Advanced Topics in Computer Science
Exercises with (Some) Solutions

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Contents

1 TCCS Modelling and Derivations

Note: In the exercises of this section, when you are asked to show that some derivations or traces are possible you should justify your answer formally by making (some significant) formal derivations using the SOS rules of CCS and of TCCS.

For the rules of CCS use the names of the book. For the rules of TCCS use the following names: $\mathrm{ACT\text{-}T}$ for action delays, $\mathrm{SUM\text{-}T}$ for non-deterministic choice delay, $\mathrm{COM\text{-}T}$ for parallel composition delay, $\mathrm{RES\text{-}T}$ for restriction delay, $\mathrm{REL\text{-}T}$ for relabeling delay, $\mathrm{CON\text{-}T}$ for process variable delay. Moreover:

$$\mathrm{ZEROD}\ \frac{P \xrightarrow{\alpha} P'}{\varepsilon(0).P \xrightarrow{\alpha} P'} \quad \mathrm{DELAY\text{-}1}\ \frac{P \xrightarrow{d'} P'}{\varepsilon(d).P \xrightarrow{d+d'} P'} \quad \mathrm{DELAY\text{-}2}\ \frac{d' \le d}{\varepsilon(d).p \xrightarrow{d'} \varepsilon(d-d').P}$$

Graphically, you can render the derivation with the *tree notation* or with the following linear notation:

P
$\xrightarrow{\alpha} \{$
    justification of the step
    including sub derivations $d_i : Q_i \xrightarrow{\alpha'} Q'_i$
    $\ldots\}$
P'

d_1 :
Q_1
$\xrightarrow{\alpha} \{$
    justification of the step
    including sub derivations $d_i : R_i \xrightarrow{\alpha'} R'_i$
    $\ldots\}$
Q'_1

$\ldots$

Exercise 1.1 *Consider the following TCCS process definitions:*

$P_1 = c.\epsilon(2).\epsilon(1).\bar{b}.\mathbf{0}$
$P_2 = \epsilon(2).\bar{c}.\epsilon(1).b.d.\mathbf{0}$
$P_3 = (P_1 \mid P_2)\backslash\{b, c\}$
$P_4 = \epsilon(1).a.P_3 + \epsilon(6).\tau.P_4$

Show formally that P_4 can generate the timed trace $(2, a)(8, d)$ and the timed trace $(7, a)(12, d)$. Show formally that P_4 can not generate the timed trace $(1, a)(5, d)$.

Exercise 1.2 *Consider the following TCCS process definitions:*

$P_1 = \epsilon(3).a.\mathbf{0}$
$P_2 = \epsilon(1).b.\mathbf{0} + \epsilon(2).\bar{a}.\mathbf{0}$
$P_3 = (P_1 \mid P_2)\backslash\{b\}$

Show that P_3 can generate the timed trace $(4, a)$. Show that P_3 can not generate the timed trace $(1, a)$.

Exercise 1.3 *Consider the following TCCS process definitions:*

$P_1 = \varepsilon(1).a.\mathbf{0} + \varepsilon(2).b.\mathbf{0} + c.\mathbf{0}$
$P_2 = \varepsilon(2).\bar{a}.\mathbf{0} + \varepsilon(1).\bar{b}.\mathbf{0}$
$P_3 = (P_1 \mid P_2)\backslash\{a, b\}$

Determine if the following timed traces belong to the timed language of process P_3:

- $(1, c)$
- $(2, c)$
- $(3, c)$

Exercise 1.4 *Consider the following TCCS process definitions:*
$P_1 = b.a.c.\mathbf{0} + a.b.c.\mathbf{0}$
$P_2 = \varepsilon(1).\bar{a}.\mathbf{0} \mid \varepsilon(2).\bar{b}.\mathbf{0}$
$P_3 = (P_1 \mid P_2)\backslash\{a, b\}$
Determine if the following timed traces belong to the timed language of process P_3:

- $(2, c)$
- $(3, c)$

Exercise 1.5 *Consider the following TCCS process definitions:*

$P_1 = \epsilon(4).a.d.P_1 + \epsilon(3).b.d.P_1 + \epsilon(2).c.d.P_1$
$P_2 = \bar{a}.P_2 + \bar{b}.P_2 + \bar{c}.P_2 + \epsilon(1).\tau.P_2$
$P_3 = (P_1 \mid P_2)\backslash\{a, b, c\}$

1. *Determine the first timestamp at which a d action can be seen.*
2. *Is it possible to have the timed trace $(3, d)(4, d)$?*

