CTL vs LTL

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Topics

- Definition of equivalence of two CTL and LTL formulas.
- CTL formulas that cannot be expressed in LTL
- LTL formulas that cannot be expressed in CTL
- Examples and exercises

Material

Reading:

Chapter 6 of the book: Section 6.3

More:

The slides in the following pages are taken from the material of the course "Introduction to Model Checking" held by Prof. Dr. Ir. Joost-Pieter Katoen at Aachen University.

Overview

Equivalence of CTL and LTL formulas

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Let Φ be a **CTL** formula and φ an **LTL** formula.

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e.g.,

CTL formula Φ	LTL formula φ
a	a
$\forall \bigcirc a$	$\bigcirc a$
$\forall (a \, \mathsf{U} \, b)$	$a \, \mathsf{U} \, b$

$a, b \in AP$

More examples

CTL formula Φ	LTL formula φ
a	a
$\forall \bigcirc a$	$\bigcirc a$
$\forall (a \, \mathsf{U} \, b)$	$a \, \mathsf{U} \, b$
$\forall \Box a$	$\Box a$
$\forall \Diamond a$	$\Diamond a$

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$\forall \Diamond a$	$\Diamond a$
$\forall(a \, \mathsf{W} \, b)$	$a \, \mathsf{W} \, b$

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$\forall \Box \forall \Diamond a$	$\Box \Diamond a$

More examples

CTL formula Φ	LTL formula φ
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$\forall(a \, \mathsf{U} \, b)$	$a \, \mathsf{U} \, b$
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$\forall \Diamond a$	$\Diamond a$
$\forall(a \, \mathsf{W} \, b)$	$a \, \mathsf{W} \, b$
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$\nwarrow$ $\nearrow$

infinitely often a

More examples

CTL formula Φ	LTL formula φ
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$\forall\Box a$	$\Box a$
$\forall\Diamond a$	$\Diamond a$
$\forall(a \,\mathsf{W}\, b)$	$a \,\mathsf{W}\, b$
$\forall\Box\forall\Diamond a$	$\Box\Diamond a$

but: $\forall\Diamond\forall\Box a \not\equiv \Diamond\Box a$

infinitely often a (↖ $\forall\Box\forall\Diamond a$, ↗ $\Box\Diamond a$)

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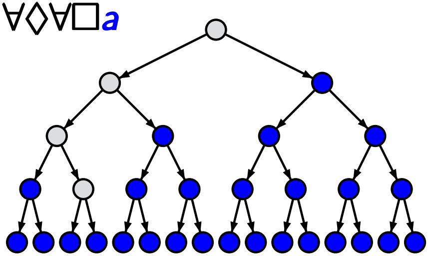

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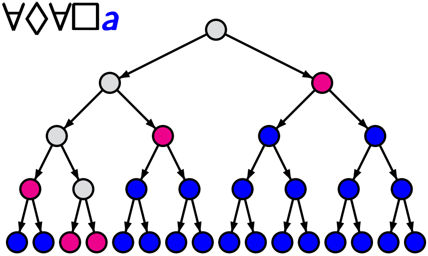

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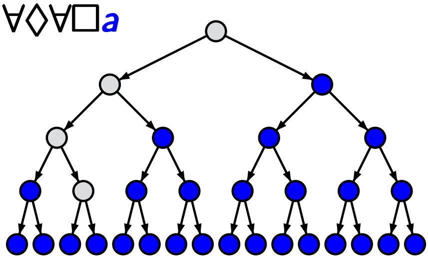
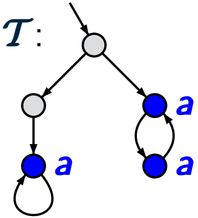




$\mathcal{T} \models \forall\Diamond\forall\Box a$

$\Diamond\Box a \not\equiv \forall\Diamond\forall\Box a$

To prove that

$$\forall\Diamond\forall\Box a \;\not\equiv\; \Diamond\Box a$$

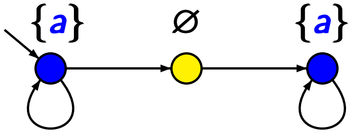
we provide an example for a TS $\mathcal{T}$ s.t.

$$\mathcal{T} \models_{\text{LTL}} \Diamond\Box a$$

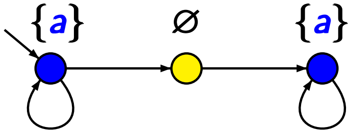
$$\mathcal{T} \not\models_{\text{CTL}} \forall\Diamond\forall\Box a$$

$\Diamond\Box a \not\equiv \forall\Diamond\forall\Box a$

transition system $\mathcal{T}$



transition system $\mathcal{T}$



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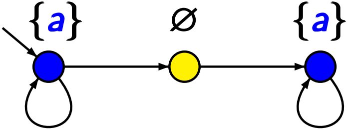

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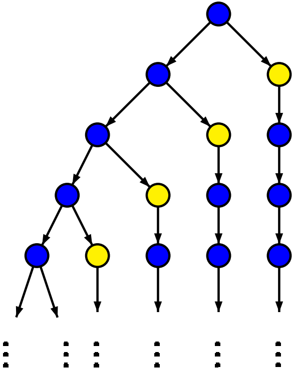
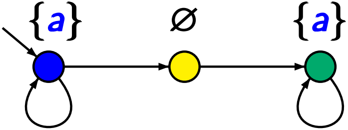
computation tree



transition system $\mathcal{T}$

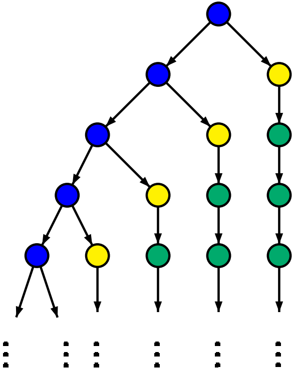


$\mathcal{T} \models_{\mathrm{LTL}} \Diamond\Box a$

$\mathcal{T} \not\models_{\mathrm{CTL}} \forall\Diamond\forall\Box a$

$Sat(\forall\Box a) = \{\bullet\}$

computation tree



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For each **CTL** formula Φ the following holds:

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- or ...

without proof

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For each **CTL** formula Φ the following holds:

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$$\downarrow$$

$$\varphi = \Box\Diamond a$$

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without proof

$\Phi = \forall\Box\forall\Diamond a$

$\downarrow$

$\varphi = \Box\Diamond a \equiv \Phi$ "infinitely often a"

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$\Diamond(a \wedge \bigcirc a) \not\equiv \forall\Diamond(a \wedge \forall\bigcirc a)$

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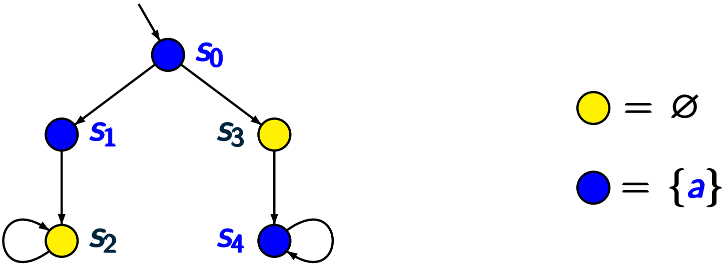

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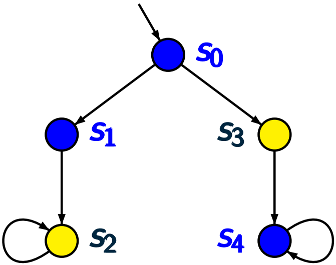
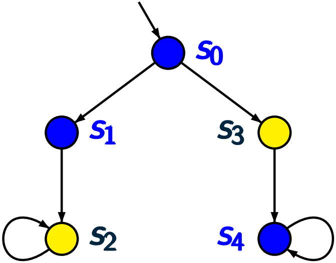


$\bigcirc = \varnothing$ (yellow)

$\bigcirc = \{a\}$ (blue)

$\mathcal{T} \models_{\mathsf{LTL}} \Diamond(a \wedge \bigcirc a)$

$\Diamond(a \wedge \bigcirc a) \not\equiv \forall\Diamond(a \wedge \forall\bigcirc a)$

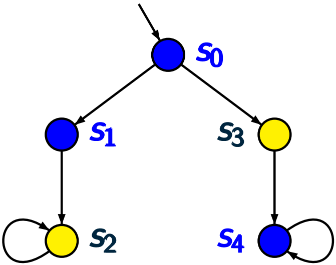


$\mathcal{T} \models_{\mathsf{LTL}} \Diamond(a \wedge \bigcirc a)$ $\longleftarrow$

$$trace(s_0\, s_1\, s_2^{\omega}) = \{a\}\,\{a\}\,\varnothing^{\omega}$$

$$trace(s_0\, s_3\, s_4^{\omega}) = \{a\}\,\varnothing\,\{a\}^{\omega}$$

$\Diamond(a \wedge \bigcirc a) \not\equiv \forall\Diamond(a \wedge \forall\bigcirc a)$



$\bigcirc$ (yellow) $= \varnothing$

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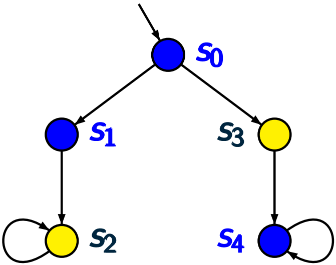

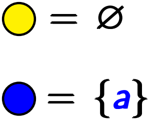

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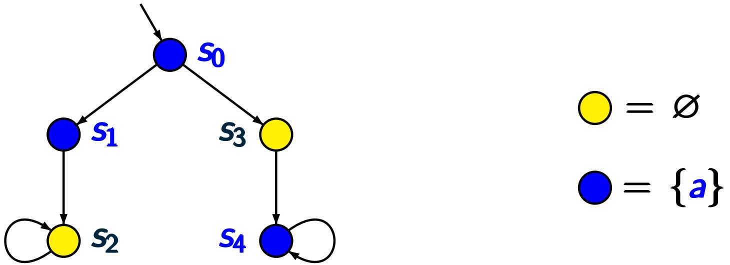
$$\mathcal{T} \models_{\text{LTL}} \Diamond(a \wedge \bigcirc a) \longleftarrow$$

$$Sat(a \wedge \forall\bigcirc a) = \{s_4\}$$

$$\mathcal{T} \not\models_{\text{CTL}} \forall\Diamond(a \wedge \forall\bigcirc a) \longleftarrow$$

$\Diamond(a \wedge \bigcirc a) \not\equiv \forall\Diamond(a \wedge \forall\bigcirc a)$



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$$s_0\, s_1\, s_2^{\omega} \not\models_{\mathrm{CTL}} \Diamond(a \wedge \forall\bigcirc a)$$

$$\mathcal{T} \not\models_{\mathrm{CTL}} \forall\Diamond(a \wedge \forall\bigcirc a) \longleftarrow$$

