LTL Equivalences and Laws

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Topics

- Equivalence of LTL formulas.
- Self-duality of *next* operator. Expansion law for the *until* operator and its derivatives.
- Expansion laws as fixpoint equations.
- The *weak until* operator. Duality of *until* and *weak until*.
- Positive Normal Form

Material

Reading:

Chapter 5 of the book, Sections 5.1.4 and 5.1.5.

More:

The slides in the following pages are taken from the material of the course "Introduction to Model Checking" held by Prof. Dr. Ir. Joost-Pieter Katoen at Aachen University.

Equivalence of LTL formulas

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Examples:

$$\varphi_1 \vee \varphi_2 \equiv \varphi_2 \vee \varphi_1$$

$$\neg\neg\varphi \equiv \varphi$$

$$\vdots$$

all equivalences
from propositional logic

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all equivalences from propositional logic

Self-duality of the next operator

$$\varphi_1 \equiv \varphi_2 \quad \text{iff} \quad \mathit{Words}(\varphi_1) = \mathit{Words}(\varphi_2)$$

Claim: $\neg\bigcirc\varphi \equiv \bigcirc\neg\varphi$ "self-duality of next"

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Proof: $A_0\, A_1\, A_2\, A_3 \ldots \models \neg \bigcirc \varphi$

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iff $A_0\, A_1\, A_2\, A_3 \ldots \not\models \bigcirc \varphi$

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Proof:

$$\begin{aligned} & A_0 A_1 A_2 A_3 \ldots \models \neg\bigcirc\varphi \\ \text{iff}\quad & A_0 A_1 A_2 A_3 \ldots \not\models \bigcirc\varphi \\ \text{iff}\quad & A_1 A_2 A_3 \ldots \not\models \varphi \end{aligned}$$

Self-duality of the next operator

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iff $A_1 A_2 A_3 \ldots \not\models \varphi$

iff $A_1 A_2 A_3 \ldots \models \neg \varphi$

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Proof:

$$A_0\, A_1\, A_2\, A_3 \ldots \models \neg \bigcirc \varphi$$

iff $A_0\, A_1\, A_2\, A_3 \ldots \not\models \bigcirc \varphi$

iff $A_1\, A_2\, A_3 \ldots \not\models \varphi$

iff $A_1\, A_2\, A_3 \ldots \models \neg \varphi$

iff $A_0\, A_1\, A_2\, A_3 \ldots \models \bigcirc \neg \varphi$

Correct or wrong?

$$\Diamond(\varphi \vee \psi) \equiv \Diamond\varphi \vee \Diamond\psi$$

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correct

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$$\Diamond(\varphi \wedge \psi) \equiv \Diamond\varphi \wedge \Diamond\psi$$

Correct or wrong?

$$\Diamond(\varphi \vee \psi) \;\equiv\; \Diamond\varphi \vee \Diamond\psi$$

correct

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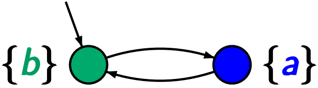
wrong,
e.g.,



$\models\; \Diamond b \wedge \Diamond a$

$\not\models\; \Diamond(b \wedge a)$

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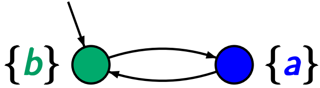

$\models \Diamond b \wedge \Diamond a$

$\not\models \Diamond(b \wedge a)$

---

similarly: $\Box(\varphi \wedge \psi) \equiv \Box\varphi \wedge \Box\psi$

$\Box(\varphi \vee \psi) \not\equiv \Box\varphi \vee \Box\psi$

Correct or wrong?

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correct Analogous: $\Box\Box\varphi \equiv \Box\varphi$

$\bigcirc\Box\varphi \equiv \Box\bigcirc\varphi \stackrel{\text{def}}{=} \psi$

correct

note that:

$A_0\, A_1\, A_2 \ldots \models \psi$ iff $A_i\, A_{i+1} \ldots \models \varphi$ for all $i \geq 1$

Correct or wrong?

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$\Diamond\Box\varphi \equiv \Box\Diamond\varphi$

$\Box\Diamond \;\widehat{=}\;$ infinitely often

$\Diamond\Box \;\widehat{=}\;$ eventually forever

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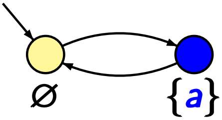

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$\Diamond\Box \;\widehat{=}\;$ eventually forever

$\models \Box\Diamond a$

$\not\models \Diamond\Box a$

Expansion laws

LTLSF3.1-28

Expansion law for U

until: $\varphi \mathsf{U} \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \mathsf{U} \psi))$

Expansion laws for U and $\Diamond$

until: $\varphi \, \mathsf{U} \, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$

eventually: $\Diamond \psi \equiv \psi \vee \bigcirc \Diamond \psi$

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note: $\Diamond\psi = \mathit{true} \, \mathsf{U} \, \psi$

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Expansion laws for U, $\Diamond$ and $\Box$

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always: $\Box\psi \;\equiv\;$ **?**

