Expressing Fairness in LTL

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Topics

- Fairness in LTL. State-based approach.
- Action-based LTL fairness. Ad-hoc and general construction.

Material

Reading:

Chapter 5 of the book, Section 5.1.6

More:

The slides in the following pages are taken from the material of the course "Introduction to Model Checking" held by Prof. Dr. Ir. Joost-Pieter Katoen at Aachen University.

Fairness in LTL

Recall: action-based fairness

fairness assumption for TS $\mathcal{T} = (S, Act, \to, S_0, AP, L)$:

$$\mathcal{F} = (\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak})$$

where $\mathcal{F}_{ucond}$, $\mathcal{F}_{strong}$, $\mathcal{F}_{weak} \subseteq 2^{Act}$

- $\mathcal{F}_{ucond}$ unconditional fairness assumption
- $\mathcal{F}_{strong}$ strong fairness assumption
- $\mathcal{F}_{weak}$ weak fairness assumption

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- for all $A \in \mathcal{F}_{ucond}$: $\overset{\infty}{\exists} i \geq 1. \alpha_i \in A$

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- for all $A \in \mathcal{F}_{strong}$:

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Recall: action-based fairness

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- for all $A \in \mathcal{F}_{weak}$:
  $$\overset{\infty}{\forall} i \geq 1. A \cap Act(s_i) \neq \varnothing \implies \overset{\infty}{\exists} i \geq 1. \alpha_i \in A$$

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fairness assumption for TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$:

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satisfaction relation for LT-properties under fairness:

$\mathcal{T} \models_{\mathcal{F}} E$ iff for all $\mathcal{F}$-fair paths π of $\mathcal{T}$: $trace(\pi) \in E$

Process fairness is LTL-definable

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$$\varphi \;::=\; \textit{true} \;\big|\; a \;\big|\; \varphi_1 \wedge \varphi_2 \;\big|\; \neg\varphi \;\big|\; \bigcirc\varphi \;\big|\; \varphi_1 \,\mathsf{U}\, \varphi_2$$

eventually $\Diamond\varphi \overset{\text{def}}{=} \textit{true}\,\mathsf{U}\,\varphi$

always $\Box\varphi \overset{\text{def}}{=} \neg\Diamond\neg\varphi$

infinitely often $\Box\Diamond\varphi$

eventually forever $\Diamond\Box\varphi$

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e.g., unconditional fairness $\Box\Diamond \mathit{crit}_i$

strong fairness $\Box\Diamond \mathit{wait}_i \rightarrow \Box\Diamond \mathit{crit}_i$

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LTL fairness assumptions

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... are conjunctions of LTL formulas of the form:

- unconditional fairness $\Box\Diamond\phi$
- strong fairness $\Box\Diamond\phi_1 \rightarrow \Box\Diamond\phi_2$
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If *fair* is a LTL fairness assumption, s a state in a TS, and φ an LTL formula then

$s \models_{fair} \varphi$ iff for all $\pi \in Paths(s)$:
if $\pi \models fair$ then $\pi \models \varphi$

LTL fairness assumptions

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$$
\begin{aligned}
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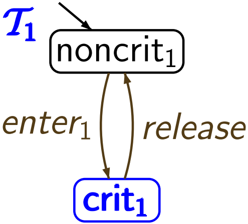
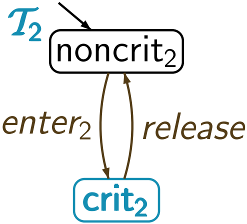
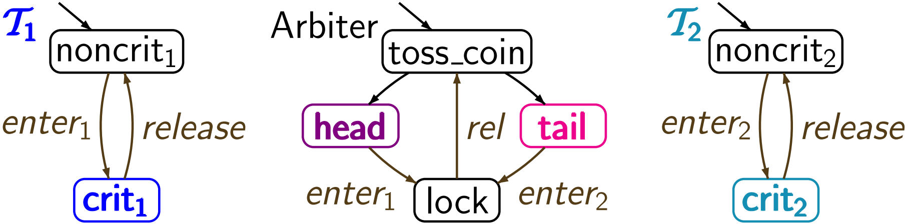
&\qquad\qquad \text{if } \pi \models fair \text{ then } \pi \models \varphi \\
&\text{iff} \quad s \models fair \rightarrow \varphi
\end{aligned}
$$

