s_{2,4} \\
\downarrow & & \downarrow \\
\vdots & & \vdots
\end{array}
$$

Properties of bisimulation equivalence

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$\uparrow$

If S is the state space of $\mathcal{T}$ then

$$\mathcal{R} = \{(s, s) : s \in S\}$$

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Properties of bisimulation equivalence

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If $\mathcal{R}$ is a bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$ then

$$\mathcal{R}^{-1} = \{(s_2, s_1) : (s_1, s_2) \in \mathcal{R}\}$$

is a bisimulation for $(\mathcal{T}_2, \mathcal{T}_1)$

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Let $\mathcal{R}_{1,2}$ be a bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$,
$\mathcal{R}_{2,3}$ be a bisimulation for $(\mathcal{T}_2, \mathcal{T}_3)$.

$\sim$ is an equivalence, i.e.,

- reflexivity: $\mathcal{T} \sim \mathcal{T}$ for all transition systems $\mathcal{T}$
- symmetry: $\mathcal{T}_1 \sim \mathcal{T}_2$ implies $\mathcal{T}_2 \sim \mathcal{T}_1$
- transitivity: if $\mathcal{T}_1 \sim \mathcal{T}_2$ and $\mathcal{T}_2 \sim \mathcal{T}_3$ then $\mathcal{T}_1 \sim \mathcal{T}_3$

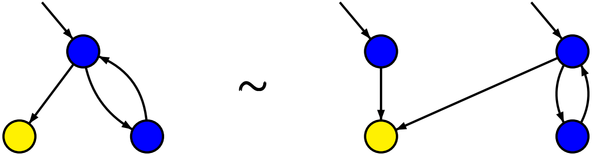
↑

Let $\mathcal{R}_{1,2}$ be a bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$,
$\mathcal{R}_{2,3}$ be a bisimulation for $(\mathcal{T}_2, \mathcal{T}_3)$.

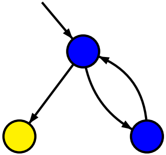
$$\mathcal{R} \stackrel{\text{def}}{=} \{ (s_1, s_3) : \exists s_2 \text{ s.t. } (s_1, s_2) \in \mathcal{R}_{1,2} \text{ and } (s_2, s_3) \in \mathcal{R}_{2,3} \}$$

is a bisimulation for $(\mathcal{T}_1, \mathcal{T}_3)$

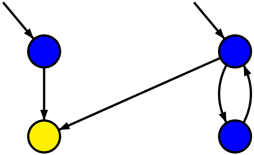
Correct or wrong?



Correct or wrong?

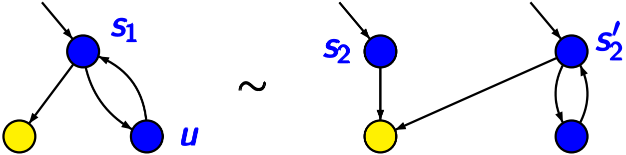


$\sim$



wrong

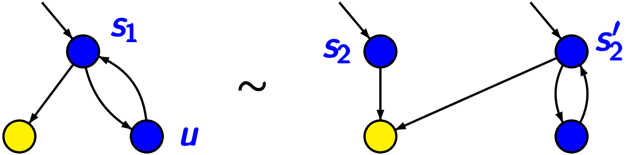
Correct or wrong?



wrong

$s_1 \longrightarrow u$, but $s_2 \not\longrightarrow blue$ (thus $s_1 \not\sim s_2$)

Correct or wrong?



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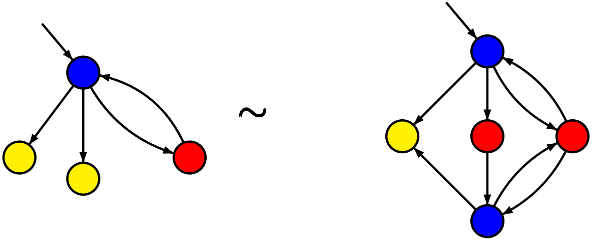