Justify your answers formally by making derivations using SOS rules. For CCS rules use the names of the book, while for TCCS rules about time passing use the following names: ACT-T *for action delays,* SUM-T *for non-deterministic choice delay,* COM-T *for parallel composition delay,* RES-T *for restriction delay,* REL-T *for relabeling delay,* CON-T *for process variable delay. Moreover:*

$$\text{ZERODELAY}\ \frac{P \xrightarrow{\alpha} P'}{\varepsilon(0).P \xrightarrow{\alpha} P'} \quad \text{DELAY-1}\ \frac{P \xrightarrow{d'} P'}{\varepsilon(d).p \xrightarrow{d+d'} P'} \quad \text{DELAY-2}\ \frac{d' \leq d}{\varepsilon(d).p \xrightarrow{d'} \varepsilon(d-d').P}$$

Exercise 1.6 *Let T be a timed transition system. Let us consider a labelled transition system T' where every time-delay action $d \in \mathbb{R}^{\geq 0}$ is replaced with the silent action τ. We now define that two states p and q from the timed transition system T are* time abstracted bisimilar *if and only if p and q are weakly bisimilar in T'.*

- *Is the notion of time abstracted bisimilarity equivalent to untimed bisimilarity?*
- *If yes, prove your claim. If no, give a counter example.*

Exercise 1.7 *Consider the following TCCS process definitions:*

$P_1 = \epsilon(1).a.b.P_1 + \epsilon(2).\tau.P_1$
$P_2 = \epsilon(2).\bar{a}.P_2 + \epsilon(3).\tau.c.P_2$
$P_3 = (P_1 \mid P_2)\backslash\{a\}$

1. **(3 points)** *Determine the lowest timestamp at which a b action can be seen.*
2. **(4 points)** *Is it possible to have the timed trace $(5, c)$?*

Justify your answers formally by making derivations using SOS rules. For CCS rules use the names of the book, while for TCCS rules about time passing use the following names: ACT-T *for action delays,* SUM-T *for non-deterministic choice delay,* COM-T *for parallel composition delay,* RES-T *for restriction delay,* REL-T *for relabeling delay,* CON-T *for process variable delay. Moreover:*

$$\text{ZERODELAY}\ \frac{P \xrightarrow{\alpha} P'}{\varepsilon(0).P \xrightarrow{\alpha} P'} \quad \text{DELAY-1}\ \frac{P \xrightarrow{d'} P'}{\varepsilon(d).p \xrightarrow{d+d'} P'} \quad \text{DELAY-2}\ \frac{d' \leq d}{\varepsilon(d).p \xrightarrow{d'} \varepsilon(d-d').P}$$

Solutions

Solution of Exercise 1.1

7.1

1) $\boxed{\xrightarrow{2}}$

$$
\text{CONT}\ \dfrac{\text{SUM}_T\ \dfrac{\text{DELAY}_1\ \dfrac{\text{ACT}_T\ \dfrac{a \in Act,\ d \in \mathbb{R}^{\geq 0}\ a \neq \tau}{a.P_3 \xrightarrow{1} a.P_3}}{\varepsilon(1).a.P_3 \xrightarrow{2} a.P_3} \qquad \text{DELAY}_2\ \dfrac{\dfrac{2 \leq 6}{\varepsilon(6).\tau.P_4 \xrightarrow{2} \varepsilon(4).\tau.P_4}}{\varepsilon(6).\tau.P_4 \xrightarrow{2} \varepsilon(4).\tau.P_4}}{\varepsilon(1).a.P_3 + \varepsilon(6).\tau.P_4 \xrightarrow{2} a.P_3 + \varepsilon(4).\tau.P_4}}{P_4 \xrightarrow{2} a.P_3 + \varepsilon(4).\tau.P_4}
$$

$\boxed{\xrightarrow{a}}$

$$
\text{SUM}_1\ \dfrac{\text{ACT}\ \dfrac{\times}{a.P_3 \xrightarrow{a} P_3}}{a.P_3 + \varepsilon(4).\tau.P_4 \xrightarrow{a} P_3}
$$

$(2, a)$ ok

$\boxed{\xrightarrow{2}}$

$$
P_3 \xrightarrow{2} (\overbrace{c.\varepsilon(2).\varepsilon(1).\bar{b}.0}^{P_1} \mid \bar{c}.\varepsilon(1).b.d.0)\setminus\{b,c\}
$$

$\boxed{\xrightarrow{\tau}}$

$$
\text{REST}_T\ \dfrac{\text{COM}_3\ \dfrac{\text{ACT}\ \dfrac{\times}{c.\varepsilon(2).\varepsilon(1).\bar{b}.0 \xrightarrow{c} \varepsilon(2).\varepsilon(1).\bar{b}.0} \qquad \text{ACT}\ \dfrac{\times}{\bar{c}.\varepsilon(1).b.d \xrightarrow{\bar{c}} \varepsilon(1).b.d}}{(c.\varepsilon(2).\varepsilon(1).\bar{b}.0 \mid \bar{c}.\varepsilon(1).b.d) \xrightarrow{\tau} (\varepsilon(2).\varepsilon(1).\bar{b}.0 \mid \varepsilon(1).b.d)}}{(c.\varepsilon(2).\varepsilon(1).\bar{b}.0 \mid \bar{c}.\varepsilon(1).b.d)\setminus\{b,c\} \xrightarrow{\tau} (\varepsilon(2).\varepsilon(1).\bar{b}.0 \mid \varepsilon(1).b.d)\setminus\{b,c\}}
$$