Randomized arbiter for MUTEX

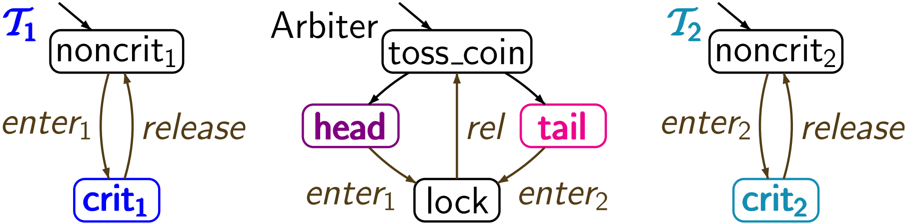




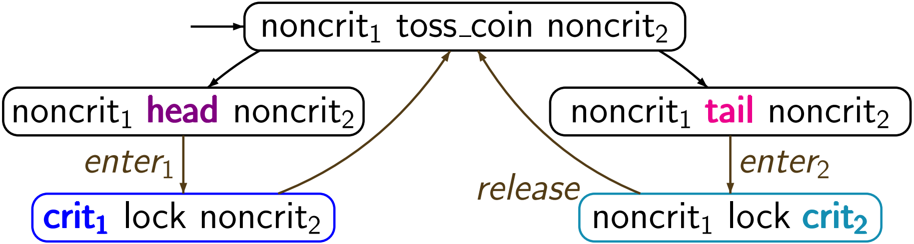
Randomized arbiter for MUTEX



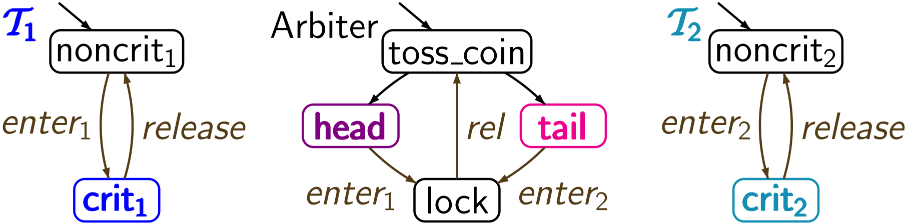
Randomized arbiter for MUTEX



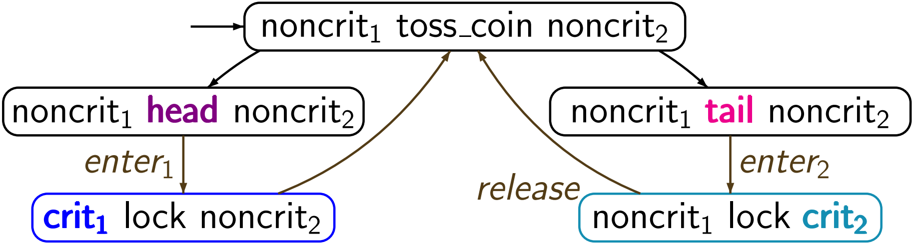
$(\mathcal{T}_1 \,|||\, \mathcal{T}_2) \,\|\, \mathit{Arbiter}$



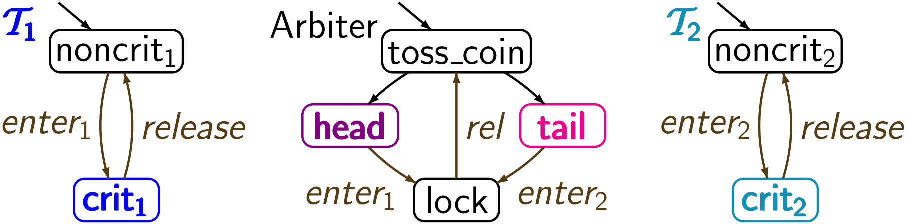
Randomized arbiter for MUTEX



$$(\mathcal{T}_1 \; ||| \; \mathcal{T}_2) \; \| \; \mathit{Arbiter} \not\models \Box\Diamond \mathsf{crit}_1 \wedge \Box\Diamond \mathsf{crit}_2$$