Expansion laws for U, $\Diamond$ and $\Box$

until:	$\varphi \, \mathsf{U} \, \psi$	$\equiv$	ψ	$\vee$	$(\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$
eventually:	$\Diamond \psi$	$\equiv$	ψ	$\vee$	$\bigcirc \Diamond \psi$
always:	$\Box \psi$	$\equiv$	ψ	$\wedge$	$\bigcirc \Box \psi$

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always: $\Box\psi \;\equiv\; \psi \;\wedge\; \bigcirc\Box\psi$

$\Box\psi = \neg\Diamond\neg\psi$

Expansion laws for U, $\Diamond$ and $\Box$

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$$\Box\psi = \neg\Diamond\neg\psi$$

$$\equiv \neg(\neg\psi \vee \bigcirc\Diamond\neg\psi) \longleftarrow \text{expansion law for } \Diamond$$

Expansion laws for U, $\Diamond$ and $\Box$

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always: $\Box \psi \equiv \psi \wedge \bigcirc \Box \psi$

$$
\begin{aligned}
\Box \psi &= \neg \Diamond \neg \psi \\
&\equiv \neg(\neg \psi \vee \bigcirc \Diamond \neg \psi) \\
&\equiv \neg\neg \psi \wedge \neg \bigcirc \Diamond \neg \psi \quad \longleftarrow \boxed{\text{de Morgan}}
\end{aligned}
$$

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&\equiv \psi \wedge \bigcirc\neg\Diamond\neg\psi \quad \longleftarrow
\end{aligned}
$$

self duality of $\bigcirc$

Expansion laws for U, $\Diamond$ and $\Box$

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&\equiv \psi \wedge \bigcirc\neg\Diamond\neg\psi \\
&\equiv \psi \wedge \bigcirc\Box\psi \qquad \longleftarrow \boxed{\text{definition of } \Box}
\end{aligned}
$$

Expansion laws are fixed point equations

until:	$\varphi \mathbf{U} \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \mathbf{U} \psi))$
eventually:	$\Diamond \psi \equiv \psi \vee \bigcirc \Diamond \psi$
always:	$\Box \psi \equiv \psi \wedge \bigcirc \Box \psi$

Expansion laws are fixed point equations

until: $\boxed{\varphi \mathsf{U} \psi} \equiv \psi \vee (\varphi \wedge \bigcirc \boxed{\varphi \mathsf{U} \psi})$

eventually: $\boxed{\Diamond \psi} \equiv \psi \vee \bigcirc \boxed{\Diamond \psi}$

always: $\boxed{\Box \psi} \equiv \psi \wedge \bigcirc \boxed{\Box \psi}$

Expansion laws are fixed point equations

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always: $\boxed{\Box \psi} \equiv \psi \wedge \bigcirc \boxed{\Box \psi}$

...don't yield a complete characterization, e.g.,

$$\begin{array}{|lcl|} \hline \mathit{false} & \equiv & a \wedge \bigcirc \mathit{false} \\ \Box a & \equiv & a \wedge \bigcirc \Box a \\ \hline \end{array}$$

consider $\psi = a$

Expansion laws are fixed point equations

until: $\boxed{\varphi \, \mathsf{U} \, \psi} \equiv \psi \vee (\varphi \wedge \bigcirc \boxed{\varphi \, \mathsf{U} \, \psi})$

eventually: $\boxed{\Diamond \psi} \equiv \psi \vee \bigcirc \boxed{\Diamond \psi}$

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although $\Box a \not\equiv \mathit{false}$

Expansion laws are fixed point equations

until: $\varphi \mathsf{U} \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \mathsf{U} \psi))$

least fixed point

eventually: $\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$

least fixed point

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least fixed point

always: $\Box\psi \equiv \psi \wedge \bigcirc\Box\psi$

greatest fixed point

...don't yield a complete characterization, e.g.,

$$false \equiv a \wedge \bigcirc false$$
$$\Box a \equiv a \wedge \bigcirc\Box a$$

although $\Box a \not\equiv false$

Expansion law for U

The LTL formula $\chi = \varphi \, \mathbf{U} \, \psi$ is the least solution of

$$\chi \equiv \psi \vee (\varphi \wedge \bigcirc \chi)$$

Expansion law for U

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$$\chi \equiv \psi \vee (\varphi \wedge \bigcirc \chi)$$

i.e., $\mathit{Words}(\varphi \, \mathbf{U} \, \psi)$ least LT-property E s.t.

$$E = \mathit{Words}(\psi) \cup \left\{ A_0 A_1 A_2 \ldots \in \mathit{Words}(\varphi) : A_1 A_2 \ldots \in E \right\}$$

Expansion law for U

The LTL formula $\chi = \varphi \,\mathbf{U}\, \psi$ is the least solution of

$$\chi \equiv \psi \lor (\varphi \land \bigcirc \chi)$$

i.e., $\mathit{Words}(\varphi \,\mathbf{U}\, \psi)$ least LT-property E s.t.

$$E = \mathit{Words}(\psi) \cup \{A_0 A_1 A_2 \ldots \in \mathit{Words}(\varphi) : A_1 A_2 \ldots \in E\}$$

It even holds that $\mathit{Words}(\varphi \,\mathbf{U}\, \psi)$ least LT-property E s.t.

