RPSV 16|17

2/2/2017

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σ-ALGEBRA (Ω, X)

Ω insieme OUTCOMES

$X \subseteq 2^{\Omega}$ insieme degli EVENTI

$\emptyset \in X$

$\Omega \in X$

$E = \{x_1, x_2, x_3\} \in X$

$E' = \{x_1\}$

1) Chiusura rispetto alla unione infinita numerabile

$$\bigcup_{i \geq 0} E_i \in X$$

2) Chiusura rispetto al complemento

$$E \in X \Rightarrow \bar{E} = \Omega - E \in X$$

(Ω, X)

$Pr : X \to [0,1]$

$Pr(\bar{E}) = 1 - Pr(E)$

$Pr\left(\bigcup_{i \in I} E_i\right) = \sum_{i \in I} Pr(E_i)$ dove E_i eventi disgiunti

cioè $E_i \cap E_j = \emptyset$ $\forall i, j \in I,\ i \neq j$

DTMC $M = \langle S, i, P, AP, L \rangle$

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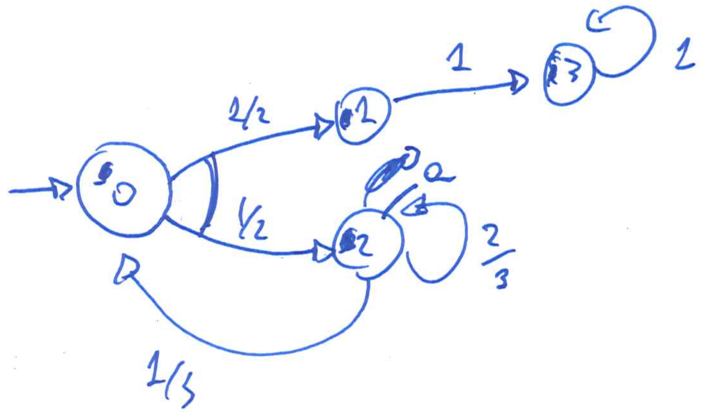

$\pi : \; s_0 \, s_1 \, s_2 \, \dots$

t.c. $\quad P(s_i, s_{i+1}) > 0 \quad \forall i \geq 0$

$i(s_0) > 0$

0 1 3 3 3 ...

0 2 0 2 2 2 0 2 0 1 3 3 3

$s \qquad Paths(s) = \{ \pi \mid \pi = s \dots \}$

$$\boxed{\Omega = \bigcup_{s_0 : i(s_0) > 0} Paths(s_0)}$$

~~$X = \{ Paths_{fin}(s_0) \mid \ldots \}$~~

$X = \{ S \in 2^{\Omega} \mid S = Cyl(\hat{\pi}), \hat{\pi} \in Paths_{fin}(M) \}$

$\underbrace{s_0\, s_1 \ldots s_n}$ prefix dw

$\hat{\pi} = 0\,2\,0$

$Cyl(\hat{\pi}) = \{ \pi \in Paths(M) \mid \hat{\pi} \text{ is a prefix of } \pi \}$

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$Cyl(0\,2\,0) = \{ 0\,2\,0 \mid 2\,2\,0\,1\,3\,3\,3\,3 \ldots ;$
$0\,2\,0 \mid 1\,3\,3 \ldots$
$0\,2\,0$
$0\,2\,0$
$\vdots$
$\}$

$Cyl(0\,2\,0) \subseteq Cyl(0\,2)$

$Cyl(0\,2) = Cyl(0\,2\,2) \cup Cyl(0\,2\,0)$

$Cyl(0\,1\,3) = \{ 0\,1\,3\,3\,3 \ldots \}$

$$Pr(Cyl(\hat{\pi})) = Pr(Cyl(s_0 \dots s_n)) = i(s_0) \cdot P(s_0, s_1) \cdot P(s_1, s_2) \dots$$

$$\dots P(s_{n-1}, s_n)$$

$$P = \begin{pmatrix} \cdot & - & - \\ \cdot & - & \\ \cdot & \_\_ & \\ \cdot & - & \end{pmatrix}$$

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$$Pr(Cyl(020)) = 1 \cdot P(0,2) \cdot P(2,0) =$$

$$i(0) = 1 \qquad = 1 \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

PCTL

$$\Phi ::= \mathrm{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid \mathbb{P}_J(\varphi)$$

$\exists \varphi \quad \mathbb{P}_{>0}(\varphi)$

$a \in AP$ $\qquad J \subseteq [0,1]$ $\qquad J$ intervallo

CTL $\not\subseteq$ PCTL

$$\varphi := \bigcirc \Phi \mid \Phi_1 \, U \, \Phi_2 \mid \Phi_1 \, U^{\leq n} \, \Phi_2$$

$\forall \Diamond \Phi$

$\neq$

$\mathbb{P}_{\geq 1}(\Diamond \Phi)$

$$\mathbb{P}_J(\Diamond \Phi) = \mathbb{P}_J(\mathrm{true} \, U \, \Phi)$$

$$\mathbb{P}_J(\Box^{\leq n} \Phi) = \mathbb{P}_J(\neg \Diamond^{\leq n} \neg \Phi) = \mathbb{P}_J(\neg(\mathrm{true} \, U^{\leq n} \neg \Phi))$$

$M = \langle S, i, P, AP, L \rangle$

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$s \models true$

$s \models a$ iff. $a \in L(s)$

$s \models \Phi_1 \wedge \Phi_2$ iff $s \models \Phi_1$ and $s \models \Phi_2$

$s \models \neg \Phi$ iff $s \not\models \Phi$

$s \models \mathbb{P}_J(\varphi)$ iff: $Pr\Big(\underbrace{\{\pi \models \varphi\}}_{\subseteq\, 2^{\Omega} \in X}\Big) \in J$

$\pi[i] \models \Phi_1$

$\pi \models \bigcirc \Phi$ iff $\pi[1] \models \Phi$

$\pi \models \Phi_1 \cup \Phi_2$ iff. $\exists j \geq 0 : \pi[j] \models \Phi_2 \wedge \forall i : 0 \leq i \leq j$

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$\pi \models \Phi_1 \, U^{\leq m} \, \Phi_2$ iff $\exists j \leq m : \ldots\ldots$

$\mathbb{P}_{\geq 0.6}(\bigcirc a)$ iff $\mathbb{P}r(\underbrace{\pi \models \bigcirc a}_{= Cyl(O2)}) \in J$

$\downarrow$

$J = [0.6, 1]$

$Cyl(O\,1)$

$Pr(Cyl(O\,2)) = 1 \cdot \frac{1}{2} = 0.5$

$P_{=?}$