$\boxed{\xrightarrow{3}}$

$$
(\varepsilon(2).\varepsilon(1).\bar{b}.0 \mid \varepsilon(1).b.d.0)\setminus\{b,c\} \xrightarrow{3} (\varepsilon(0).\bar{b}.0 \mid b.d.0)\setminus\{b,c\}
$$

$\boxed{\xrightarrow{\tau}}$

$$
(\varepsilon(0).\bar{b}.0 \mid b.d.0)\setminus\{b,c\} \xrightarrow[\;]{\tau} (0 \mid d.0)\setminus\{b,c\} \qquad (b,\bar{b})
$$

$\boxed{\xrightarrow{1}}$ $(0 \mid d.0)\setminus\{b,c\} \xrightarrow{1} (0 \mid d.0)\setminus\{b,c\}$

$\boxed{\xrightarrow{d}}$

$(0 \mid d.0)\setminus\{b,c\} \xrightarrow{d} (0 \mid 0)\setminus\{b,c\}$ $\quad(8, d)$ ok

2) $\boxed{\xrightarrow{6}}$

$P_4 \xrightarrow{6} a.P_3 + \mathcal{E}(0).\tau.P_4$

$\boxed{\xrightarrow{\tau}}$

$a.P_3 + \mathcal{E}(0).\tau.P_4 \xrightarrow{\tau} P_4$

$\boxed{\xrightarrow{1}}$

$P_4 \xrightarrow{1} \mathcal{E}(0).a.P_3 + \mathcal{E}(5).\tau.P_4$

$\boxed{\xrightarrow{a}}$

$\mathcal{E}(0).a.P_3 + \mathcal{E}(5).\tau.P_4 \xrightarrow{a} P_3$ (7, a) ok

$\boxed{P_3 \xrightarrow{2} (P_1 | \overline{c}.\mathcal{E}(1).b.d.0)\backslash\{b,c\} \xrightarrow{\tau} (\mathcal{E}(2).\mathcal{E}(1).\overline{b}.0 | \mathcal{E}(1).b.d.0)\backslash\{b,c\}}$

$\boxed{\mathcal{E}(2).\mathcal{E}(1).\overline{b}.0 | \mathcal{E}(1).b.d.0 \backslash\{b,c\} \xrightarrow{3} (\overline{b}.0 | b.d.0)\backslash\{b,c\} \xrightarrow{\tau} (0 | d.0)\backslash\{b,c\}}$

$\boxed{\xrightarrow{d}}$

$(0 | d.0)\backslash\{b,c\} \xrightarrow{d} (0|0)\backslash\{b,c\}$ (12, d) ok

3)

$P_4 \xrightarrow{1} a.P_3 + \mathcal{E}(5).\tau.P_4 \xrightarrow{a} P_3$ (1, a) ok

IT IS IMPOSSIBLE TO HAVE A TIMED TRACE (5, d) BECAUSE THE FIRST TIMED TRACE THAT I CAN SHOWN A d IS (6, d)