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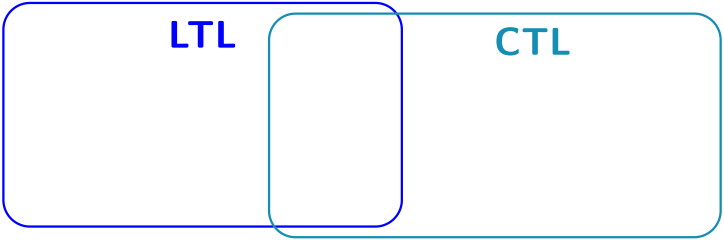

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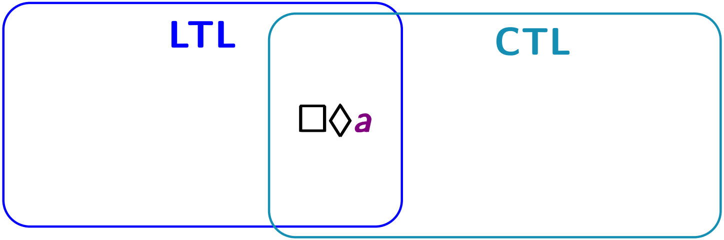

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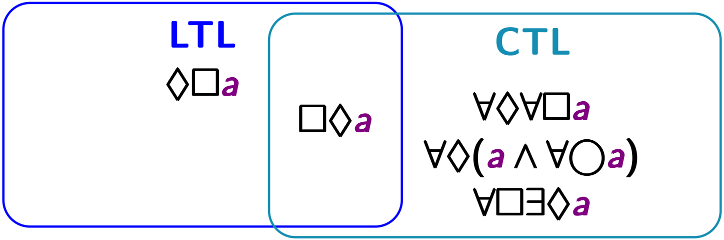

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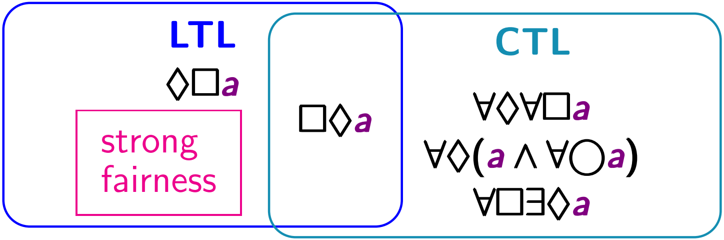
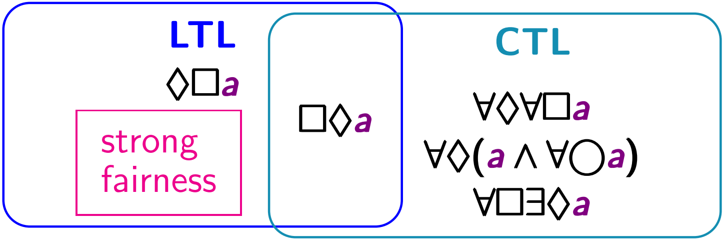


Expressiveness of LTL and CTL

The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas $\forall\lozenge(a \wedge \forall\bigcirc a)$, $\forall\lozenge\forall\square a$ and $\forall\square\exists\lozenge a$ have no equivalent LTL formula

- The **LTL** formula $\lozenge\square a$ has no equivalent CTL formula



CTL properties that are not LTL-definable

The **CTL** formulas

$$\forall\Diamond(a \wedge \forall\bigcirc a)$$

$$\forall\Diamond\forall\Box a$$

$$\forall\Box\exists\Diamond a$$

have no equivalent **LTL** formula

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Proof uses the fact that for each **CTL** formula Φ:

- either there is **no** equivalent **LTL** formula
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CTL properties that are not LTL-definable

The **CTL** formulas

$\forall\Diamond(a \wedge \forall\bigcirc a)$ ⟵ already considered

$\forall\Diamond\forall\Box a$ ⟵ already considered

$\forall\Box\exists\Diamond a$

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The **CTL** formulas

$$\forall\Diamond(a \wedge \forall\bigcirc a)$$

$$\forall\Diamond\forall\Box a$$

$$\forall\Box\exists\Diamond a \longleftarrow \text{alternative (direct) proof}$$

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There is no LTL formula equivalent to $\forall\Box\exists\Diamond a$

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suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall\Box\exists\Diamond a$

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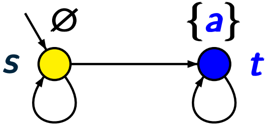
consider the following TS $\mathcal{T}_1$:



There is no LTL formula equivalent to $\forall\Box\exists\Diamond a$

suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall\Box\exists\Diamond a$

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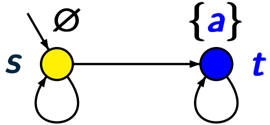

$Sat(\exists\Diamond a) = \{s, t\}$

There is no LTL formula equivalent to $\forall\Box\exists\Diamond a$

suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall\Box\exists\Diamond a$

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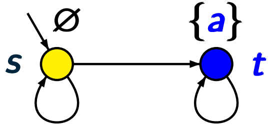

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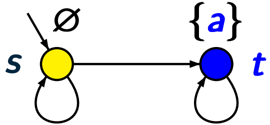

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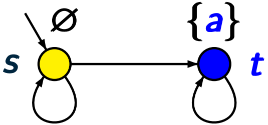

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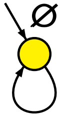

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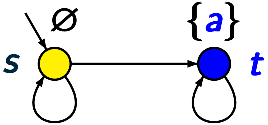

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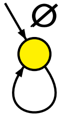

$Traces(\mathcal{T}_2) = \{\varnothing^\omega\}$

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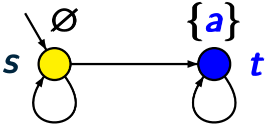

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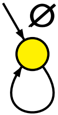

$$Traces(\mathcal{T}_2) = \{\varnothing^\omega\} \subseteq Traces(\mathcal{T}_1)$$

