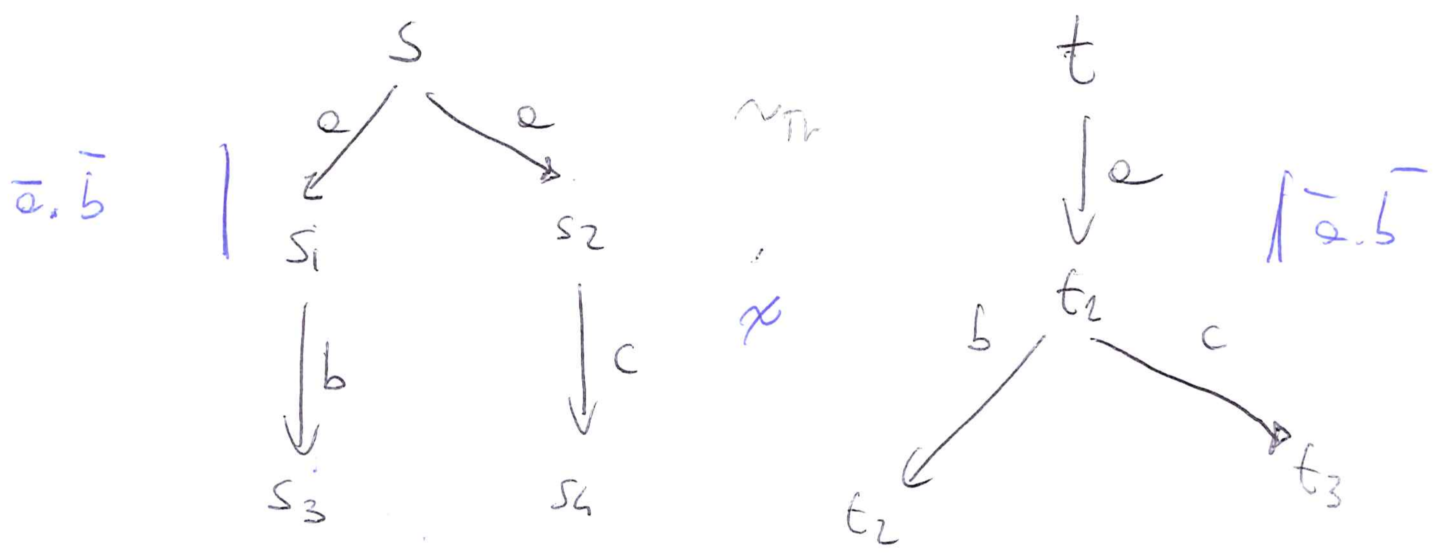
$L(s) = \{a, ab, ac\}$ $L(t) = \{a, ab, ac\}$

11/01/17 - 1



TIMED BISIMILARITY TLTS $\mathcal{T} = \langle S, \Sigma \cup \mathbb{R}^{\geq 0}, \xrightarrow{a} \cup \xrightarrow{d} \rangle$

R is a TIMED BISIMULATION IFF

$R \subseteq S \times S$ whenever $\boxed{((s,v),(t,v')) \in R}$

$(s,t) \in R$

1) $s \xrightarrow{a} s'$ then $t \xrightarrow{a} t'$ for some $t' \in S$ and $(s',t') \in R$ $a \in \Sigma \cup \{\tau\}$
2) $s \xrightarrow{d} s'$ then $t \xrightarrow{d} t'$ for some $t' \in S$ and $(s',t') \in R$ $d \in \mathbb{R}^{\geq 0}$
3) $t \xrightarrow{a} t'$ then $s \xrightarrow{a} s'$ " " $s' \in S$ and $(s',t') \in R$
4) $t \xrightarrow{d} t'$ then $s \xrightarrow{d} s'$ " " $s' \in S$ and $(s',t') \in R$