Solution of Exercise 1.4

7.4 $\boxed{\xrightarrow{1}}$

$$\text{CON}_T\ \dfrac{\text{RES}_T\ \dfrac{\text{COM}_T\ \dfrac{\text{CON}_T\ \dfrac{\text{SUM}_T\ \dfrac{\text{DELAY}_1\ \dfrac{\text{ACT}_T\ \dfrac{b \in Act \quad d \in \mathbb{R}^{\geq 0} \quad b \neq \tau}{b.a.c.0 \xrightarrow{1} b.a.c.0}}{b.a.c.0 \xrightarrow{1} b.a.c.0} \quad \text{DELAY}_1\ \dfrac{\dfrac{a \in Act \quad d \in \mathbb{R}^{\geq 0} \quad a \neq \tau}{a.b.c.0 \xrightarrow{1} a.b.c.0}}{a.b.c.0 \xrightarrow{1} a.b.c.0}}{b.a.c.0 + a.b.c.0 \xrightarrow{1} b.a.c.0 + a.b.c.0}}{P_1 \xrightarrow{1} b.a.c.0 + a.b.c.0} \quad Nosynch(P_1, P_2, 1) \quad \text{CON}_T\ \dfrac{\text{COM}_T\ \dfrac{\text{DELAY}_2\ \dfrac{1 \leq 1}{\mathcal{E}(1).\bar{a}.0 \xrightarrow{1} \mathcal{E}(0).\bar{a}.0} \quad \text{DELAY}_2\ \dfrac{1 \leq 2}{\mathcal{E}(2).\bar{b}.0 \xrightarrow{1} \mathcal{E}(1).\bar{b}.0}}{\mathcal{E}(1).\bar{a}.0 \mid \mathcal{E}(2).\bar{b}.0 \xrightarrow{1} \mathcal{E}(0).\bar{a}.0 \mid \mathcal{E}(1).\bar{b}.0}}{P_2 \xrightarrow{1} \mathcal{E}(0).\bar{a}.0 \mid \mathcal{E}(1).\bar{b}.0}}{(P_1 \mid P_2) \xrightarrow{1} (b.a.c.0 + a.b.c.0) \mid (\mathcal{E}(0).\bar{a}.0 \mid \mathcal{E}(1).\bar{b}.0)}}{(P_1 \mid P_2) \setminus \{a,b\} \xrightarrow{1} (b.a.c.0 + a.b.c.0) \mid (\mathcal{E}(0).\bar{a}.0 \mid \mathcal{E}(1).\bar{b}.0) \setminus \{a,b\}}}{P_3 \xrightarrow{1} (b.a.c.0 + a.b.c.0) \mid (\mathcal{E}(0).\bar{a}.0 \mid \mathcal{E}(1).\bar{b}.0) \setminus \{a,b\}}$$

$\boxed{\xrightarrow{1}}$

$$\text{RES}_T\ \dfrac{\text{COM}_T\ \dfrac{\text{SUM}_T\ \dfrac{\text{DELAY}_1\ \dfrac{\text{ACT}_T\ \dfrac{b \in Act \quad d \in \mathbb{R}^{\geq 0} \quad b \neq \tau}{b.a.c.0 \xrightarrow{1} b.a.c.0}}{b.a.c.0 \xrightarrow{1} b.a.c.0} \quad \text{DELAY}_1\ \dfrac{\text{ACT}_T\ \dfrac{a \in Act \quad d \in \mathbb{R}^{\geq 0} \quad a \neq \tau}{a.b.c.0 \xrightarrow{1} a.b.c.0}}{a.b.c.0 \xrightarrow{1} a.b.c.0}}{b.a.c.0 + a.b.c.0 \xrightarrow{1} b.a.c.0 + a.b.c.0} \quad Nosynch(P_1, P_2, 1) \quad \text{COM}_T\ \dfrac{\text{DELAY}_1\ \dfrac{\text{ACT}_T\ \dfrac{\bar{a} \in Act \quad d \in \mathbb{R}^{\geq 0} \quad \bar{a} \neq \tau}{\bar{a}.0 \xrightarrow{1} \bar{a}.0}}{\mathcal{E}(0).\bar{a}.0 \xrightarrow{1} \bar{a}.0} \quad \text{DELAY}_2\ \dfrac{1 \leq 1}{\mathcal{E}(1).\bar{b}.0 \xrightarrow{1} \mathcal{E}(0).\bar{b}.0}}{\mathcal{E}(0).\bar{a}.0 \mid \mathcal{E}(1).\bar{b}.0 \xrightarrow{1} \bar{a}.0 \mid \mathcal{E}(0).\bar{b}.0}}{(b.a.c.0 + a.b.c.0) \mid (\mathcal{E}(0).\bar{a}.0 \mid \mathcal{E}(1).\bar{b}.0) \not\xrightarrow{1}}}{(b.a.c.0 + a.b.c.0) \mid (\mathcal{E}(0).\bar{a}.0 \mid \mathcal{E}(1).\bar{b}.0) \setminus \{a,b\} \not\xrightarrow{1}}$$

TRUE

FALSE

TRUE

CAN'T PASS 1 TIME STEP BECAUSE IT'S POSSIBLE A τ ON $\bar{a}, a$

$\boxed{\xrightarrow{\tau}}$

$$\text{RES}\ \dfrac{\text{COM}_3\ \dfrac{\text{SUM}_2\ \dfrac{\text{ACT}\ \dfrac{\times}{a.b.c.0 \xrightarrow{a} b.c.0}}{(b.a.c.0 + a.b.c.0) \xrightarrow{a} b.c.0} \quad \text{COM}_1\ \dfrac{\text{DELAY}_0\ \dfrac{\text{ACT}\ \dfrac{\times}{\bar{a}.0 \xrightarrow{\bar{a}} 0}}{\mathcal{E}(0).\bar{a}.0 \xrightarrow{\bar{a}} 0}}{(\mathcal{E}(0).\bar{a}.0 \mid \mathcal{E}(1).\bar{b}.0) \xrightarrow{\bar{a}} 0 \mid \mathcal{E}(1).\bar{b}.0}}{(b.a.c.0 + a.b.c.0) \mid (\mathcal{E}(0).\bar{a}.0 \mid \mathcal{E}(1).\bar{b}.0) \xrightarrow{\tau} (b.c.0 \mid (0 \mid \mathcal{E}(1).\bar{b}.0)}}{(b.a.c.0 + a.b.c.0) \mid (\mathcal{E}(0).\bar{a}.0 \mid \mathcal{E}(1).\bar{b}.0) \setminus \{a,b\} \xrightarrow{\tau} (b.c.0 \mid (0 \mid \mathcal{E}(1).\bar{b}.0)}$$