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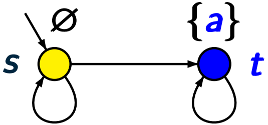

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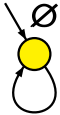

$$Traces(\mathcal{T}_2) = \{\varnothing^\omega\} \subseteq Traces(\mathcal{T}_1) \subseteq Words(\varphi)$$

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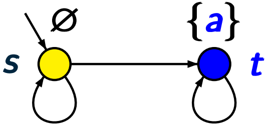

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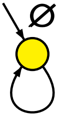
consider the following TS $\mathcal{T}_2$:



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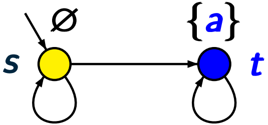

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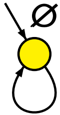

$Traces(\mathcal{T}_2) = \{\varnothing^\omega\} \subseteq Traces(\mathcal{T}_1) \subseteq Words(\varphi)$

Hence: $\mathcal{T}_2 \models \varphi$

$\implies \mathcal{T}_2 \models \forall\Box\exists\Diamond a$

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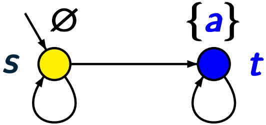

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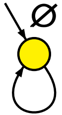


$$Traces(\mathcal{T}_2) = \{\varnothing^\omega\} \subseteq Traces(\mathcal{T}_1) \subseteq Words(\varphi)$$

Hence: $\mathcal{T}_2 \models \varphi$

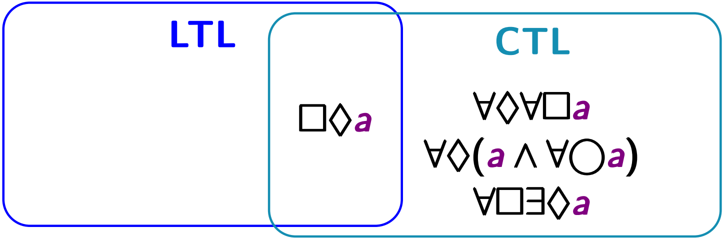
$\implies$ $\mathcal{T}_2 \models \forall\Box\exists\Diamond a$ contradiction !!

Expressiveness of LTL and CTL

The expressive powers of **LTL** and **CTL** are incomparable

> The **CTL** formulas $\forall\Diamond(a \wedge \forall\bigcirc a)$, $\forall\Diamond\forall\Box a$ and $\forall\Box\exists\Diamond a$ have no equivalent **LTL** formula

The **LTL** formula $\Diamond\Box a$ has no equivalent **CTL** formula



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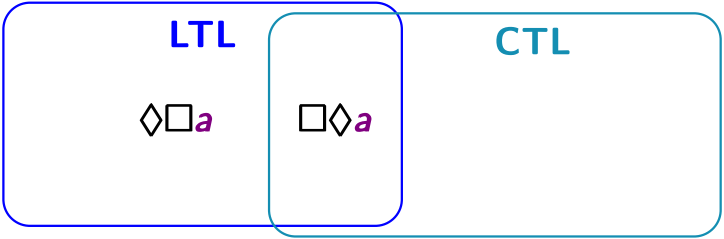

LTL formula $\Diamond\Box a$

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Proof (sketch): provide sequences $(\mathcal{T}_n)_{n\geq 0}$, $(\mathcal{T}'_n)_{n\geq 0}$ of transition systems such that for all $n \geq 0$:

(1) $\mathcal{T}_n \not\models \Diamond\Box a$

(2) $\mathcal{T}'_n \models \Diamond\Box a$

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(1) $\mathcal{T}_n \not\models \Diamond\Box a$

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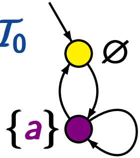
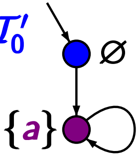
(3) $\mathcal{T}_n$ and $\mathcal{T}'_n$ satisfy the same **CTL** formulas length $\leq n$

Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$

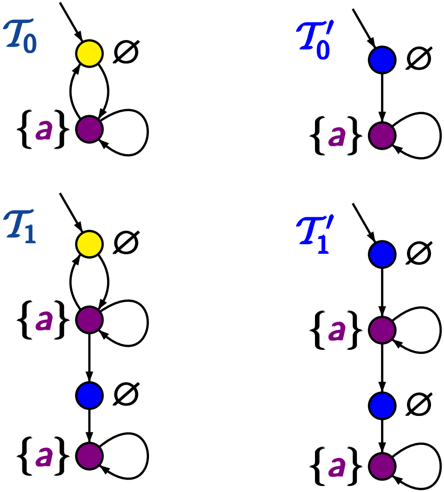
$\mathcal{T}_0$



$\mathcal{T}'_0$



Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$



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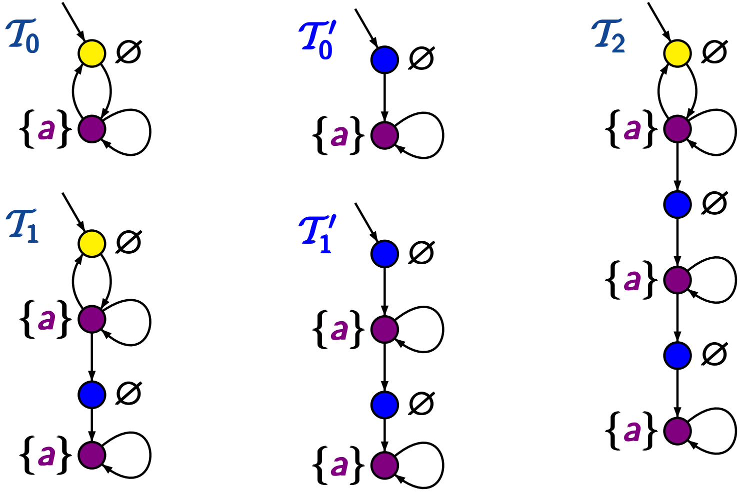