(1) $\mathit{Words}(\psi) \subseteq E$

(2) $\{A_0 A_1 A_2 \ldots \in \mathit{Words}(\varphi) : A_1 A_2 \ldots \in E\} \subseteq E$

The weak until operator W

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$$\varphi \, \mathsf{W} \, \psi \stackrel{\text{def}}{=} (\varphi \, \mathsf{U} \, \psi) \vee \Box\varphi$$

The weak until operator W

$$\varphi \,\mathsf{W}\, \psi \overset{\text{def}}{=} (\varphi \,\mathsf{U}\, \psi) \vee \Box\varphi$$

deriving "always" and "until" from "weak until":

$$\Box\varphi \quad \equiv \quad ?$$

The weak until operator W

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deriving "always" and "until" from "weak until":

$$\Box\varphi \equiv \varphi \, \mathsf{W} \, \mathit{false}$$

The weak until operator W

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deriving "always" and "until" from "weak until":

$$\Box\varphi \equiv \varphi \mathsf{W}\, \mathit{false}$$

$$\varphi \mathsf{U} \psi \equiv ?$$

The weak until operator W

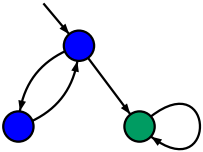
$$\varphi \, \mathsf{W} \, \psi \stackrel{\text{def}}{=} (\varphi \, \mathsf{U} \, \psi) \vee \Box\varphi$$

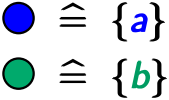
deriving "always" and "until" from "weak until":

$$\Box\varphi \equiv \varphi \, \mathsf{W} \, \mathit{false}$$

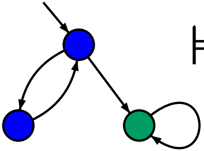
$$\varphi \, \mathsf{U} \, \psi \equiv (\varphi \, \mathsf{W} \, \psi) \wedge \Diamond\psi$$

Does $\mathcal{T} \models a \, \mathrm{W} \, b$ hold?





Does $\mathcal{T} \models a \,\mathsf{W}\, b$ hold?

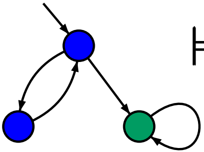


$\models a \,\mathsf{W}\, b$

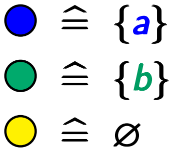
● $\widehat{=}$ $\{a\}$

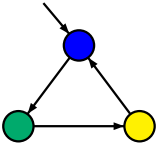
● $\widehat{=}$ $\{b\}$

Does $\mathcal{T} \models a \mathsf{W}\, b$ hold?



$\models a \mathsf{W}\, b$





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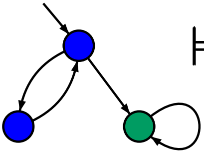
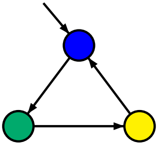


$\models a \, \mathsf{W} \, b$

(blue circle) $\widehat{=}$ $\{a\}$

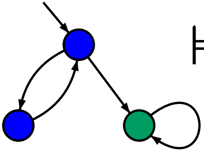
(green circle) $\widehat{=}$ $\{b\}$

(yellow circle) $\widehat{=}$ $\varnothing$



$\models \; a \, \mathsf{W} \, b$ (even $a \, \mathsf{U} \, b$)

Does $\mathcal{T} \models a \, \mathsf{W} \, b$ hold?

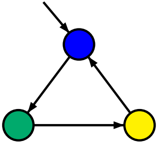


$\models a \, \mathsf{W} \, b$

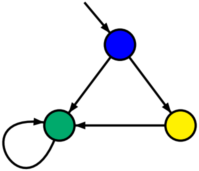
$\widehat{=} \; \{a\}$

$\widehat{=} \; \{b\}$

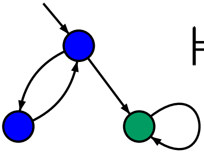
$\widehat{=} \; \varnothing$



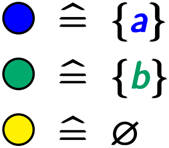
$\models \; a \, \mathsf{W} \, b \quad$ (even $a \, \mathsf{U} \, b$)

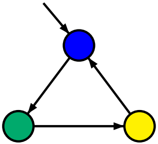


Does $\mathcal{T} \models a \, \mathsf{W} \, b$ hold?