NOW CAN PASS 1 TIME STEP $\qquad$ τ ON $b, \bar{b}$

$$b.c.0 \mid (0 \mid \mathcal{E}(1).\bar{b}.0) \xrightarrow{1} b.c.0 \mid (0 \mid \mathcal{E}(0).\bar{b}.0) \xrightarrow{\tau} c.0 \mid 0 \mid 0$$

NOW WE HAVE $(2, c)$ AND CAN PASS 1 MORE TIME STEP FOR $(3, c)$

Solution of Exercise 1.5

~~The first state~~

$P_3 \stackrel{d}{\not\to}$ and $P_3 \stackrel{\tau}{\not\to}$ so we have to let time pass.

No more than 1 time unit can pass because after 1 time unit a τ action becomes enabled. The following is the derivation tree of the $\xrightarrow{1}$ step.

EX-1

$$\frac{1 \le 4}{\varepsilon(4).a.d.P_1 \xrightarrow{1} \varepsilon(3).a.d.P_1}\ \text{DELAY-2} \qquad \frac{1 \le 3}{\varepsilon(3).b.d.P_1 \xrightarrow{1} \varepsilon(2).b.d.P_1}\ \text{DELAY-2} \qquad \frac{1 \le 2}{\varepsilon(2).c.d.P_1 \xrightarrow{1} \varepsilon(1).c.d.P_1}\ \text{DELAY-2}$$

$$\text{CON}\ \ \varepsilon(4).a.d.P_1 + \varepsilon(3).b.d.P_1 + \varepsilon(2).c.d.P_1 \xrightarrow{1} \varepsilon(3).a.d.P_1 + \varepsilon(2).b.d.P_1 + \varepsilon(1).c.d.P_1$$

$$\text{COM-T}\ \ P_1 \xrightarrow{1} \varepsilon(3).a.d.P_1 + \varepsilon(2).b.d.P_1 + \varepsilon(1).c.d.P_1 \qquad \boxed{P_2 \xrightarrow{1} (*)\ \text{IN OTHER SHEET}} \quad \text{Nosynch(P1,P2,1)}$$

$$\text{RES}\ \ P_1 | P_2 \xrightarrow{1} \varepsilon(3).a.d.P_1 + \varepsilon(2).b.d.P_1 + \varepsilon(1).c.d.P_1 \mid \bar{a}.P_2 + \bar{b}.P_2 + \bar{c}.P_2 + \varepsilon(0).\tau.P_2$$

$$\text{CON}\ \ (P_1 | P_2) \setminus \{a,b,c\} \xrightarrow{1} (\ldots) \setminus \{a,b,c\}$$

$$P_3 \xrightarrow{1} (\varepsilon(3).a.d.P_1 + \varepsilon(2).b.d.P_1 + \varepsilon(1).c.d.P_1 \mid \bar{a}.P_2 + \bar{b}.P_2 + \bar{c}.P_2 + \varepsilon(0).\tau.P_2) \setminus \{a,b,c\}$$

EX-2

$$\frac{}{\bar{a}.P_2 \xrightarrow{1} \bar{a}.P_2}\ \text{ACT-T} \qquad \frac{}{\bar{b}.P_2 \xrightarrow{1} \bar{b}.P_2}\ \text{ACT-T} \qquad \frac{}{\bar{c}.P_2 \xrightarrow{1} \bar{c}.P_2}\ \text{ACT-T} \qquad \frac{1 \le 1}{\varepsilon(1).\tau.P_2 \xrightarrow{1} \varepsilon(0).\tau.P_2}\ \text{DELAY-2}$$

$$\text{SUM-T}\ \ \bar{a}.P_2 + \bar{b}.P_2 + \bar{c}.P_2 + \varepsilon(1).\tau.P_2 \xrightarrow{1} \bar{a}.P_2 + \bar{b}.P_2 + \bar{c}.P_2 + \varepsilon(0).\tau.P_2$$

$$\text{CON}\ \ (*)\ \ P_2 \xrightarrow{1} \bar{a}.P_2 + \bar{b}.P_2 + \bar{c}.P_2 + \varepsilon(0).\tau.P_2$$