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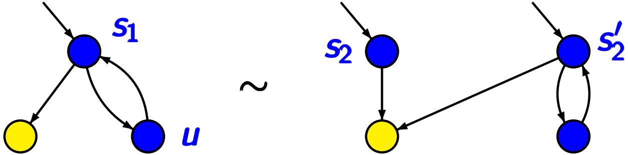

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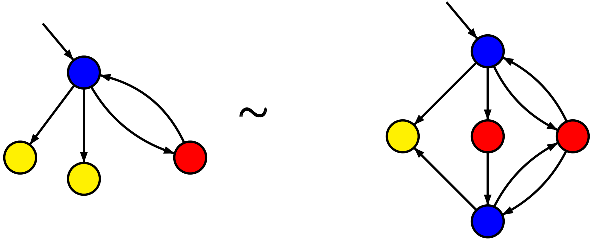
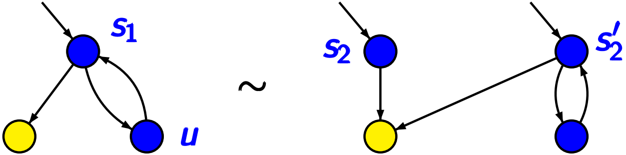


correct

Correct or wrong?

BSEQOR5.1-19



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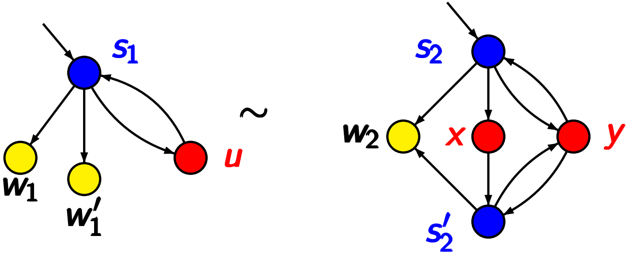
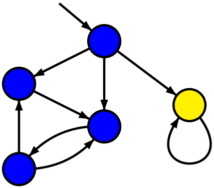


correct

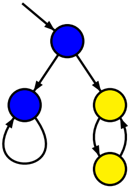
bisimulation:

$$\{(w_1, w_2), (w_1', w_2), (s_1, s_2), (s_1, s_2'), (u, x), (u, y)\}$$

Correct or wrong?



$\sim$



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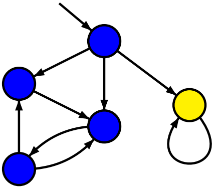

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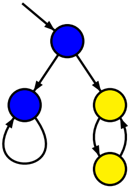

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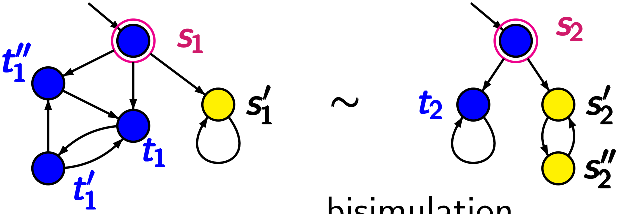
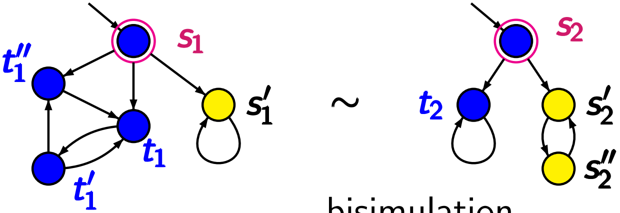


correct

bisimulation

$$\{(s_1, s_2), (s_1', s_2'), (s_1', s_2''), (t_1, t_2), (t_1', t_2), (t_1'', t_2)\}$$

Correct or wrong?