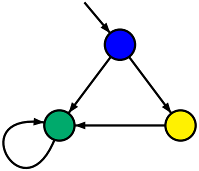


$\models a \, \mathsf{W} \, b$



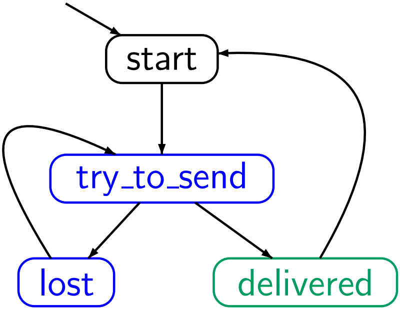


$\models a \, \mathsf{W} \, b$ (even $a \, \mathsf{U} \, b$)

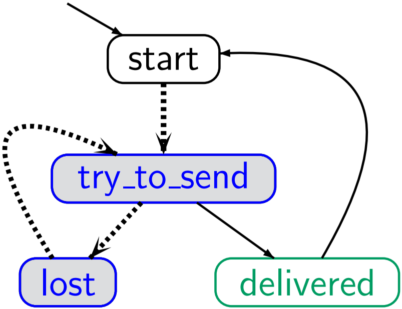


$\not\models a \, \mathsf{W} \, b$

Example: simple communication protocol

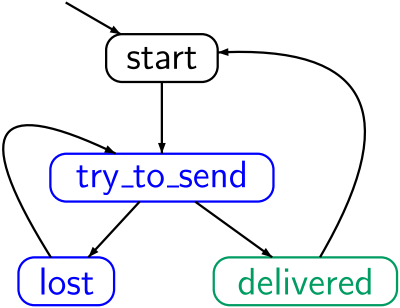


Example: simple communication protocol



$$\mathcal{T} \not\models \Box(\textit{blue} \longrightarrow \textit{blue}\ \mathsf{U}\ \textit{delivered})$$

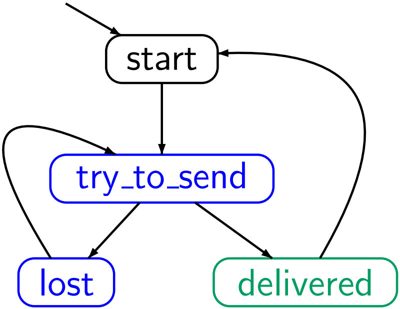
Example: until versus weak until



$$\mathcal{T} \not\models \Box(\mathit{blue} \longrightarrow \mathit{blue}\, \mathsf{U}\, \mathit{delivered})$$

$$\mathcal{T} \models \Box(\mathit{blue} \longrightarrow \mathit{blue}\, \mathsf{W}\, \mathit{delivered})$$

Example: until versus weak until



constrained liveness:

$$\mathcal{T} \not\models \Box(\textit{blue} \longrightarrow \textit{blue} \,\mathsf{U}\, \textit{delivered})$$

safety:

$$\mathcal{T} \models \Box(\textit{blue} \longrightarrow \textit{blue} \,\mathsf{W}\, \textit{delivered})$$

Duality of U and W

$$\varphi \, \mathsf{W} \, \psi \;\stackrel{\text{def}}{=}\; (\varphi \, \mathsf{U} \, \psi) \vee \Box\varphi$$

goal: express $\neg(\varphi \, \mathsf{U} \, \psi)$ via **W**, and vice versa

Duality of U and W

$$\varphi \, \mathsf{W} \, \psi \;\stackrel{\text{def}}{=}\; (\varphi \, \mathsf{U} \, \psi) \;\vee\; \Box\varphi$$

$$\begin{aligned} & \neg(\varphi \, \mathsf{U} \, \psi) \\ \equiv\; & \big((\varphi \wedge \neg\psi) \; \mathsf{U} \; (\neg\varphi \wedge \neg\psi)\big) \;\vee\; \Box(\varphi \wedge \neg\psi) \end{aligned}$$

Duality of U and W

$$\varphi \,\mathsf{W}\, \psi \;\stackrel{\text{def}}{=}\; (\varphi \,\mathsf{U}\, \psi) \vee \Box\varphi$$

$$\neg(\varphi \,\mathsf{U}\, \psi)$$

$$\equiv \; ((\varphi \wedge \neg\psi) \;\mathsf{U}\; (\neg\varphi \wedge \neg\psi)) \vee \Box(\varphi \wedge \neg\psi)$$

$$\equiv \; (\varphi \wedge \neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi)$$

to be incorporated into the syntax of PNF formulae. The propositional logical primitives of the positive normal form for LTL are the constant true and its dual constant false $= \neg$true, as well as conjunction $\wedge$ and its dual, $\vee$. De Morgan's rules $\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi$ and $\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi$ yield the duality of conjunction and disjunction. According to the duality rule $\neg \bigcirc \varphi \equiv \bigcirc \neg\varphi$, the next-step operator is a dual of itself. Therefore, no extra operator is necessary for $\bigcirc$. Now consider the until operator. First we observe that

$$\neg(\varphi \,\mathsf{U}\, \psi) \quad \equiv \quad ((\varphi \wedge \neg\psi) \,\mathsf{U}\, (\neg\varphi \wedge \neg\psi)) \ \vee \ \Box(\varphi \wedge \neg\psi).$$

The first disjunct on the right-hand side asserts that φ stops to hold "too early", i.e., before ψ becomes valid. The second disjunct states that φ always holds but never ψ. Clearly, in both cases, $\neg(\varphi \,\mathsf{U}\, \psi)$ holds.