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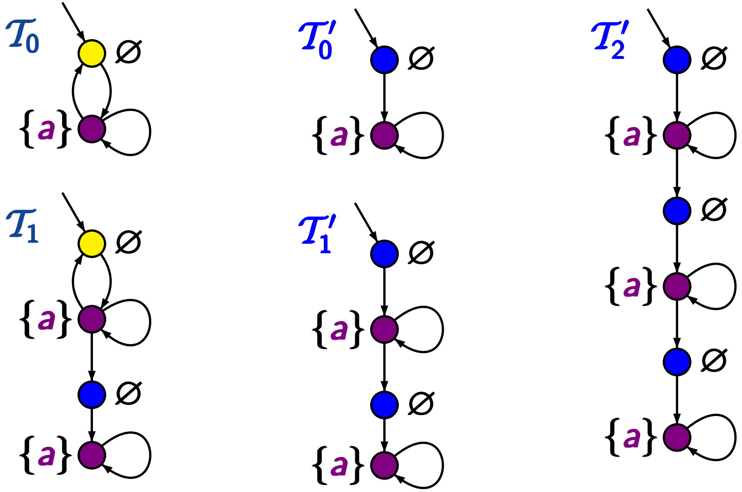

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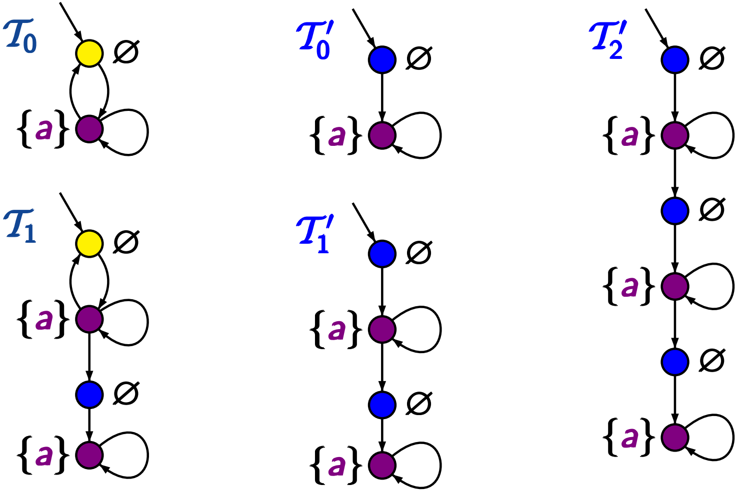
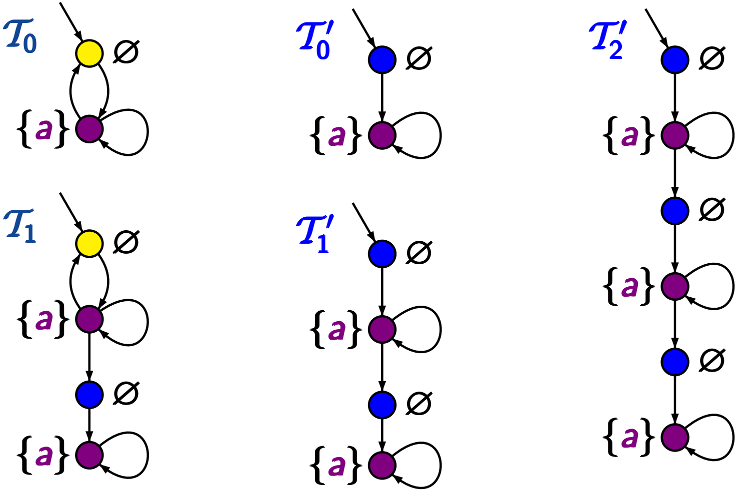


$$\mathcal{T}_n \not\models \Diamond\Box a$$

Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$



$\mathcal{T}_n \not\models \Diamond\Box a$ $\qquad$ $\mathcal{T}'_n \models \Diamond\Box a$

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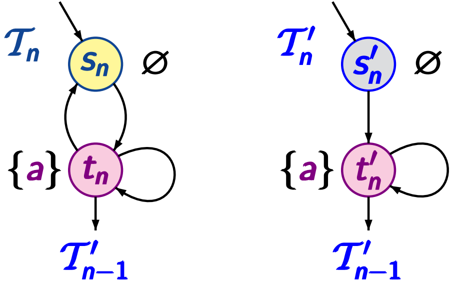

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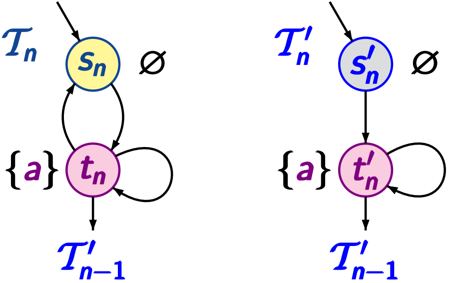

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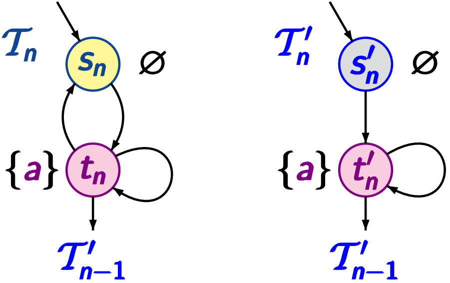