$s \sim t$ sse $\exists R \subseteq S \times S$ R timed bisimulation

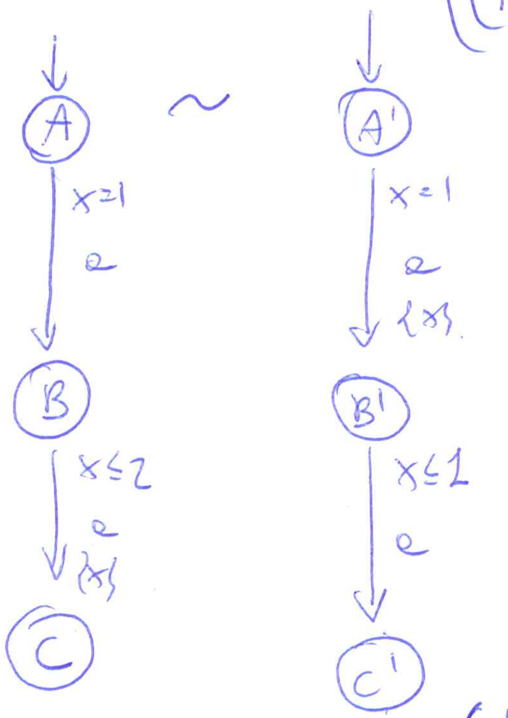
2

$*$ d x

$x > d$, $0 \leq d \leq 1$

$\boxed{\sim} = \bigcup R$ $\sim$ is the TIMED BISIMILARITY

$((A,[x=0]),(A',[x=0])) \in R \Rightarrow (A,[x=0]) \sim (A',[x=0]) \Rightarrow A \sim A'$



$R = \{((A,[0 \leq x \leq 1]),(A',[0 \leq x \leq 1]))\}$

$\cup$

$\{((A,[x=1]),(A',[x=1]))\} \cup \{((B,[x=1]),(B',[x=0]))\}$

$\cup$ $\{((A,[x>1]),(A',[x>1]))\} \cup \{((B,[1 < x \leq 2]), B'[0 < x \leq 1])\}$

$\cup$

$\{((C, x=0),(C', 0 \leq x \leq 1))\}$

$\{((B,[x>2]),(B',[x>1]))\}$

$\{((C, x>0),(C', x>1))\}$

TLTS

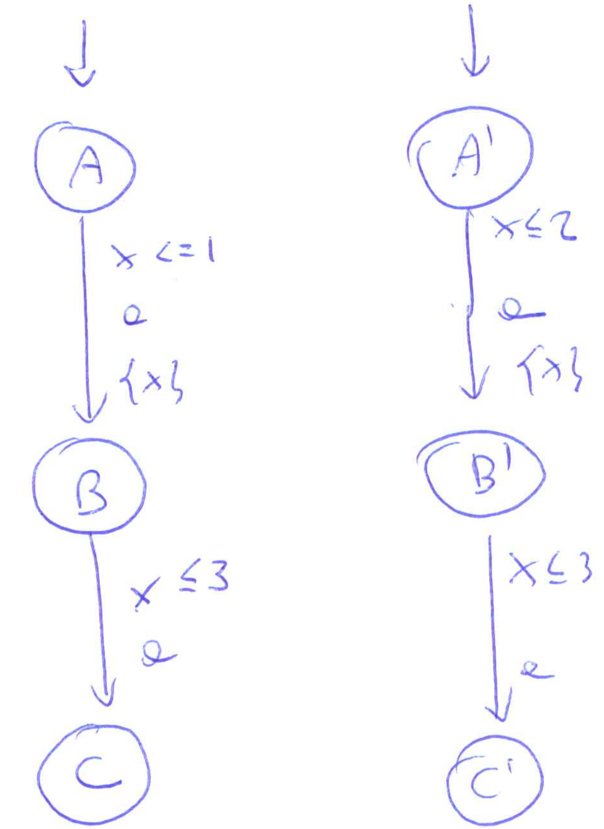
$(A,[0 \leq x \leq 1]) \sim (A',[0 \leq x < 1])$

$(A,[0 \leq x \leq 1]) \xrightarrow{d} (A,[x>1])$ $\quad$ $(A',[0 \leq x < 1]) \xrightarrow{d} (A',[x>2])$