This observation provides the motivation to introduce the operator W (called *weak until* or *unless*) as the dual of U. It is defined by:

$$\varphi \,\mathsf{W}\, \psi \ \stackrel{\text{def}}{=} \ (\varphi \,\mathsf{U}\, \psi) \ \vee \ \Box\varphi.$$

The semantics of $\varphi \,\mathsf{W}\, \psi$ is similar to that of $\varphi \,\mathsf{U}\, \psi$, except that $\varphi \,\mathsf{U}\, \psi$ requires a state to be reached for which ψ holds, whereas this is not required for $\varphi \,\mathsf{W}\, \psi$. Until U and weak until W are dual in the following sense:

$$\begin{aligned} \neg(\varphi \,\mathsf{U}\, \psi) \quad &\equiv \quad (\varphi \wedge \neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi) \\ \neg(\varphi \,\mathsf{W}\, \psi) \quad &\equiv \quad (\varphi \wedge \neg\psi) \,\mathsf{U}\, (\neg\varphi \wedge \neg\psi) \end{aligned}$$

Expansion laws for U and W

Expansion laws for U and W

$$\varphi \, \mathsf{U} \, \psi \quad \equiv \quad \psi \lor (\varphi \land \bigcirc(\varphi \, \mathsf{U} \, \psi))$$

$$\varphi \, \mathsf{W} \, \psi \quad \equiv \quad ?$$

Expansion laws for U and W

$$\varphi \,\mathsf{U}\, \psi \equiv \psi \lor (\varphi \land \bigcirc(\varphi \,\mathsf{U}\, \psi))$$

$$\varphi \,\mathsf{W}\, \psi \equiv \psi \lor (\varphi \land \bigcirc(\varphi \,\mathsf{W}\, \psi))$$

Expansion laws for U and W

$$\varphi \,\mathsf{U}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$$

smallest solution

$$\varphi \,\mathsf{W}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$$

Expansion laws for U and W

$\varphi \, \mathsf{U} \, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$	smallest solution
$\varphi \, \mathsf{W} \, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{W} \, \psi))$	largest solution

Expansion laws for U and W

$\varphi \, \mathsf{U} \, \psi$	$\equiv$	$\psi \lor (\varphi \land \bigcirc(\varphi \, \mathsf{U} \, \psi))$	smallest solution
$\varphi \, \mathsf{W} \, \psi$	$\equiv$	$\psi \lor (\varphi \land \bigcirc(\varphi \, \mathsf{W} \, \psi))$	largest solution

$\mathit{Words}(\varphi \, \mathsf{U} \, \psi)$ smallest LT-property E s.t.

Expansion laws for U and W

$\varphi \mathsf{U} \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \mathsf{U} \psi))$	smallest solution
$\varphi \mathsf{W} \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \mathsf{W} \psi))$	largest solution

$Words(\varphi \mathsf{U} \psi)$ smallest LT-property E s.t.

(1) $Words(\psi) \subseteq E$

(2) $\{A_0 A_1 A_2 \ldots \in Words(\varphi) : A_1 A_2 \ldots \in E\} \subseteq E$

Expansion laws for U and W

$\varphi \, \mathsf{U} \, \psi$	$\equiv$	$\psi \lor (\varphi \land \bigcirc(\varphi \, \mathsf{U} \, \psi))$	smallest solution
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$$\Updownarrow$$

$$Words(\psi) \cup \{A_0 A_1 A_2 \ldots \in Words(\varphi) : A_1 A_2 \ldots \in E\} \subseteq E$$

$\varphi \,\mathsf{U}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
$\varphi \,\mathsf{W}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$	largest solution

$Words(\varphi \,\mathsf{U}\, \psi)$ smallest LT-property E s.t.

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$\mathit{Words}(\varphi \, \mathsf{W} \, \psi)$ largest LT-property E s.t.

Expansion laws for U and W

$\varphi \,\mathbf{U}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathbf{U}\, \psi))$	smallest solution
$\varphi \,\mathbf{W}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathbf{W}\, \psi))$	largest solution

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$$Words(\psi) \cup \{ A_0 A_1 A_2 \ldots \in Words(\varphi) : A_1 A_2 \ldots \in E \} \subseteq E$$

$Words(\varphi \, \mathsf{W} \, \psi)$ largest LT-property E s.t.

$$E \subseteq Words(\psi) \cup \{ A_0 A_1 A_2 \ldots \in Words(\varphi) : A_1 A_2 \ldots \in E \}$$

Expansion laws for U and W

$$\varphi \, \mathsf{U} \, \psi \quad \equiv \quad \psi \lor (\varphi \land \bigcirc(\varphi \, \mathsf{U} \, \psi))$$

smallest solution

$$\varphi \, \mathsf{W} \, \psi \quad \equiv \quad \psi \lor (\varphi \land \bigcirc(\varphi \, \mathsf{W} \, \psi))$$

largest solution

Expansion laws for U, W, $\Diamond$, and $\Box$

$$\varphi \,\mathsf{U}\, \psi \;\equiv\; \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$$

smallest solution

$$\Diamond\psi \;\equiv\; \psi \vee \bigcirc\Diamond\psi$$

smallest solution

$$\varphi \,\mathsf{W}\, \psi \;\equiv\; \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$$

largest solution

$$\Box\varphi \;\equiv\; \varphi \wedge \bigcirc\Box\varphi$$

largest solution

---

remind: $\Diamond\psi = \mathit{true} \,\mathsf{U}\, \psi$, $\Box\varphi \equiv \varphi \,\mathsf{W}\, \mathit{false}$