Now, $(\mathcal{E}(3).a.d.P_1 + \mathcal{E}(2).b.d.P_1 + \mathcal{E}(1).c.d.P_1 \mid \bar{a}.P_2 + \bar{b}.P_2 + \bar{c}.P_2 + \mathcal{E}(0).\tau.P_2) \setminus \{a,b,c\}$

$\not\xrightarrow{1}$ Time can not pass because there is a τ enabled;

The following derivation tree shows the transition τ:

EX-3

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\tau.P_2 \xrightarrow{\tau} P_2}\text{ACT}}{\mathcal{E}(0).\tau.P_2 \xrightarrow{\tau} P_2}\text{ZERO D}}{\bar{a}.P_2 + \bar{b}.P_2 + \bar{c}.P_2 + \mathcal{E}(0).\tau.P_2 \xrightarrow{\tau} P_2}\text{SUM}}{(\ldots|\ldots) \xrightarrow{\tau} \mathcal{E}(3).a.d.P_1 + \mathcal{E}(2).b.d.P_1 + \mathcal{E}(1).c.d.P_1 \mid P_2}\text{COM2}}{(\mathcal{E}(3).a.d.P_1 + \mathcal{E}(2).b.d.P_1 + \mathcal{E}(1).c.d.P_1 \mid \bar{a}.P_2 + \bar{b}.P_2 + \bar{c}.P_2 + \mathcal{E}(0).\tau.P_2) \setminus \{a,b,c\} \xrightarrow{\tau} \ldots}\text{RES}
$$

$$\ldots (\mathcal{E}(3).a.d.P_1 + \mathcal{E}(2).b.d.P_1 + \mathcal{E}(1).c.d.P_1 \mid P_2) \setminus \{a,b,c\}$$

Now, we can let 1 more time unit pass. Note that no $\xrightarrow{d}$ is enabled at this point.
The derivation tree of the next $\xrightarrow{1}$ step is similar to the one already shown, thus we report only the derivative process:

$(\mathcal{E}(3).a.d.P_1 + \mathcal{E}(2).b.d.P_1 + \mathcal{E}(1).c.d.P_1 \mid P_2) \setminus \{a,b,c\}$

$\xrightarrow{1}$

$(\mathcal{E}(2).a.d.P_1 + \mathcal{E}(1).b.d.P_1 + \mathcal{E}(0).c.d.P_1 \mid \bar{a}.P_2 + \bar{b}.P_2 + \bar{c}.P_2 + \mathcal{E}(0).\tau.P_2) \setminus \{a,b,c\} \equiv P_4$

Note that in P_4 there is again a τ enabled, thus time cannot pass.
However, in P_4 there are two ways of producing a τ: one is by the time-out $\mathcal{E}(0).\tau.P_2$, the other is by an internal communication on channel c.
We select such a communication:

EX-4

Ex-5

$$\dfrac{\dfrac{\dfrac{\dfrac{}{c.d.P_2 \xrightarrow{c} d.P_2}\,\text{ACT}}{\varepsilon(0).c.d.P_2 \xrightarrow{c} d.P_2}\,\text{ZERO}}{\varepsilon(2).a.d.P_1+\varepsilon(1).b.d.P_2+\varepsilon(0).c.d.P_1 \xrightarrow{c} d.P_1}\,\text{SUM} \qquad \dfrac{\dfrac{}{\bar{c}.P_2 \xrightarrow{\bar{c}} P_2}\,\text{ACT}}{\bar{a}.P_2+\bar{b}.P_2+\bar{c}.P_2+\varepsilon(0).\tau.P_2 \xrightarrow{\bar{c}} P_2}\,\text{SUM}}{(\ldots\,|\,\ldots) \xrightarrow{\tau} (d.P_2\,|\,P_2)}\,\text{COM3}$$

$$\dfrac{(\ldots\,|\,\ldots) \xrightarrow{\tau} (d.P_2\,|\,P_2)}{(\varepsilon(2).a.d.P_1+\varepsilon(2).b.d.P_2+\varepsilon(0).c.d.P_1\,|\,\bar{a}.P_2+\bar{b}.P_2+\bar{c}.P_2+\varepsilon(0).\tau.P_2)\backslash\{a,b,c\} \xrightarrow{\tau} (d.P_1\,|\,P_2)\backslash\{a,b,c\}}\,\text{RES}$$

Finally, the process $(d.P_1|P_2)\backslash\{a,b,c\}$ is able to produce a $\underline{d}$

$$(d.P_1|P_2)\backslash\{a,b,c\} \xrightarrow{d} (P_1|P_2)\backslash\{a,b,c\}$$

Summing up the two $\xrightarrow{1}$ delays elapsed so far, we can produce the timed trace $(2,d)$. The first time at which d can be seen is $\underline{2}$.