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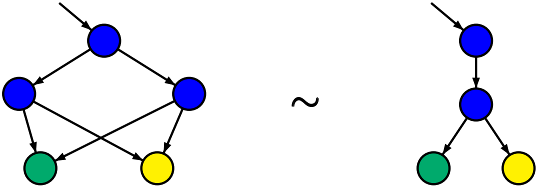

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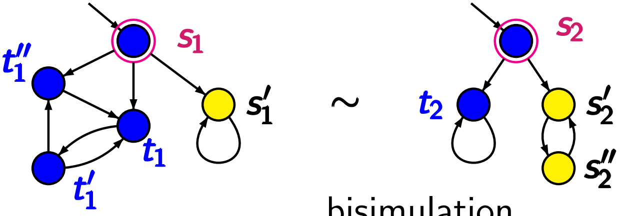

correct

bisimulation

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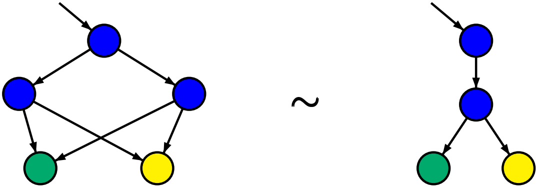

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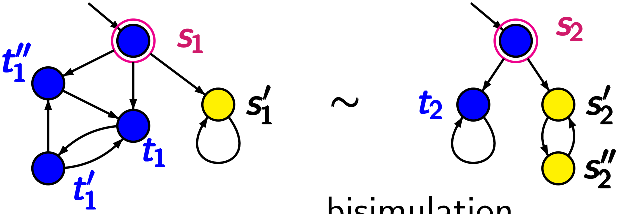
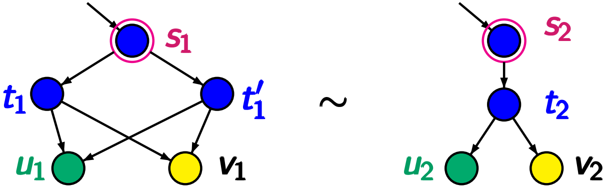


correct

bisimulation

$$\{(s_1, s_2), (s_1', s_2'), (s_1', s_2''), (t_1, t_2), (t_1', t_2), (t_1'', t_2)\}$$



correct

bisimulation: $\{(s_1, s_2), (t_1, t_2), (t_1', t_2), (u_1, u_2), (v_1, v_2)\}$

Bisimulation vs. trace equivalence

Bisimulation vs. trace equivalence

$$\mathcal{T}_1 \sim \mathcal{T}_2 \implies \mathit{Traces}(\mathcal{T}_1) = \mathit{Traces}(\mathcal{T}_2)$$

Bisimulation vs. trace equivalence

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proof: ... path fragment lifting ...

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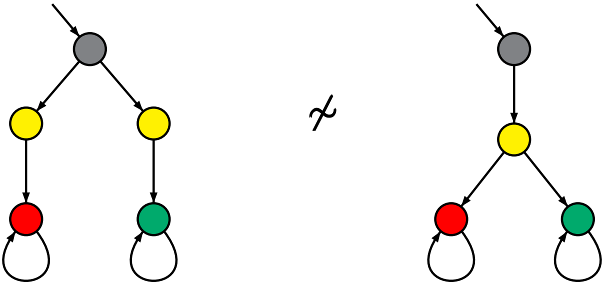

trace equivalent, but not bisimulation equivalent

Bisimulation vs. trace equivalence

$$\mathcal{T}_1 \sim \mathcal{T}_2 \implies Traces(\mathcal{T}_1) = Traces(\mathcal{T}_2)$$

proof: ... path fragment lifting ...

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Trace equivalence is strictly coarser than bisimulation equivalence.

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Bisimulation equivalent transition systems satisfy the same **LT properties** (e.g., **LTL** formulas).

Bisimulation equivalence ...

- as a relation that compares 2 transition systems

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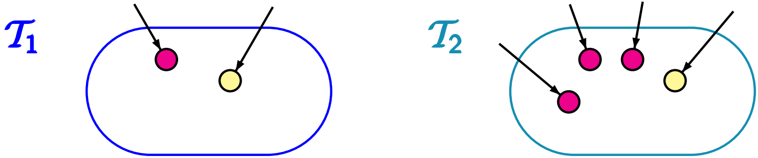

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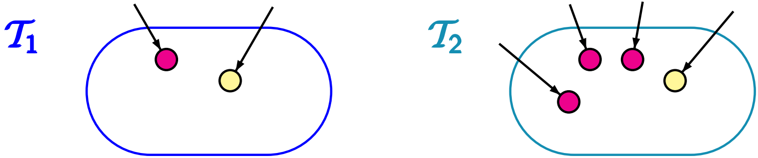