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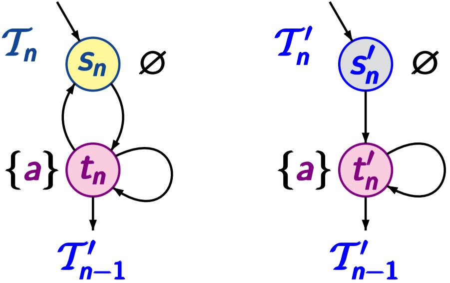
For all **CTL** formulas Φ of length $|\Phi| \leq n$:

$$s_n \models \Phi \quad \text{iff} \quad s'_n \models \Phi$$
$$t_n \models \Phi \quad \text{iff} \quad t'_n \models \Phi$$

Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$



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$s_n \models \Phi$ iff $s'_n \models \Phi$

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Hence: $\mathcal{T}_n$ and $\mathcal{T}'_n$ fulfill the same **CTL** formulas of length $\leq n$

CTL vs LTL

Does $\forall\Diamond(a \land \exists\bigcirc a) \equiv \Diamond(a \land \bigcirc a)$ hold ?

Does $\forall\Diamond(a \wedge \exists\bigcirc a) \equiv \Diamond(a \wedge \bigcirc a)$ hold ?

answer: **no.**

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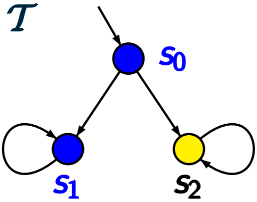

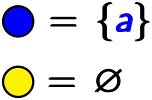

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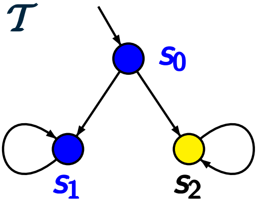

$\mathcal{T} \not\models \Diamond(a \wedge \bigcirc a)$

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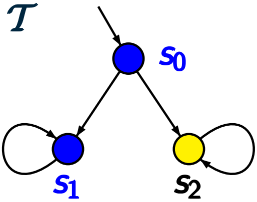


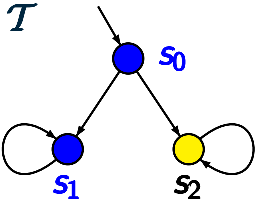
$\mathcal{T} \not\models \Diamond(a \wedge \bigcirc a)$

note: $\pi = s_0\, s_2\, s_2\, s_2 \ldots$ is a path in $\mathcal{T}$ with

$trace(\pi) = \{a\}\, \varnothing\, \varnothing\, \varnothing \ldots \notin Words(\Diamond(a \wedge \bigcirc a))$

Does $\forall\Diamond(a \wedge \exists\bigcirc a) \equiv \Diamond(a \wedge \bigcirc a)$ hold ?

answer: **no.**



$\mathcal{T} \not\models \Diamond(a \wedge \bigcirc a)$

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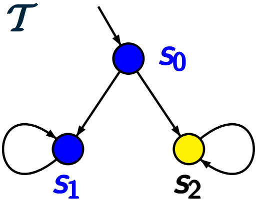

$\mathcal{T} \not\models \Diamond(a \wedge \bigcirc a)$

$\mathcal{T} \models \forall\Diamond(a \wedge \exists\bigcirc a)$

$$Sat(\exists\bigcirc a) = \{s_0, s_1\}$$
$$Sat(\forall\Diamond(a \wedge \exists\bigcirc a)) = \{s_0, s_1\}$$

Correct or wrong?

For each **NBA** $\mathcal{A}$ there is a **CTL** formula Φ such that for all transition systems $\mathcal{T}$:

$$\mathcal{T} \models \Phi \quad \text{iff} \quad \mathit{Traces}(\mathcal{T}) \subseteq \mathcal{L}_\omega(\mathcal{A})$$

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wrong. consider, e.g., an NBA $\mathcal{A}$ with

$$\mathcal{L}_\omega(\mathcal{A}) = Words(\Diamond\Box a)$$

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$$\mathcal{L}_\omega(\mathcal{A}) = Words(\Diamond\Box a)$$

But there is no CTL formula Φ such that $\Phi \equiv \Diamond\Box a$

Correct or wrong?

If Φ is **CTL** formula and φ an **LTL** formula such that $\Phi \equiv \varphi$ then $\neg\Phi \equiv \neg\varphi$

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$$\Phi = \forall\Box\forall\Diamond a, \quad \varphi = \Box\Diamond a$$

- $\Phi \equiv \varphi$
- there is no CTL formula that is equivalent to

$$\neg\varphi \equiv \Diamond\Box\neg a$$

Correct or wrong?

$s \models \exists\Box\exists\Diamond a$ iff there is a path $\pi \in \mathit{Paths}(s)$ with $\pi \models \Box\Diamond a$

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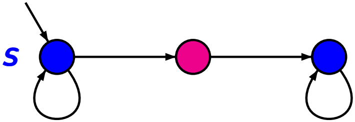

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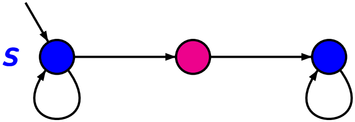

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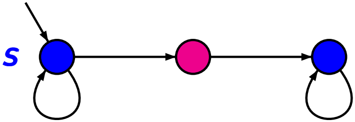
$s \models \exists\Box\exists\Diamond a$ iff there is a path $\pi \in \mathit{Paths}(s)$ with $\pi \models \Box\Diamond a$

wrong.