To answer to the second question, we can start again in the process

$$(d.P_2|P_2)\backslash\{a,b,c\}$$

Such a process can delay 1 time unit more:

Ex-6

SEE OTHER SHEET

$$\dfrac{\dfrac{}{d.P_2 \xrightarrow{1} d.P_1}\,\text{ACT-T} \qquad \dfrac{(*)}{P_2 \xrightarrow{1} \bar{a}.P_2+\bar{b}.P_2+\bar{c}.P_2+\varepsilon(0).\tau.P_2} \quad \text{NoSynch(d.P1,P2,1)}}{\dfrac{d.P_1|P_2 \xrightarrow{1} d.P_1\,|\,\bar{a}.P_2+\bar{b}.P_2+\bar{c}.P_2+\varepsilon(0).\tau.P_2}{(d.P_1|P_2)\backslash\{a,b,c\} \xrightarrow{1} (d.P_1\,|\,\bar{a}.P_2+\bar{b}.P_2+\bar{c}.P_2+\varepsilon(0).\tau.P_2)\backslash\{a,b,c\} \equiv P_5}\,\text{RES}}\,\text{COM-T}$$

In process P_5 a d can be seen, to produce the timed trace $(3, d)$

$$P_5 \xrightarrow{d} (P_1 \mid \bar{a}.P_2 + \bar{b}.P_2 + \bar{c}.P_2 + \varepsilon(0).\tau.P_2)\backslash\{a,b,c\} \equiv P_6$$

Now, as we shown in the previous steps, P_6 must produce a τ before time goes on

$$P_6 \xrightarrow{\tau} (P_1 \mid P_2)\backslash\{a,b,c\} \equiv P_3$$

We already know that from P_3 at least 2 time units have to pass before another d can be seen, therefore it is not possible to produce the timed trace $(3,d)(4,d)$.

One may wonder ~~whe~~ whether is possible to produce such a trace if in process P_4 we select the time-out τ instead of the communication τ.
The effect ~~wo~~ would be the same: there is a ~~p~~ the possibility to see a d at time 3, but then the process starts back in $(P_1 \mid \cdots)\backslash\{a,b,c\}$ and from this process ~~the~~ to see another d one must wait at least 2 more time units.

EX-7

Solution of Exercise 1.6

Let T be a timed transition system. Let us consider a labelled transition system T' where every time-delay action $d \in \mathbb{R}^{\geq 0}$ is replaced with the silent action τ. We now define that two states p and q from the timed transition system T are *time abstracted bisimilar* if and only if p and q are weakly bisimilar in T'.

- Is the notion of time abstracted bisimilarity equivalent to untimed bisimilarity?
  - No, see next bullet.
- If yes, prove your claim. If no, give a counter example.
  - A counter example is the following timed transition system

$$\begin{array}{ccc} A & \qquad & B \\ \downarrow a & & \downarrow 3 \\ A' & & B' \\ & & \downarrow a \\ & & B'' \end{array}$$

Now the initial states are time abstracted bisimilar since they are weakly bisimilar in the following labelled transition system:

$$
\begin{array}{ccc}
A & \quad & B \\
\downarrow a & & \downarrow \tau \\
A' & & B' \\
 & & \downarrow a \\
 & & B''
\end{array}
$$

On the other hand they can not be untimed bisimilar since $A \xrightarrow{a} A'$, but $B \stackrel{a}{\not\rightarrow}$.

Solution of Exercise 1.7

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After two time units both a synchronization between a and $\bar{a}$ and an internal τ become enabled:

$$\frac{\text{ACT-T} \ \times}{a.b.P_1 \xrightarrow{1} a.b.P_1} \ \text{DELAY-1}$$

$$\frac{}{\varepsilon(1).a.b.P_1 \xrightarrow{2} a.b.P_1} \ \text{SUM-T} \qquad \frac{\text{DELAY-2} \ \times}{\varepsilon(2).\tau.P_1 \xrightarrow{2} \varepsilon(0).\tau.P_1}$$

$$\frac{}{\varepsilon(1).a.b.P_1 + \varepsilon(2).\tau.P_1 \xrightarrow{2} a.b.P_1 + \varepsilon(0).\tau.P_1} \ \text{CON}$$

$$\frac{\text{DELAY-2} \ \times}{\varepsilon(2).\bar{a}.P_2 \xrightarrow{2} \varepsilon(0).\bar{a}.P_2} \ \text{SUM-T} \qquad \frac{\text{DELAY2} \ \times}{\varepsilon(3).\tau.c.P_2 \xrightarrow{2} \varepsilon(1).\tau.c.P_2}$$

$$\frac{}{\varepsilon(2).\bar{a}.P_2 + \varepsilon(3).\tau.c.P_2 \xrightarrow{2} \varepsilon(0).\bar{a}.P_2 + \varepsilon(1).\tau.c.P_2} \ \text{CON}$$