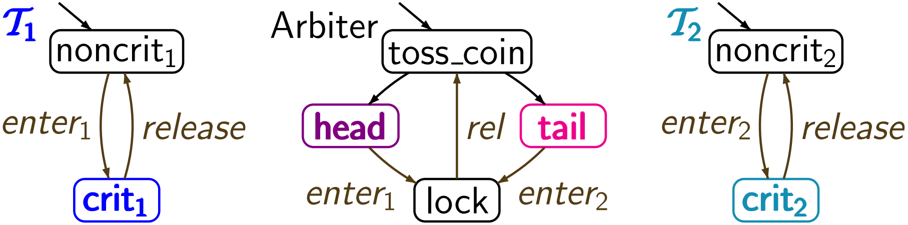
Randomized arbiter for MUTEX



unconditional LTL-fairness:

$$fair = \Box\Diamond \mathbf{head} \wedge \Box\Diamond \mathbf{tail}$$

Randomized arbiter for MUTEX



unconditional LTL-fairness:

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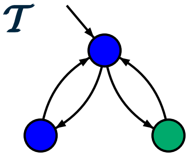
$$(\mathcal{T}_1 \;|||\; \mathcal{T}_2) \,\|\, Arbiter \;\models_{fair}\; \Box\Diamond\mathsf{crit}_1 \;\wedge\; \Box\Diamond\mathsf{crit}_2$$

Correct or wrong?

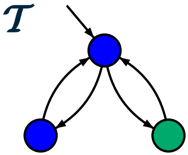


LTL fairness assumption

$fair = \Diamond\Box a \rightarrow \Box\Diamond b$

● $\widehat{=} \{a\}$ ● $\widehat{=} \{b\}$

Correct or wrong?



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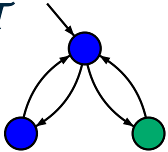
$\mathcal{T} \models_{fair} \bigcirc b$ **?**

Correct or wrong?

$\mathcal{T}$



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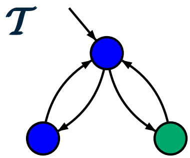
● $\widehat{=}\{a\}$ ● $\widehat{=}\{b\}$

$\mathcal{T} \not\models_{fair} \bigcirc b$ as ●→●→●→●→●→●→ ... is fair

Correct or wrong?



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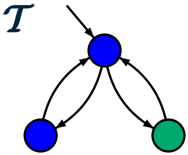
$\mathcal{T} \models_{fair} a \,\mathsf{U}\, b$?

Correct or wrong?



LTL fairness assumption

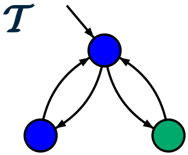
$fair = \Diamond\Box a \rightarrow \Box\Diamond b$

● (blue) $\widehat{=} \{a\}$ ● (green) $\widehat{=} \{b\}$

$\mathcal{T} \not\models_{fair} \bigcirc b$ as ● → ● → ● → ● → ● → ● → ... is fair

$\mathcal{T} \models_{fair} a \, \mathsf{U} \, b$ √

Correct or wrong?



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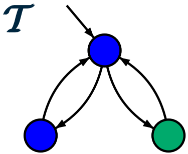
$\mathcal{T} \models_{fair} a \,\mathsf{U}\, b$ √

$\mathcal{T} \models_{fair} a \,\mathsf{U}\, \Box(b \leftrightarrow \bigcirc a)$?

Correct or wrong?



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$\mathcal{T} \not\models_{fair} \bigcirc b$ as ● → ● → ● → ● → ● → ● → ... is fair

$\mathcal{T} \models_{fair} a \,\mathsf{U}\, b$ ✓

$\mathcal{T} \not\models_{fair} a \,\mathsf{U}\, \Box(b \leftrightarrow \bigcirc a)$

as ● → ● → ● → ● → ● → ● → ... is fair

LTL-fairness assumptions

- can be necessary to prove liveness properties, e.g., mutual exclusion with arbiter/semaphore

$$\mathcal{T}_{sem} \not\models \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

$$\mathcal{T}_{sem} \models_{fair} \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

for appropriate fairness condition

LTL-fairness assumptions

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for appropriate fairness condition, e.g.,

$$fair = \bigwedge_{i=1,2} ((\Box\Diamond wait_i \rightarrow \Box\Diamond crit_i) \wedge (\Diamond\Box noncrit_i \rightarrow \Box\Diamond wait_i))$$

LTL-fairness assumptions

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$$\mathcal{T}_{sem} \models_{fair} \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

for appropriate fairness condition

- can be verifiable system properties

  e.g., Peterson algorithm guarantees strong fairness

$$\mathcal{T}_{Pet} \models \Box\Diamond wait_1 \rightarrow \Box\Diamond crit_1$$