Positive normal form (PNF)

Positive normal form (PNF)

- negation only on the level of literals
- uses for each operator its dual

PNF for propositional logic

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syntax of propositional formulas in PNF:

$$\varphi \;::=\; \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2$$

PNF for propositional logic

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syntax of propositional formulas in PNF:

$$\varphi ::= \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2$$

$\neg \mathit{true} \equiv \mathit{false}$	duality of the constant truth values
$\neg(\varphi_1 \wedge \varphi_2) \equiv \neg\varphi_1 \vee \neg\varphi_2$	duality of $\vee$ and $\wedge$ (de Morgan's law)

LTL in positive normal form (PNF)

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using duality of constants and duality of $\vee$ and $\wedge$

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$$\varphi \;::=\; \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid$$

$\bigcirc\varphi$ + dual operator for $\bigcirc$

using duality of constants and duality of $\vee$ and $\wedge$

LTL in positive normal form (PNF)

- negation only on the level of literals
- uses for each operator its dual

$$\varphi ::= \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid$$
$$\bigcirc \varphi \longleftarrow$$ no new operator needed for $\neg\bigcirc$

using duality of constants and duality of $\vee$ and $\wedge$

$\neg \bigcirc \varphi \equiv \bigcirc \neg \varphi$ self-duality of the next operator

LTL in positive normal form (PNF)

- negation only on the level of literals
- uses for each operator its dual

$$\varphi \; ::= \; \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid$$
$$\bigcirc\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2 \;+\; \text{dual operator for } \mathsf{U}$$

using duality of constants and duality of $\vee$ and $\wedge$

$\neg \bigcirc \varphi \equiv \bigcirc \neg \varphi$ self-duality of the next operator

LTL in positive normal form (PNF)

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$$\varphi \; ::= \; \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \bigcirc \varphi \mid \varphi_1 \mathsf{U} \varphi_2 \mid \varphi_1 \mathsf{W} \varphi_2$$

using duality of constants and duality of $\vee$ and $\wedge$

$\neg \bigcirc \varphi \equiv \bigcirc \neg \varphi$ self-duality of the next operator

$\neg(\varphi_1 \mathsf{U} \varphi_2) \equiv (\neg \varphi_2) \mathsf{W} (\neg \varphi_1 \wedge \neg \varphi_2)$
duality of **U** and **W**

Derivation of $\Diamond$ and $\Box$ in LTL-PNF

$$\varphi \quad ::= \quad \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid$$
$$\bigcirc \varphi \mid \varphi_1 \, \mathsf{U} \, \varphi_2 \mid \varphi_1 \, \mathsf{W} \, \varphi_2$$

Derivation of $\Diamond$ and $\Box$ in LTL-PNF

$$\varphi \quad ::= \quad \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid$$
$$\bigcirc\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2 \mid \varphi_1 \,\mathsf{W}\, \varphi_2 \mid \Diamond\varphi \mid \Box\varphi$$

$\Diamond$ and $\Box$ can (still) be derived:

$$\Diamond\varphi \quad \stackrel{\text{def}}{=} \quad \mathit{true}\,\mathsf{U}\,\varphi$$
$$\Box\varphi \quad \stackrel{\text{def}}{=} \quad \varphi\,\mathsf{W}\,\mathit{false}$$

Universality of LTL-PNF

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Each LTL formula can be transformed into an equivalent LTL formula in **PNF**

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LTL formula $\varphi \rightsquigarrow$ LTL formula in PNF φ'
by successive application of the following rules:

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LTL formula $\varphi \rightsquigarrow$ LTL formula in PNF φ'
by successive application of the following rules:

$$\begin{array}{lcl}
\neg \textit{true} & \rightsquigarrow & \textit{false} \\
\neg\neg\varphi & \rightsquigarrow & \varphi \\
\neg(\varphi_1 \wedge \varphi_2) & \rightsquigarrow & \neg\varphi_1 \ \vee \ \neg\varphi_2 \\
\neg \bigcirc \varphi & \rightsquigarrow & \bigcirc\neg\varphi \\
\neg(\varphi_1 \, \mathsf{U} \, \varphi_2) & \rightsquigarrow & (\neg\varphi_2) \, \mathsf{W} (\neg\varphi_1 \wedge \neg\varphi_2)
\end{array}$$

Universality of LTL-PNF

Each LTL formula can be transformed into an equivalent LTL formula in **PNF**

LTL formula $\varphi \rightsquigarrow$ LTL formula in PNF φ'
by successive application of the following rules:

$$\begin{array}{lcl}
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\neg\neg\varphi & \rightsquigarrow & \varphi \\
\neg(\varphi_1 \wedge \varphi_2) & \rightsquigarrow & \neg\varphi_1 \vee \neg\varphi_2 \\
\neg \bigcirc \varphi & \rightsquigarrow & \bigcirc\neg\varphi \\
\neg(\varphi_1 \, \mathsf{U} \, \varphi_2) & \rightsquigarrow & (\neg\varphi_2) \, \mathsf{W} (\neg\varphi_1 \wedge \neg\varphi_2)
\end{array}$$

exponential-blow up is possible

Example: LTL $\rightsquigarrow$ LTL-PNF

$\neg \mathit{true}$	$\rightsquigarrow$	false
$\neg\neg\varphi$	$\rightsquigarrow$	φ
$\neg(\varphi_1 \wedge \varphi_2)$	$\rightsquigarrow$	$\neg\varphi_1 \vee \neg\varphi_2$
$\neg \bigcirc \varphi$	$\rightsquigarrow$	$\bigcirc\neg\varphi$
$\neg(\varphi_1 \,\mathsf{U}\, \varphi_2)$	$\rightsquigarrow$	$(\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2)$

