Timed Traces

Let $A = (L, \ell_0, E, I)$ be a timed automaton over a set of clocks C and a set of labels N.

Timed Traces

A sequence $(t_1, a_1)(t_2, a_2)(t_3, a_3)\ldots$ where $t_i \in \mathbb{R}^{\geq 0}$ and $a_i \in N$ is called a timed trace of A iff there is a transition sequence

$$(\ell_0, v_0) \xrightarrow{d_1} . \xrightarrow{a_1} . \xrightarrow{d_2} . \xrightarrow{a_2} . \xrightarrow{d_3} . \xrightarrow{a_3} \ldots$$

in A such that $v_0(x) = 0$ for all $x \in C$ and

$$t_i = t_{i-1} + d_i \qquad \text{where } t_0 = 0.$$

Intuition: t_i is the absolute time (time-stamp) when a_i happened since the start of the automaton A.

Timed and Untimed Language Equivalence

The set of all timed traces of an automaton A is denoted by $L(A)$ and called the timed language of A.

Theorem [Alur, Courcoubetis, Dill, Henzinger'94]

Timed language equivalence (the problem whether $L(A_1) = L(A_2)$ for given timed automata A_1 and A_2) is undecidable.

We say that $a_1 a_2 a_3 \ldots$ is an untimed trace of A iff there exist $t_1, t_2, t_3, \ldots \in \mathbb{R}^{\geq 0}$ such that $(t_1, a_1)(t_2, a_2)(t_3, a_3) \ldots$ is a timed trace of A.

Theorem [Alur, Dill'94]

Untimed language equivalence for timed automata is decidable.

Timed Bisimilarity

Let A_1 and A_2 be timed automata.

Timed Bisimilarity

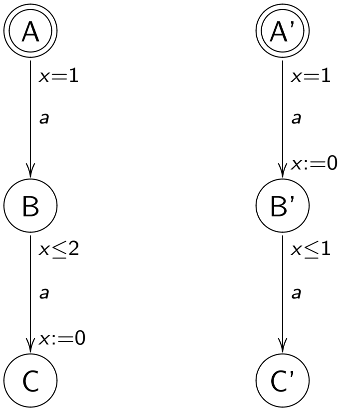
We say that A_1 and A_2 are timed bisimilar iff the transition systems $T(A_1)$ and $T(A_2)$ generated by A_1 and A_2 are strongly bisimilar.

Remark: both

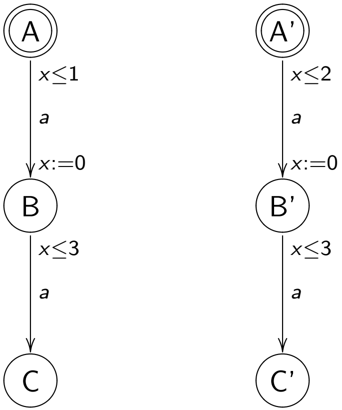
- $\xrightarrow{a}$ for $a \in N$ and
- $\xrightarrow{d}$ for $d \in \mathbb{R}^{\geq 0}$

are considered as normal (visible) transitions.

Example of Timed Bisimilar Automata



Example of Timed Non-Bisimilar Automata



Untimed Bisimilarity

Let A_1 and A_2 be timed automata. Let ϵ be a new (fresh) action.

Untimed Bisimilarity

We say that A_1 and A_2 are untimed bisimilar iff the transition systems $T(A_1)$ and $T(A_2)$ generated by A_1 and A_2 where every transition of the form $\xrightarrow{d}$ for $d \in \mathbb{R}^{\geq 0}$ is replaced with $\xrightarrow{\epsilon}$ are strongly bisimilar.

Remark:

- $\xrightarrow{a}$ for $a \in N$ is treated as a visible transition, while
- $\xrightarrow{d}$ for $d \in \mathbb{R}^{\geq 0}$ are all labelled by a single visible action $\xrightarrow{\epsilon}$.

Corollary

Any two timed bisimilar automata are also untimed bisimilar.

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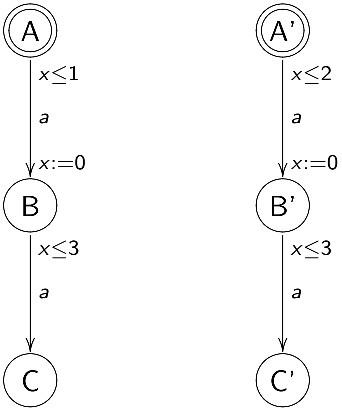
Timed Non-Bisimilar but Untimed Bisimilar Automata



Decidability of Timed and Untimed Bisimilarity

Theorem [Cerans'92]

Timed bisimilarity for timed automata is decidable in EXPTIME (deterministic exponential time).

Theorem [Larsen, Wang'93]

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