LTL-fairness assumptions

- can be necessary to prove liveness properties, e.g.,

  $$\mathcal{T}_{sem} \not\models \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

  $$\mathcal{T}_{sem} \models_{fair} \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

  for appropriate fairness condition

- can be verifiable system properties, e.g.,

  $$\mathcal{T}_{Pet} \models \Box\Diamond wait_1 \rightarrow \Box\Diamond crit_1$$

- are irrelevant for verifying safety properties

$\mathcal{T} \models \varphi_{safe}$ iff $\mathcal{T} \models_{fair} \varphi_{safe}$

if *fair* is realizable

Correct or wrong?

Each strong **LTL** fairness assumption

$$fair \;=\; \Box\Diamond a \rightarrow \Box\Diamond b$$

is realizable for each TS over $AP = \{a, b, \ldots\}$.

Correct or wrong?

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recall: a fairness condition is called realizable if for each reachable state s there exists a fair path starting in s

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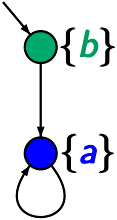
is realizable for each TS over $AP = \{a, b, \ldots\}$.

wrong



$fair = \Box\Diamond a \rightarrow \Box\Diamond b$

is <u>not</u> realizable

Action-based fairness ⇝ LTL-fairness

Action-based fairness $\rightsquigarrow$ LTL-fairness

idea: use new atomic propositions ***enabled***(A) and ***taken***(A) and extend the labeling function:

$$enabled(A) \in L(s) \quad \text{iff} \quad s \xrightarrow{\alpha} \ldots \text{ for some } \alpha \in A$$

$$taken(A) \in L(s) \quad \text{iff} \quad \text{for all transitions } \ldots \xrightarrow{\alpha} s\text{: } \alpha \in A$$

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- unconditional A-fairness: $\Box\Diamond taken(A)$
- strong A-fairness: $\Box\Diamond enabled(A) \rightarrow \Box\Diamond taken(A)$
- weak A-fairness: $\Diamond\Box enabled(A) \rightarrow \Box\Diamond taken(A)$

Action-based fairness $\rightsquigarrow$ LTL-fairness

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$$taken(A) \in L(s) \quad \text{iff} \quad \text{for } \boxed{\text{all}} \text{ transitions } \ldots \xrightarrow{\alpha} s: \alpha \in A$$

problem: each state s can have several incoming transitions

$$t \xrightarrow{\alpha} s, \quad u \xrightarrow{\beta} s, \ \ldots$$

Action-based fairness $\rightsquigarrow$ LTL-fairness

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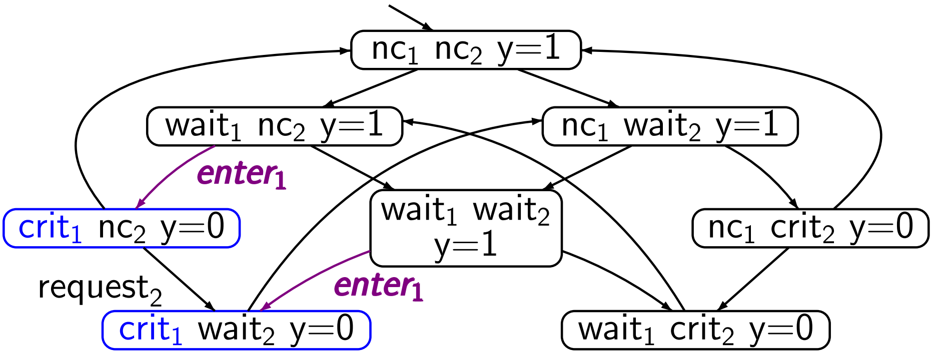
alternative 1: ad-hoc choice of "***taken***-predicate"