$(A,[0 \leq x \leq 1]) \xrightarrow{d} (A,[x=1]) \xrightarrow{e} (B, x=1)$

$(A',[0 \leq x < 1]) \xrightarrow{d} (A',[x=1]) \xrightarrow{e} (B', x=0)$

3



$$((A,[x=0]),\ (A',[x=0]))$$

$$A:\ (A',[x=0]) \xrightarrow{1.7} (A',[x=1.7])$$

$$D:\ (A,[x=0]) \xrightarrow{1.7} (A,[x=1.7])$$

$$((A,[x=1.7]),\ (A',[x=1.7]))$$

$$A:\ (A',[x=1.7]) \xrightarrow{e} (B',[x=0])$$

$$D:\ (A,[x=1.7]) \overset{e}{\not\rightarrow}$$

$A \not\approx A'$

$A \sim_u A'$

$$R = \{(\overset{\varepsilon^{0.7}\circlearrowright}{(A,[0\le x\le 1])},\ \overset{\varepsilon^{1.7}\circlearrowright}{(A',[0\le x\le 2])})\} \xrightarrow{\varepsilon} \cup \{(\overset{\varepsilon\circlearrowright}{(A,[x>1])}),\ \overset{\varepsilon\circlearrowright}{(A',[x>2])})\}$$

$$\downarrow e$$

$$\{((B,[x=0]),\ (B',[x=0]))\} \cup \{((B,[x\ge 0]),(B',[x\ge 0]))\} \cup$$

$$\cup \{((C,[x\ge 0]),\ (C',[x\ge 0]))\}$$

WEAK TIMED BISIMILARITY

$\stackrel{\alpha}{\Rightarrow} \cup \stackrel{d}{\Rightarrow}$

$\stackrel{d}{\Rightarrow} = (\xrightarrow{\tau})^* \xrightarrow{d_1} (\xrightarrow{\tau})^* \xrightarrow{d_2} \dots$

$\dots (\xrightarrow{\tau})^* \xrightarrow{d_n} (\xrightarrow{\tau})^*$

t.c. $\sum_{i=1}^{n} d_i = d$

UNTIMED

$$\stackrel{\alpha}{\Rightarrow} = \begin{cases} (\xrightarrow{\tau})^* & \alpha = \tau \\ (\xrightarrow{\tau})^* \xrightarrow{\alpha} (\xrightarrow{\tau})^* & \alpha \neq \tau \end{cases}$$

1) $s \xrightarrow{\alpha} s' \Rightarrow t \stackrel{\alpha}{\Rightarrow} t'$

2)

$\stackrel{0}{\Rightarrow} \quad (\xrightarrow{\tau})^* \xrightarrow{0} (\xrightarrow{\tau})^* \xrightarrow{0} \dots$

$\xrightarrow{0}$ è sempre un self loop

$\boxed{\stackrel{0}{\Rightarrow} \equiv \stackrel{\tau}{\Rightarrow}}$

R è una WEAK TIMED BISIMULATION sse

whenever $(s,t) \in R$

1) $s \xrightarrow{\alpha} s'$ ~~⇒~~ allora $t \stackrel{\alpha}{\Rightarrow} t'$ e $(s',t') \in R$

2) $s \xrightarrow{d} s'$ allora $s \stackrel{d}{\Rightarrow} t'$ e $(s',t') \in R$)

3) $\lhd$

4) $\sim$

TIME - ABSTRACTED BISIMILARITY $\sim_{ta}$

every transition $\xrightarrow{d}$ in the TLTS is replaced with $\xrightarrow{\tau}$, then the definition of weak bisimilarity is applied

$$\sim_{ta} \overset{?}{\equiv} \sim_u$$

$\sim_{ta}$: $\xrightarrow{d}$ ~~⇝~~ $\xrightarrow{\tau}$ e Def. weak

$\sim_u$: $\xrightarrow{d}$ ⇝ $\xrightarrow{\varepsilon}$ e Def. strong

$\xrightarrow{5} \sim \xrightarrow{\varepsilon}$

$\xrightarrow{3}\xrightarrow{2}$ $\quad$ $\xrightarrow{\varepsilon}\xrightarrow{\varepsilon}$

$\xrightarrow{2}\xrightarrow{3}$ $\quad$ $\xrightarrow{\varepsilon}\xrightarrow{\varepsilon}$

$$\boxed{\sim_{ta} \not\equiv \sim_u}$$

→ Counter example

A

$A \xrightarrow{e} A'$

$B \xrightarrow{3} B' \xrightarrow{e} B''$

$B \xrightarrow{\tau} B' \xrightarrow{e} B''$ → $A \sim_{ta} B$

$R = \{(A,B)\ (A', B''), (A, B')\}$

$A \not\sim_u B$

~~$R = \{(A,B),$~~

$B \xrightarrow{\varepsilon} B' \xrightarrow{e} B''$