- as a relation on the states of **1** transition system

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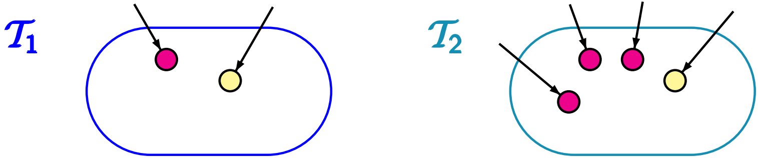

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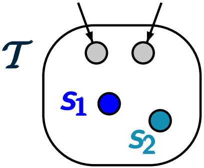

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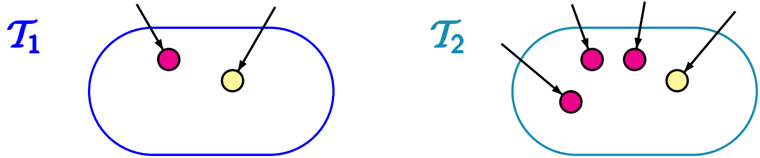

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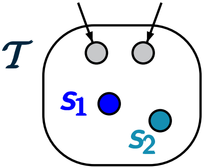

$s_1 \sim s_2$ iff $\mathcal{T}_{s_1} \sim \mathcal{T}_{s_2}$

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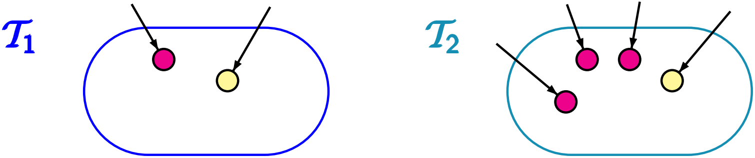

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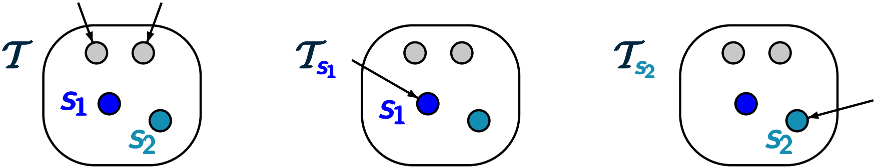

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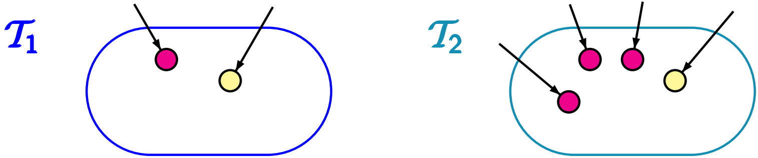

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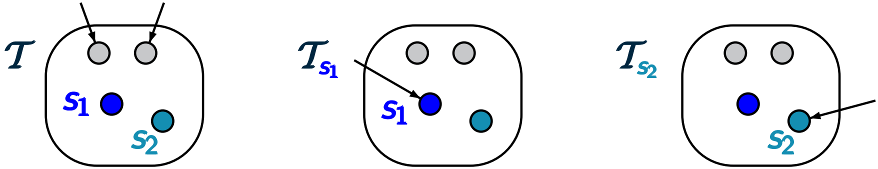

$s_1 \sim s_2$ iff $\mathcal{T}_{s_1} \sim \mathcal{T}_{s_2}$ iff
there exists a bisimulation $\mathcal{R}$ for $\mathcal{T}$ s.t. $(s_1, s_2) \in \mathcal{R}$

Bisimulations on a single TS

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Let $\mathcal{T}$ be a TS with proposition set AP.