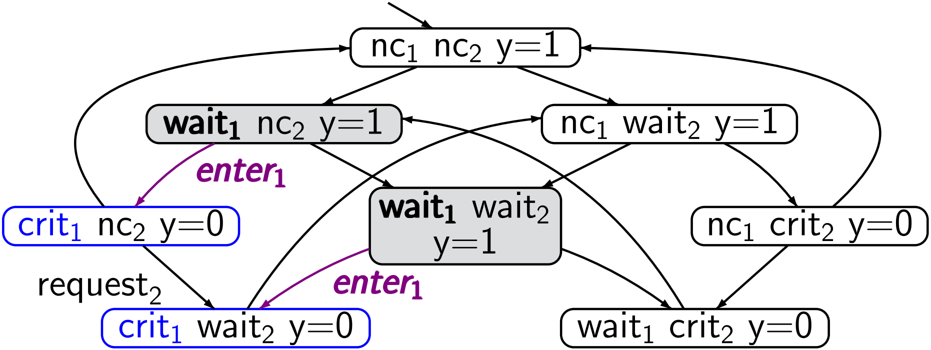
alternative 2: modify the given transition system by adding an action component to the states

Ad-hoc: action fairness ⇝ LTL-fairness



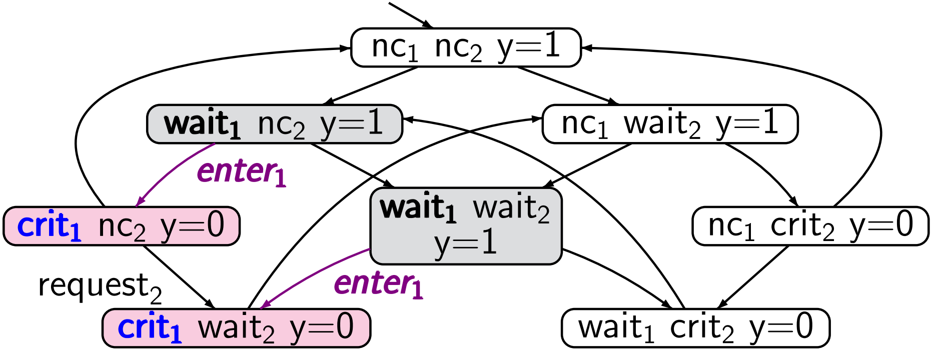
TS for mutual exclusion with semaphore

Ad-hoc: action fairness ⇝ LTL-fairness



 $\widehat{=}$ *enabled*({*enter*$_1$})

TS for mutual exclusion with semaphore

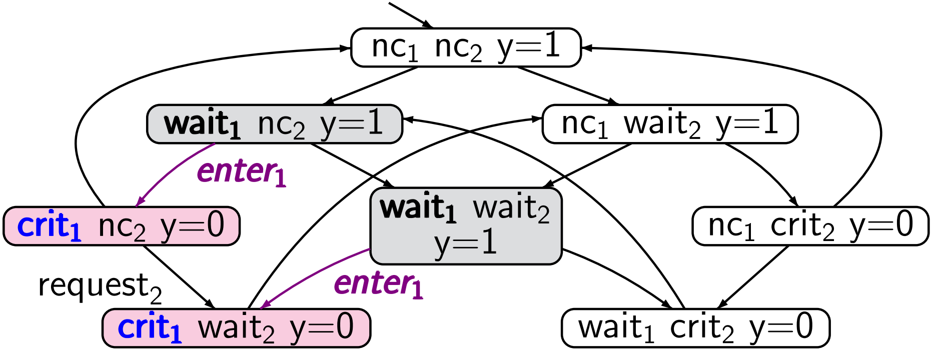


$\widehat{=}$ *taken*({*enter*$_1$})

$\widehat{=}$ *enabled*({*enter*$_1$})