Example: LTL $\rightsquigarrow$ LTL-PNF

$\neg true \rightsquigarrow false$ + analogue rule for $\neg false$

$\neg\neg\varphi \rightsquigarrow \varphi$

$\neg(\varphi_1 \wedge \varphi_2) \rightsquigarrow \neg\varphi_1 \vee \neg\varphi_2$ + analogue rule for $\neg\vee$

$\neg \bigcirc \varphi \rightsquigarrow \bigcirc\neg\varphi$

$\neg(\varphi_1 \,\mathsf{U}\, \varphi_2) \rightsquigarrow (\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2)$

Example: LTL $\rightsquigarrow$ LTL-PNF

$\neg \mathit{true}$	$\rightsquigarrow$	false	+ analogue rule for $\neg \mathit{false}$
$\neg\neg\varphi$	$\rightsquigarrow$	φ	
$\neg(\varphi_1 \wedge \varphi_2)$	$\rightsquigarrow$	$\neg\varphi_1 \vee \neg\varphi_2$	+ analogue rule for $\neg\vee$
$\neg \bigcirc \varphi$	$\rightsquigarrow$	$\bigcirc\neg\varphi$	
$\neg(\varphi_1 \mathbf{U} \varphi_2)$	$\rightsquigarrow$	$(\neg\varphi_2)\mathbf{W}(\neg\varphi_1 \wedge \neg\varphi_2)$	
$\neg\Diamond\varphi$	$\rightsquigarrow$	$\Box\neg\varphi$	$\neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi$

Example: LTL $\rightsquigarrow$ LTL-PNF

$\neg \mathit{true}$	$\rightsquigarrow$	false	+ analogue rule for $\neg \mathit{false}$
$\neg\neg\varphi$	$\rightsquigarrow$	φ	
$\neg(\varphi_1 \wedge \varphi_2)$	$\rightsquigarrow$	$\neg\varphi_1 \vee \neg\varphi_2$	+ analogue rule for $\neg\vee$
$\neg \bigcirc \varphi$	$\rightsquigarrow$	$\bigcirc\neg\varphi$	
$\neg(\varphi_1 \mathsf{U} \varphi_2)$	$\rightsquigarrow$	$(\neg\varphi_2) \mathsf{W} (\neg\varphi_1 \wedge \neg\varphi_2)$	
$\neg\Diamond\varphi$	$\rightsquigarrow$	$\Box\neg\varphi$	$\neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi$

$$\neg\Box((a \,\mathsf{U}\, b) \vee \bigcirc c)$$

Example: LTL $\rightsquigarrow$ LTL-PNF

$\neg true$	$\rightsquigarrow$	$false$	+ analogue rule for $\neg false$
$\neg\neg\varphi$	$\rightsquigarrow$	φ	
$\neg(\varphi_1 \wedge \varphi_2)$	$\rightsquigarrow$	$\neg\varphi_1 \vee \neg\varphi_2$	+ analogue rule for $\neg\vee$
$\neg \bigcirc \varphi$	$\rightsquigarrow$	$\bigcirc\neg\varphi$	
$\neg(\varphi_1 \mathsf{U} \varphi_2)$	$\rightsquigarrow$	$(\neg\varphi_2) \mathsf{W} (\neg\varphi_1 \wedge \neg\varphi_2)$	
$\neg\Diamond\varphi$	$\rightsquigarrow$	$\Box\neg\varphi$	$\neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi$

$$\neg\Box((a \,\mathsf{U}\, b) \;\vee\; \bigcirc c)$$

$$\equiv \;\Diamond\neg((a \,\mathsf{U}\, b) \;\vee\; \bigcirc c)$$

$\longleftarrow$ duality of $\Diamond$ and $\Box$

Example: LTL $\leadsto$ LTL-PNF

$\neg \mathit{true}$	$\leadsto$	false	+ analogue rule for $\neg \mathit{false}$
$\neg\neg\varphi$	$\leadsto$	φ	
$\neg(\varphi_1 \wedge \varphi_2)$	$\leadsto$	$\neg\varphi_1 \vee \neg\varphi_2$	+ analogue rule for $\neg\vee$
$\neg \bigcirc \varphi$	$\leadsto$	$\bigcirc\neg\varphi$	
$\neg(\varphi_1 \mathsf{U} \varphi_2)$	$\leadsto$	$(\neg\varphi_2) \mathsf{W} (\neg\varphi_1 \wedge \neg\varphi_2)$	
$\neg\Diamond\varphi$	$\leadsto$	$\Box\neg\varphi$	$\neg\Box\varphi \leadsto \Diamond\neg\varphi$

$$\neg\Box((a \,\mathsf{U}\, b) \vee \bigcirc c)$$

$$\equiv \Diamond\neg((a \,\mathsf{U}\, b) \vee \bigcirc c) \qquad \longleftarrow \boxed{\text{duality of } \Diamond \text{ and } \Box}$$

$$\equiv \Diamond(\neg(a \,\mathsf{U}\, b) \wedge \neg\bigcirc c) \qquad \longleftarrow \boxed{\text{duality of } \wedge \text{ and } \vee}$$