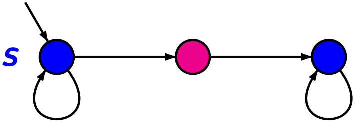


$s \models \exists\Box\exists\Diamond a$

note that: $s \models \exists\Diamond a$

$s \models \exists\Box\exists\Diamond a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Box\Diamond a$

wrong.



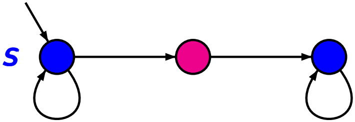
$s \models \exists\Box\exists\Diamond a$

note that: $s \models \exists\Diamond a$

thus: $s\,s\,s\,\ldots \models \Box\exists\Diamond a$

$s \models \exists\Box\exists\Diamond a$ iff there is a path $\pi \in \mathit{Paths}(s)$ with $\pi \models \Box\Diamond a$

wrong.



$s \models \exists\Box\exists\Diamond a$

note that: $s \models \exists\Diamond a$

thus: $s\,s\,s\ldots \models \Box\exists\Diamond a$

but there is no path where $\Box\Diamond a$ holds

Correct or wrong?

$s \models \exists\Box\exists\Diamond a$ iff there is a path $\pi \in \mathit{Paths}(s)$ with $\pi \models \Box\Diamond a$

wrong.

$s \models \exists\Diamond\exists\Box a$ iff there is a path $\pi \in \mathit{Paths}(s)$ with $\pi \models \Diamond\Box a$

Correct or wrong?

$s \models \exists\Box\exists\Diamond a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Box\Diamond a$

wrong.

$s \models \exists\Diamond\exists\Box a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Diamond\Box a$

correct.

Correct or wrong?

$s \models \exists\Box\exists\Diamond a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Box\Diamond a$

wrong.

$s \models \exists\Diamond\exists\Box a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Diamond\Box a$

correct.

$$\exists\Diamond\exists\Box a \equiv \neg\forall\Box\forall\Diamond\neg a$$

Correct or wrong?

$s \models \exists\Box\exists\Diamond a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Box\Diamond a$

wrong.

$s \models \exists\Diamond\exists\Box a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Diamond\Box a$

correct.

$$\exists\Diamond\exists\Box a \equiv \neg\forall\Box\forall\Diamond\neg a$$

$$s \models \exists\Diamond\exists\Box a$$

Correct or wrong?

$s \models \exists\Box\exists\Diamond a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Box\Diamond a$

wrong.

$s \models \exists\Diamond\exists\Box a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Diamond\Box a$

correct.

$$\exists\Diamond\exists\Box a \equiv \neg\forall\Box\forall\Diamond\neg a$$

$$s \models \exists\Diamond\exists\Box a \quad \text{iff} \quad s \not\models \forall\Box\forall\Diamond\neg a$$

Correct or wrong?

$s \models \exists\Box\exists\Diamond a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Box\Diamond a$

wrong.

$s \models \exists\Diamond\exists\Box a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Diamond\Box a$

correct.

$$\exists\Diamond\exists\Box a \equiv \neg\forall\Box\forall\Diamond\neg a$$

$$s \models \exists\Diamond\exists\Box a \quad \text{iff} \quad s \not\models \forall\Box\forall\Diamond\neg a$$

$$\text{iff} \quad s \not\models \Box\Diamond\neg a$$

$s \models \exists\Box\exists\Diamond a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Box\Diamond a$

wrong.

$s \models \exists\Diamond\exists\Box a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Diamond\Box a$

correct.

$$\exists\Diamond\exists\Box a \equiv \neg\forall\Box\forall\Diamond\neg a$$

$$s \models \exists\Diamond\exists\Box a \text{ iff } s \not\models \forall\Box\forall\Diamond\neg a$$

$$\text{iff } s \not\models \Box\Diamond\neg a \equiv \neg\Diamond\Box a$$

Correct or wrong?

$s \models \exists\Box\exists\Diamond a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Box\Diamond a$

wrong.

$s \models \exists\Diamond\exists\Box a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Diamond\Box a$

correct.

$$\exists\Diamond\exists\Box a \equiv \neg\forall\Box\forall\Diamond\neg a$$

$$
\begin{aligned}
s \models \exists\Diamond\exists\Box a \quad & \text{iff} \quad s \not\models \forall\Box\forall\Diamond\neg a \\
& \text{iff} \quad s \not\models \Box\Diamond\neg a \equiv \neg\Diamond\Box a \\
& \text{iff} \quad \text{there is a path } \pi \ \ldots.
\end{aligned}
$$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$ iff there is a path $\pi \in Paths(\mathcal{T})$ with $\pi \models \Box a$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$ iff there is a path $\pi \in \mathit{Paths}(\mathcal{T})$ with $\pi \models \Box a$

correct

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$ iff there is a path $\pi \in Paths(\mathcal{T})$ with $\pi \models \Box a$

correct $\mathcal{T} \not\models \neg\exists\Box a$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$ iff there is a path $\pi \in Paths(\mathcal{T})$ with $\pi \models \Box a$

correct $\mathcal{T} \not\models \neg\exists\Box a$

iff there is an initial state s with $s \not\models \neg\exists\Box a$

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$ iff there is a path $\pi \in Paths(\mathcal{T})$ with $\pi \models \Box a$

correct $\mathcal{T} \not\models \neg\exists\Box a$

iff there is an initial state s with $s \not\models \neg\exists\Box a$

iff there is an initial state s with $s \models \exists\Box a$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$ iff there is a path $\pi \in \mathit{Paths}(\mathcal{T})$ with $\pi \models \Box a$

correct $\mathcal{T} \not\models \neg\exists\Box a$

iff there is an initial state s with $s \not\models \neg\exists\Box a$

iff there is an initial state s with $s \models \exists\Box a$

iff there is a path $\pi \in \mathit{Paths}(\mathcal{T})$ with $\pi \models \Box a$