TS for mutual exclusion with semaphore

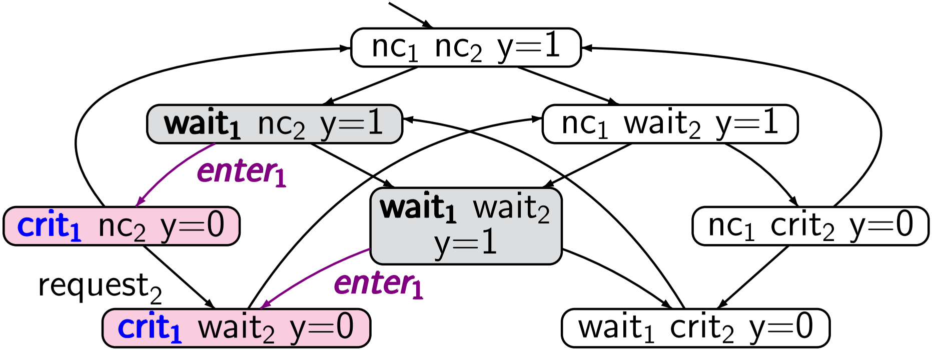
Ad-hoc: action fairness ⇝ LTL-fairness



$\widehat{=}$ *taken*({*enter*$_1$}) $\widehat{=}$ $crit_1$

$\widehat{=}$ *enabled*({*enter*$_1$}) $\widehat{=}$ $wait_1 \wedge \neg crit_2$

Ad-hoc: action fairness ⇝ LTL-fairness



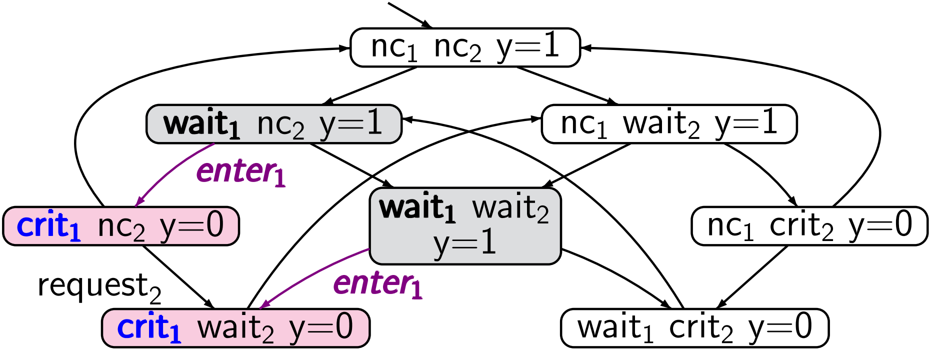
pink box $\widehat{=}$ *taken*({*enter*$_1$}) $\widehat{=}$ **crit$_1$**

grey box $\widehat{=}$ *enabled*({*enter*$_1$}) $\widehat{=}$ **wait$_1$** $\wedge \neg$**crit$_2$**

strong {*enter*$_1$}-fairness: LTL formula

$$\Box\Diamond\, enabled(\{enter_1\}) \rightarrow \Box\Diamond\, taken(\{enter_1\})$$

Ad-hoc: action fairness ⇝ LTL-fairness



(pink box)	$\widehat{=}$	*taken*({*enter*$_1$})	$\widehat{=}$	**crit$_1$**
(grey box)	$\widehat{=}$	*enabled*({*enter*$_1$})	$\widehat{=}$	**wait$_1$** $\wedge \neg$**crit$_2$**

$$\Box\Diamond\, \mathit{enabled}(\{\mathit{enter}_1\}) \;\rightarrow\; \Box\Diamond\, \mathit{taken}(\{\mathit{enter}_1\})$$

$$\widehat{=}\quad \Box\Diamond\,(\mathrm{wait}_1 \wedge \neg \mathrm{crit}_2) \;\rightarrow\; \Box\Diamond\, \mathrm{crit}_1$$

Action-based fairness $\rightsquigarrow$ LTL-fairness

idea: use new atomic propositions ***enabled*(*A*)** and ***taken*(*A*)** and extend the labeling function:

$$\mathit{enabled}(A) \in L(s) \quad \text{iff} \quad s \xrightarrow{\alpha} \ldots \text{ for some } \alpha \in A$$

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alternative 1: **ad-hoc choice** of "***taken***-predicate"

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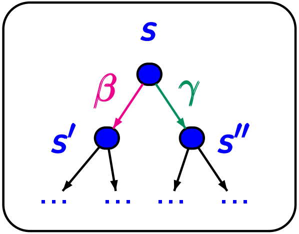

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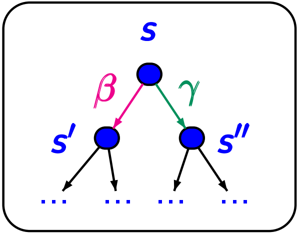

transition system
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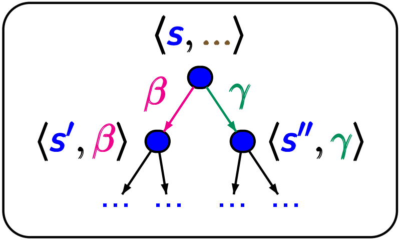

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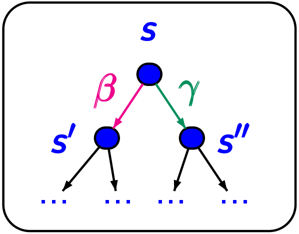