Example: LTL $\rightsquigarrow$ LTL-PNF

$\neg \mathit{true}$	$\rightsquigarrow$	false	+ analogue rule for $\neg \mathit{false}$
$\neg\neg\varphi$	$\rightsquigarrow$	φ	
$\neg(\varphi_1 \wedge \varphi_2)$	$\rightsquigarrow$	$\neg\varphi_1 \vee \neg\varphi_2$	+ analogue rule for $\neg\vee$
$\neg \bigcirc \varphi$	$\rightsquigarrow$	$\bigcirc\neg\varphi$	
$\neg(\varphi_1 \,\mathsf{U}\, \varphi_2)$	$\rightsquigarrow$	$(\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2)$	
$\neg\Diamond\varphi$	$\rightsquigarrow$	$\Box\neg\varphi$	$\neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi$

$$\neg\Box((a \,\mathsf{U}\, b) \vee \bigcirc c)$$

$$\equiv \Diamond\neg((a \,\mathsf{U}\, b) \vee \bigcirc c)$$ $\longleftarrow$ duality of $\Diamond$ and $\Box$

$$\equiv \Diamond(\neg(a \,\mathsf{U}\, b) \wedge \neg\bigcirc c)$$ $\longleftarrow$ duality of $\wedge$ and $\vee$

$$\equiv \Diamond(\neg(a \,\mathsf{U}\, b) \wedge \bigcirc\neg c)$$ $\longleftarrow$ self-duality of $\bigcirc$

Example: LTL $\rightsquigarrow$ LTL-PNF

$\neg true$	$\rightsquigarrow$	$false$	+ analogue rule for $\neg false$
$\neg\neg\varphi$	$\rightsquigarrow$	φ	
$\neg(\varphi_1 \wedge \varphi_2)$	$\rightsquigarrow$	$\neg\varphi_1 \vee \neg\varphi_2$	+ analogue rule for $\neg\vee$
$\neg \bigcirc \varphi$	$\rightsquigarrow$	$\bigcirc\neg\varphi$	
$\neg(\varphi_1 \mathsf{U} \varphi_2)$	$\rightsquigarrow$	$(\neg\varphi_2) \mathsf{W} (\neg\varphi_1 \wedge \neg\varphi_2)$	
$\neg\Diamond\varphi$	$\rightsquigarrow$	$\Box\neg\varphi$	$\neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi$

$$\neg\Box((a \,\mathsf{U}\, b) \vee \bigcirc c)$$

$$\equiv \Diamond\neg((a \,\mathsf{U}\, b) \vee \bigcirc c) \quad \longleftarrow \text{duality of } \Diamond \text{ and } \Box$$

$$\equiv \Diamond(\neg(a \,\mathsf{U}\, b) \wedge \neg\bigcirc c) \quad \longleftarrow \text{duality of } \wedge \text{ and } \vee$$

$$\equiv \Diamond((\neg b) \,\mathsf{W}\, (\neg a \wedge \neg b) \wedge \bigcirc\neg c) \quad \longleftarrow \text{duality of } \mathsf{U} \text{ and } \mathsf{W}$$

Example: LTL $\rightsquigarrow$ LTL-PNF

$\neg \mathit{true}$	$\rightsquigarrow$	false	+ analogue rule for $\neg \mathit{false}$
$\neg\neg\varphi$	$\rightsquigarrow$	φ	
$\neg(\varphi_1 \wedge \varphi_2)$	$\rightsquigarrow$	$\neg\varphi_1 \vee \neg\varphi_2$	+ analogue rule for $\neg\vee$
$\neg \bigcirc \varphi$	$\rightsquigarrow$	$\bigcirc\neg\varphi$	
$\neg(\varphi_1 \,\mathsf{U}\, \varphi_2)$	$\rightsquigarrow$	$(\neg\varphi_2) \,\mathsf{W}\, (\neg\varphi_1 \wedge \neg\varphi_2)$	
$\neg\Diamond\varphi$	$\rightsquigarrow$	$\Box\neg\varphi$	$\neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi$

$$\neg\Box((a \,\mathsf{U}\, b) \vee \bigcirc c)$$

$$\equiv \Diamond\neg((a \,\mathsf{U}\, b) \vee \bigcirc c)$$

$$\equiv \Diamond(\neg(a \,\mathsf{U}\, b) \wedge \neg\bigcirc c)$$

$$\equiv \Diamond((\neg b) \,\mathsf{W}\, (\neg a \wedge \neg b) \wedge \bigcirc\neg c) \longleftarrow \boxed{\textbf{PNF}}$$