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\varphi$ iff there is a path $\pi \in Paths(\mathcal{T})$ with $\pi \models \varphi$

correct $\mathcal{T} \not\models \neg\exists\varphi$

iff there is an initial state s with $s \not\models \neg\exists\varphi$

iff there is an initial state s with $s \models \exists\varphi$

iff there is a path $\pi \in Paths(\mathcal{T})$ with $\pi \models \varphi$

Correct or wrong?

$$\mathcal{T} \not\models \neg\forall\Box a \quad \text{iff} \quad \text{for all paths } \pi \in \mathit{Paths}(\mathcal{T}): \; \pi \models \Box a$$

Correct or wrong?

$\mathcal{T} \not\models \neg\forall\Box a$ iff for all paths $\pi \in \mathit{Paths}(\mathcal{T})$:

$$\pi \models \Box a$$

wrong.

Correct or wrong?

$\mathcal{T} \not\models \neg\forall\Box a$ iff for all paths $\pi \in \mathit{Paths}(\mathcal{T})$:
$\pi \models \Box a$

wrong.

$\mathcal{T} \not\models \neg\forall\Box a$

Correct or wrong?

$$\mathcal{T} \not\models \neg\forall\Box a \quad \text{iff} \quad \text{for all paths } \pi \in Paths(\mathcal{T}): \; \pi \models \Box a$$

wrong.

$$\mathcal{T} \not\models \neg\forall\Box a$$

iff there is an initial state s with $s \not\models \neg\forall\Box a$

Correct or wrong?

$$\mathcal{T} \not\models \neg\forall\Box a \quad \text{iff} \quad \text{for all paths } \pi \in \mathit{Paths}(\mathcal{T}):\ \pi \models \Box a$$

wrong.

$\mathcal{T} \not\models \neg\forall\Box a$

iff there is an initial state s with $s \not\models \neg\forall\Box a$

iff there is an initial state s with $s \models \forall\Box a$

$\mathcal{T} \not\models \neg\forall\Box a$ iff for all paths $\pi \in \mathit{Paths}(\mathcal{T})$:
$\pi \models \Box a$

wrong.

$\mathcal{T} \not\models \neg\forall\Box a$

iff there is an initial state s with $s \not\models \neg\forall\Box a$

iff there is an initial state s with $s \models \forall\Box a$

but there might be another initial state t
s.t. $t \not\models \forall\Box a$

Correct or wrong?

If $\mathcal{T}_1$ and $\mathcal{T}_2$ are trace equivalent TS then for all **CTL** formulas Φ: $\mathcal{T}_1 \models \Phi$ iff $\mathcal{T}_2 \models \Phi$

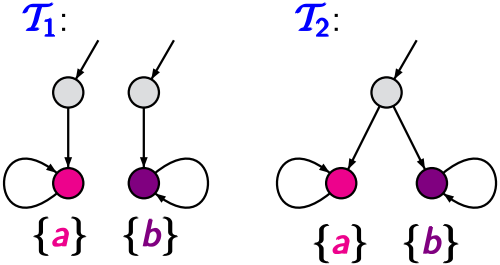
Correct or wrong?

If $\mathcal{T}_1$ and $\mathcal{T}_2$ are trace equivalent TS then for all **CTL** formulas Φ: $\mathcal{T}_1 \models \Phi$ iff $\mathcal{T}_2 \models \Phi$

wrong.

If $\mathcal{T}_1$ and $\mathcal{T}_2$ are trace equivalent TS then for all **CTL** formulas Φ: $\mathcal{T}_1 \models \Phi$ iff $\mathcal{T}_2 \models \Phi$

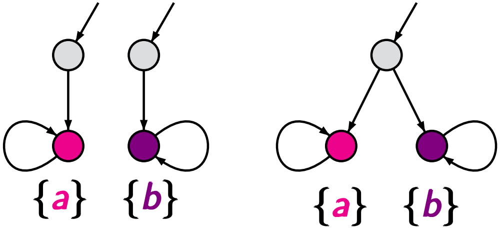
wrong.



If $\mathcal{T}_1$ and $\mathcal{T}_2$ are trace equivalent TS then for all **CTL** formulas Φ: $\mathcal{T}_1 \models \Phi$ iff $\mathcal{T}_2 \models \Phi$

wrong.

$\mathcal{T}_1$: $\mathcal{T}_2$:



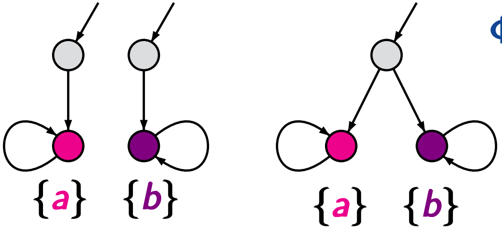
$\mathcal{T}_1$ and $\mathcal{T}_2$ are trace equivalent

If $\mathcal{T}_1$ and $\mathcal{T}_2$ are trace equivalent TS then for all **CTL** formulas Φ: $\mathcal{T}_1 \models \Phi$ iff $\mathcal{T}_2 \models \Phi$

wrong.

$\mathcal{T}_1$:

$\mathcal{T}_2$:



consider the CTL formula

$\Phi = \exists\bigcirc a \wedge \exists\bigcirc b$

$\mathcal{T}_1 \not\models \Phi$

$\mathcal{T}_2 \models \Phi$

$\mathcal{T}_1$ and $\mathcal{T}_2$ are trace equivalent