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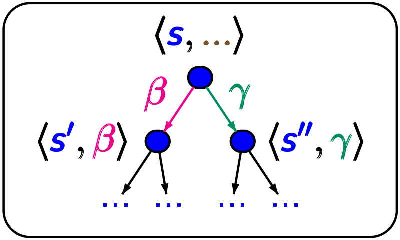

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for $A \subseteq Act$

strong **LTL**-fairness
$\Box\Diamond enabled(A) \rightarrow \Box\Diamond taken(A)$

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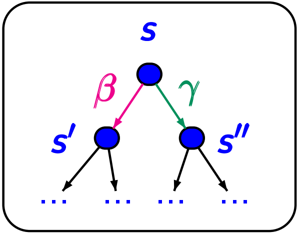

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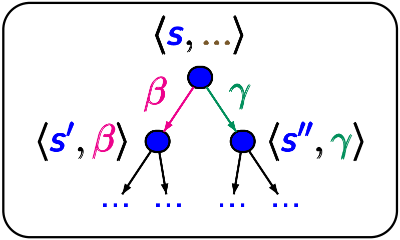

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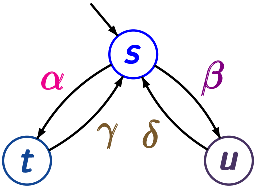
$enabled(A) \in L'(\langle s, \alpha \rangle)$ iff $s \xrightarrow{\beta} \ldots$ for some $\beta \in A$

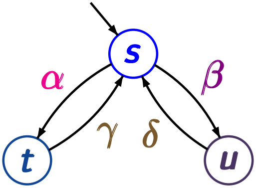
$taken(A) \in L'(\langle s, \alpha \rangle)$ iff $\alpha \in A$

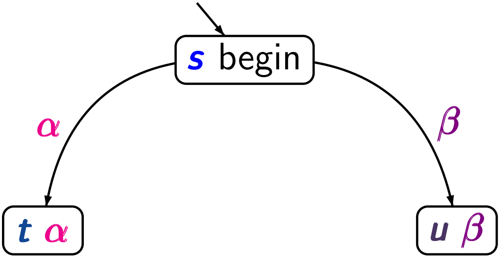
Example: action fairness $\rightsquigarrow$ LTL-fairness

action-based fairness $\rightsquigarrow$ LTL-fairness



action-based fairness ⇝ LTL-fairness





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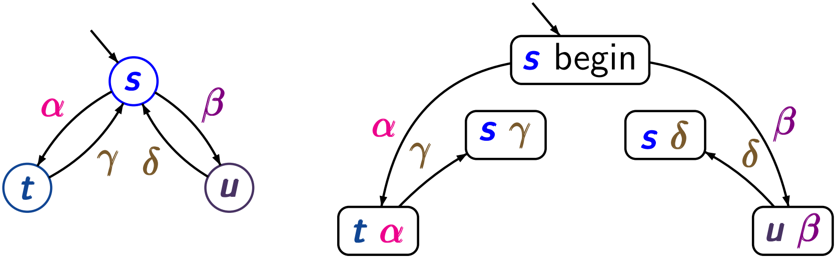

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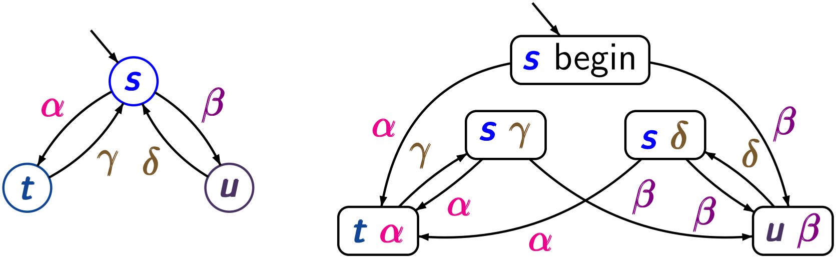

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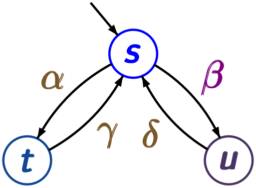