Bisimulations on a single TS

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A bisimulation for $\mathcal{T}$ is a binary relation $\mathcal{R}$ on the state space of $\mathcal{T}$ s.t. for all $(s_1, s_2) \in \mathcal{R}$:

(1) $L(s_1) = L(s_2)$

(2) $\forall\, s_1' \in Post(s_1)\ \exists\, s_2' \in Post(s_2)$ s.t. $(s_1', s_2') \in \mathcal{R}$

(3) $\forall\, s_2' \in Post(s_2)\ \exists\, s_1' \in Post(s_1)$ s.t. $(s_1', s_2') \in \mathcal{R}$

Bisimulation equivalence $\sim_{\mathcal{T}}$ on a single TS

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coinductive definition of $\sim_{\mathcal{T}}$:

$s_1 \sim_{\mathcal{T}} s_2$ iff there exists a bisimulation $\mathcal{R}$ for $\mathcal{T}$ s.t. $(s_1, s_2) \in \mathcal{R}$

Bisimulation equivalence

Let $\mathcal{T}$ be a transition system with state space S.

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Bisimulation equivalence

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$$\begin{array}{ccc} s_1 & \sim_{\mathcal{T}} & s_2 \\ \downarrow & & \\ s_1' & & \end{array} \quad \text{can be completed to} \quad \begin{array}{ccc} s_1 & \sim_{\mathcal{T}} & s_2 \\ \downarrow & & \downarrow \\ s_1' & \sim_{\mathcal{T}} & s_2' \end{array}$$

Two variants of bisimulation equivalence

$\sim$ relation that compares **2** transition systems

$\sim_{\mathcal{T}}$ equivalence on the state space of a single TS $\mathcal{T}$

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   for all states s_1 and s_2 of $\mathcal{T}$:

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where $\mathcal{T}_s$ agrees with $\mathcal{T}$, except that state s is declared to be the unique initial state

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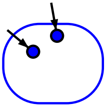
2. $\sim$ can be derived from $\sim_{\mathcal{T}}$

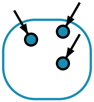
Derivation of $\sim$ from $\sim_{\mathcal{T}}$

given two transition systems $\mathcal{T}_1$ and $\mathcal{T}_2$

$\mathcal{T}_1$ with state space S_1



$\mathcal{T}_2$ with state space S_2



Derivation of $\sim$ from $\sim_{\mathcal{T}}$

given two transition systems $\mathcal{T}_1$ and $\mathcal{T}_2$

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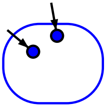
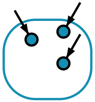
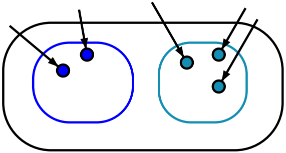


$\mathcal{T}_2$ with state space S_2



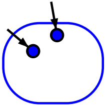
consider $\mathcal{T} = \mathcal{T}_1 \uplus \mathcal{T}_2$
(state space $S_1 \uplus S_2$)



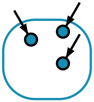
Derivation of $\sim$ from $\sim_{\mathcal{T}}$

given two transition systems $\mathcal{T}_1$ and $\mathcal{T}_2$

$\mathcal{T}_1$ with state space S_1



$\mathcal{T}_2$ with state space S_2



consider $\mathcal{T} = \mathcal{T}_1 \uplus \mathcal{T}_2$
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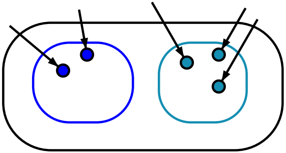
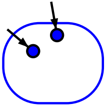


$\mathcal{T}_1 \sim \mathcal{T}_2$ iff $\forall$ initial states s_1 of $\mathcal{T}_1$
$\exists$ initial state s_2 of $\mathcal{T}_2$ s.t. $s_1 \sim_{\mathcal{T}} s_2$,

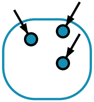
Derivation of $\sim$ from $\sim_{\mathcal{T}}$

given two transition systems $\mathcal{T}_1$ and $\mathcal{T}_2$

$\mathcal{T}_1$ with state space S_1



$\mathcal{T}_2$ with state space S_2



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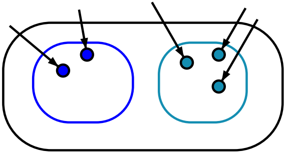

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$\exists$ initial state s_2 of $\mathcal{T}_2$ s.t. $s_1 \sim_{\mathcal{T}} s_2$,
and vice versa

Bisimulation quotient

Bisimulation quotient

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS.