$$\text{COM-T} \quad P_1 \xrightarrow{2} a.b.P_1 + \varepsilon(0).\tau.P_1 \qquad (\text{No Synch}(P_1, P_2, 2)) \qquad P_2 \xrightarrow{2} \varepsilon(0).\bar{a}.P_2 + \varepsilon(1).\tau.c.P_2$$

$$\text{RES-T} \quad P_1 \mid P_2 \xrightarrow{2} a.b.P_1 + \varepsilon(0).\tau.P_1 \mid \varepsilon(0).\bar{a}.P_2 + \varepsilon(1).\tau.c.P_2$$

$$\text{CON} \quad (P_1 \mid P_2)\backslash\{a\} \xrightarrow{2} (a.b.P_1 + \varepsilon(0).\tau.P_1 \mid \varepsilon(0).\bar{a}.P_2 + \varepsilon(1).\tau.c.P_2)\backslash\{a\}$$

$$P_3 \xrightarrow{2} (a.b.P_1 + \varepsilon(0).\tau.P_1 \mid \varepsilon(0).\bar{a}.P_2 + \varepsilon(1).\tau.c.P_2)\backslash\{a\}$$

The best strategy to see a b as soon as possible is to make a synchronize with $\bar{a}$ and then b becomes immediately enabled:

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$$\dfrac{\dfrac{\dfrac{\text{ACT}\ \checkmark}{a.b.P_1 \xrightarrow{a} b.P_1}\ \text{SUM}}{a.b.P_1+\varepsilon(0).\tau.P_1 \xrightarrow{a} b.P_1} \qquad \dfrac{\dfrac{\dfrac{\text{ACT}\ \checkmark}{\bar a.P_2 \xrightarrow{\bar a} P_2}\ \text{ZERO}}{\varepsilon(0).\bar a.P_2 \xrightarrow{\bar a} P_2}\ \text{SUM}}{\varepsilon(0).\bar a.P_2+\varepsilon(1).\tau.c.P_2 \xrightarrow{\bar a} P_2}}{a.b.P_1+\varepsilon(0).\tau.P_1 \mid \varepsilon(0).\bar a.P_2+\varepsilon(1).\tau.c.P_2 \xrightarrow{\tau} b.P_1 \mid P_2}\ \text{COM-3}$$

RES

$P' = (a.b.P_1+\varepsilon(0).\tau.P_1 \mid \varepsilon(0).\bar a.P_2+\varepsilon(1).\tau.c.P_2)\setminus\{a\} \xrightarrow{\tau} (b.P_1 \mid P_2)\setminus\{a\}$

Now $(b.P_1 \mid P_2)\setminus\{a\} \xrightarrow{b} (P_1 \mid P_2)\setminus\{a\} = P_3$. So, the first time at which b can be seen is 2.

2) Note that we have shown in the previous point that $P_3 \xrightarrow{2} P'$. To see the trace $(5, c)$ we can not perfom the b, so we have to chose to make the internal τ:

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$P_3 \xrightarrow{2} P_2' \xrightarrow{\tau} (P_1 \mid (\varepsilon(0).\bar a.P_2+\varepsilon(1).\tau.c.P_2))\setminus\{a\} = P''$ The global time is still 2

IF we make P'' wait another time unit we get

$P'' \xrightarrow{1} (\varepsilon(0).a.b.P_1+\varepsilon(1).\tau.P_1 \mid (\bar a.P_2+\varepsilon(0).\tau.c.P_2))\setminus\{a\} = P'''$
(**)

Note that in P''' a synchronization between $\bar a$ and a is enabled and an internal τ is enabled too (**). After this second τ a c becames enabled,

$P''' \xrightarrow{\tau} (\varepsilon(0).a.b.P_1+\varepsilon(1).\tau.P_1 \mid c.P_2)\setminus\{a\} = P^{(IV)}$ Now the global time is 3 and the c is enabled. We can let another time unit pass:

$P^{(IV)} \xrightarrow{1} (a.b.P_1+\varepsilon(0).\tau.P_1 \mid c.P_2)\setminus\{a\} = P^{(V)}$ The global time is 4.

In $P^{(V)}$ there is a τ enabled, thus time cannot progress without doing it.

$P^{(V)} \xrightarrow{\tau} (P_1 \mid c.P_2)\setminus\{a\}$. Finally $(P_1 \mid c.P_2)\setminus\{a\} \xrightarrow{1} (\varepsilon(0).a.b.P_1+\varepsilon(1).\tau.P_1 \mid c.P_2)\setminus\{a\}$
The time stamp of c is 5 $\boxed{\text{yes}}$ $\xrightarrow{c} (\varepsilon(0).a.b.P_1+\varepsilon(1).\tau.P_1 \mid P_2)\setminus\{a\}$