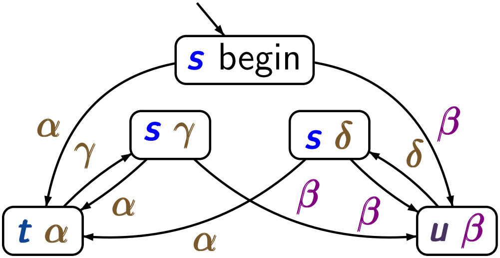

strong fairness for $\{\beta\}$:

$$\Box\Diamond\, enabled(\beta) \rightarrow \Box\Diamond\, taken(\beta)$$

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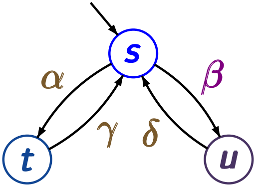
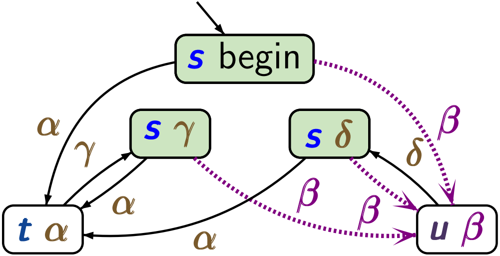
action-based fairness ⇝ LTL-fairness





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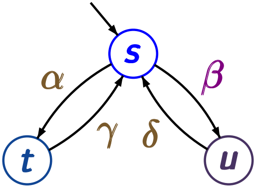

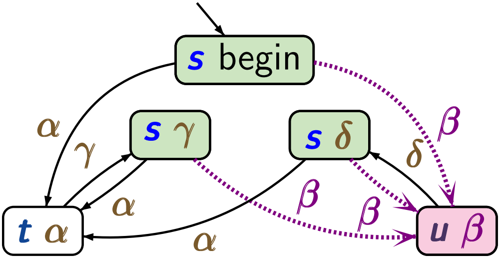

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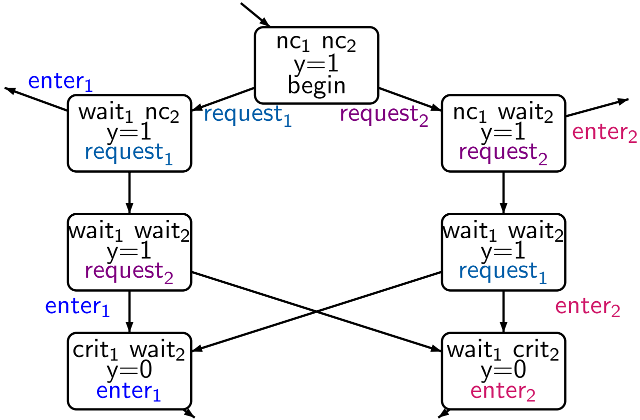
add additional variable last_action with domain Act ∪ {begin}



Example: mutual exclusion with semaphore

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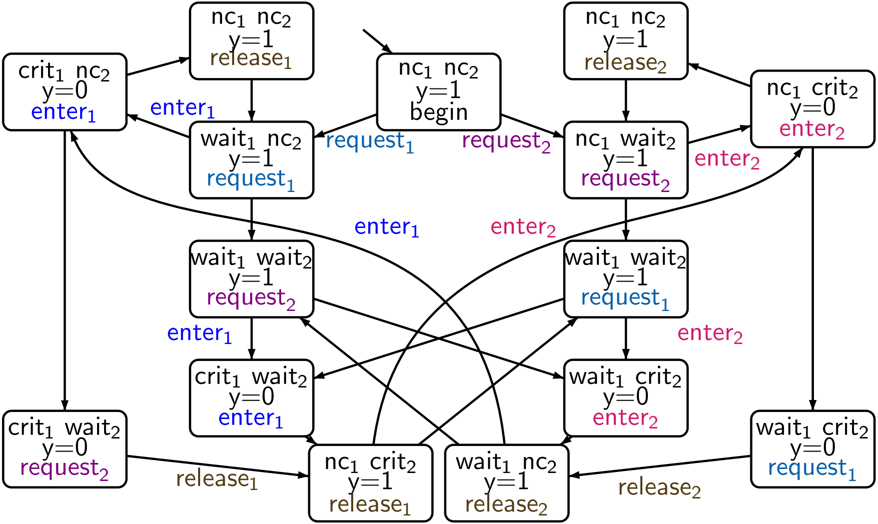

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