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bisimulation quotient $\mathcal{T}/\!\sim$ arises from $\mathcal{T}$
by collapsing bisimulation equivalent states

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set of bisimulation equivalence classes

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↑

well-defined

by the labeling condition of bisimulations

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$$\frac{s \longrightarrow s'}{[s]_{\sim_{\mathcal{T}}} \longrightarrow [s']_{\sim_{\mathcal{T}}}}$$

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action labels irrelevant

Bisimulation quotient

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS.

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$$\frac{s \xrightarrow{\alpha} s'}{[s]_{\sim_{\mathcal{T}}} \xrightarrow{\tau} [s']_{\sim_{\mathcal{T}}}}$$

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$$\mathcal{T} \sim \mathcal{T}/\sim$$

Example: interleaving of n printers

parallel system $\mathcal{T} = \underbrace{\mathit{Printer} \,|||\, \mathit{Printer} \,|||\, \ldots \,|||\, \mathit{Printer}}_{n \text{ printers}}$

Example: interleaving of *n* printers

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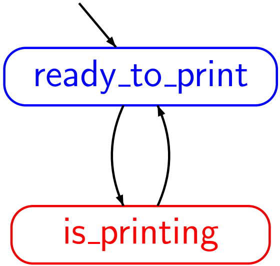
transition system
for each printer

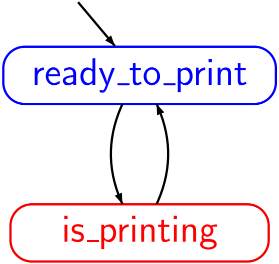


Example: interleaving of n printers

parallel system $\mathcal{T} = \underbrace{\mathit{Printer} \;|||\; \mathit{Printer} \;|||\; \ldots \;|||\; \mathit{Printer}}_{n \text{ printers}}$

$AP = \{0, 1, \ldots, n\}$ "number of available printers"

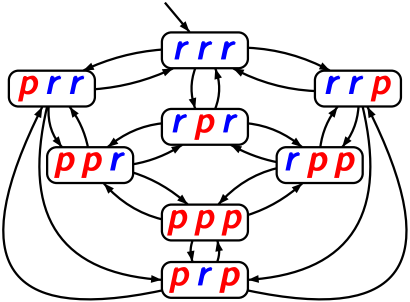
transition system for each printer



Example: n=3 printers

parallel system $\mathcal{T} = \underbrace{Printer \,|||\, Printer \,|||\, \ldots \,|||\, Printer}_{n \text{ printers}}$

$AP = \{0, 1, 2, 3\}$



p: is printing

r: ready to print

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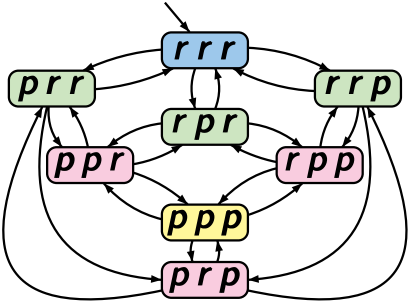

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Example: n=3 printers

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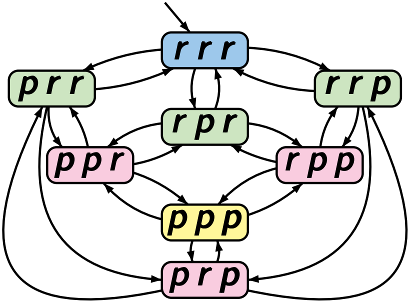

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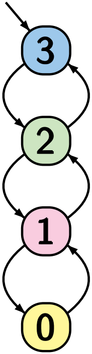

bisimulation quotient

Example: n=3 printers

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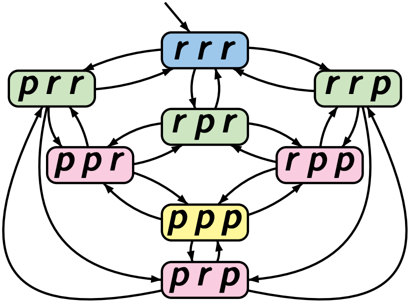

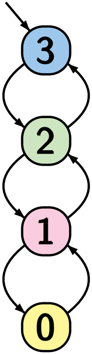

2^n states

$n+1$ states