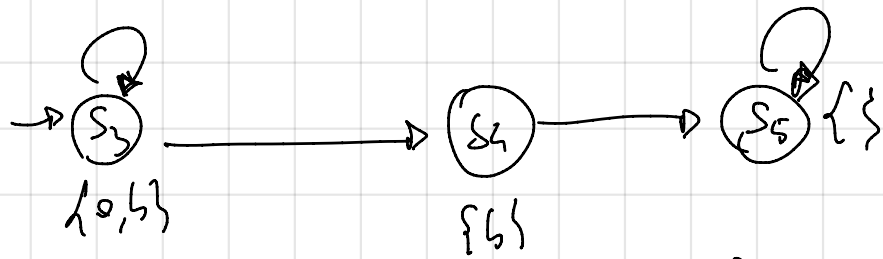
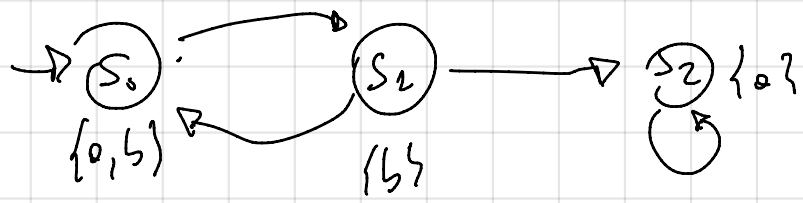


EX 2.4



fair:  $(\square \diamond (a|b) \rightarrow \square \diamond \tau c) \wedge (\square \diamond (a|b) \rightarrow \square \diamond \tau b)$

which are the fair paths?

unfair  $\pi_1 : (S_0 S_1)^\omega \rightsquigarrow (\{a,b\} \{b\})^\omega$

fair  $\pi_2 : (S_0 S_1)^+ S_2^\omega \equiv \{ (S_0 S_1)^m S_2^\omega \mid m > 0 \} \rightsquigarrow (\{a,b\} \{b\})^+ \cdot \{a\}^\omega$

unfair  $\pi_3 : (S_2)^\omega \rightsquigarrow \{a,b\}^\omega$

fair  $\pi_4 : S_3^+ S_4 S_5^\omega \rightsquigarrow \{a,b\}^+ \{b\} \{\}^\omega$

5.4 EX

$$a) \quad \Box (Peter.use \rightarrow \neg Betsy.use \wedge Betsy.use \rightarrow \neg Peter.use) \\ \equiv \Box \neg (Peter.use \wedge Betsy.use)$$

$$b) \quad \Box (B.use \rightarrow \Diamond B.release) \wedge \\ \Box (P.use \rightarrow \Diamond P.release)$$

$$c) \quad \Box (B.request \rightarrow \Diamond B.use) \wedge \\ \Box (P.request \rightarrow \Diamond P.use)$$

$$d) \quad \Box \Diamond P.request \wedge \Box \Diamond B.request$$

$$e) \quad \Box (P.release \rightarrow (\neg P.use \vee B.use)) \wedge \\ \Box (B.release \rightarrow (\neg B.use \vee P.use))$$

5.6 EX

$$\Box a \rightarrow \Box b \stackrel{?}{\equiv} a \cup (b \vee \neg a)$$

$\downarrow \equiv$

$$\neg \Box a \vee \Box b$$

|||

$$\Box \neg a \vee \Box b$$

|||

$$\Box (\neg a \vee b) \equiv \text{true} \cup (b \vee \neg a)$$

$$t_2' = \{a\}^m \quad \{b\} \quad \checkmark$$

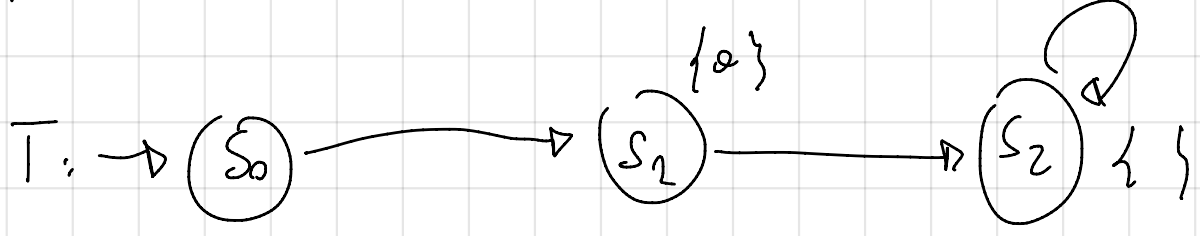
$$t_2 = \{a\} \quad \checkmark$$

Two cases:

- 1) the trace starts with  $\{a\}^m$   $n > 0$  } will wait for  $b$  or  $\neg a$
- 2) the trace starts with  $\{\neg a\}$   $\rightarrow$  both are satisfied } will wait for  $b \vee \neg a$

$\hookrightarrow$  this is exactly the same.

$$f) \Box a \wedge \Box \Box a \stackrel{?}{\equiv} \Box a$$

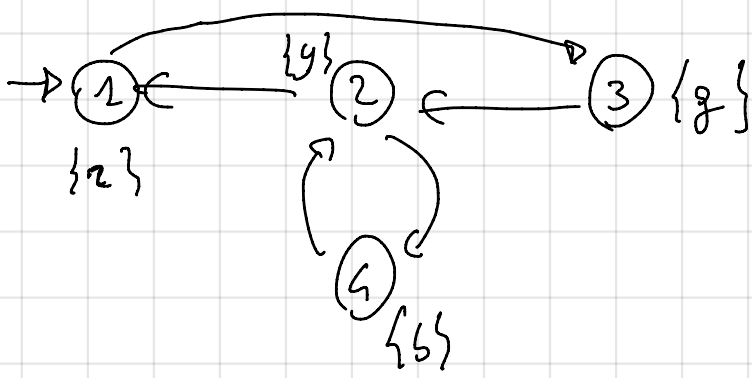


$$T \models \Box a$$

and

$$T \not\models \Box a \wedge \Box \Box a$$

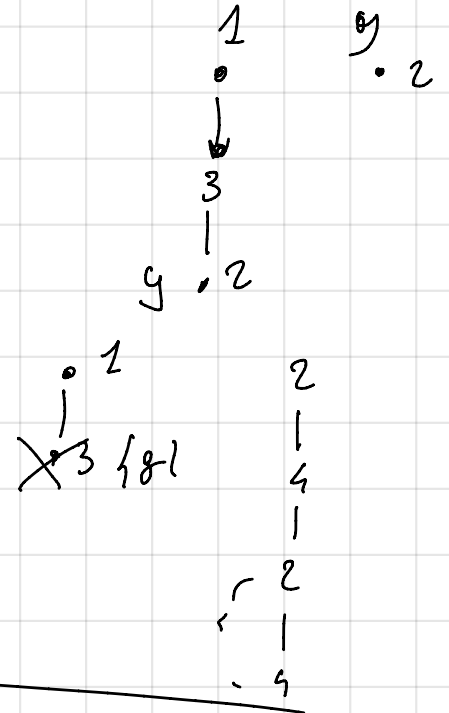
6.1  
EX



$$\text{Set}(\forall \square y) = \{1, 2, 3, 4\}$$

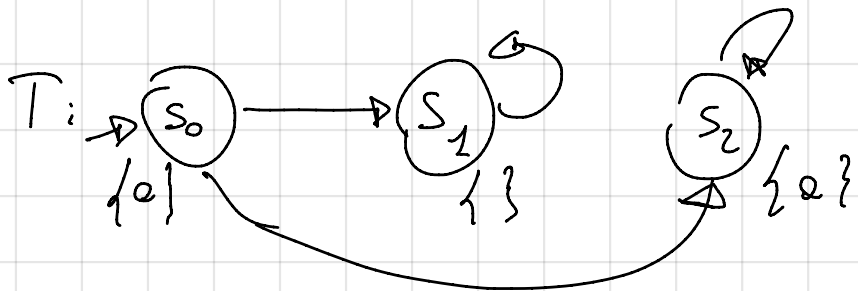
$$\text{Set}(\exists \square \neg y) = \{2, 4\}$$

$$\text{Set}(\forall (b \cup \neg b)) = \{1, 2, 3, 4\}$$



6.3 EX  $S \models \exists \square a \Rightarrow S \models \forall \square a$  NO

Counterexample

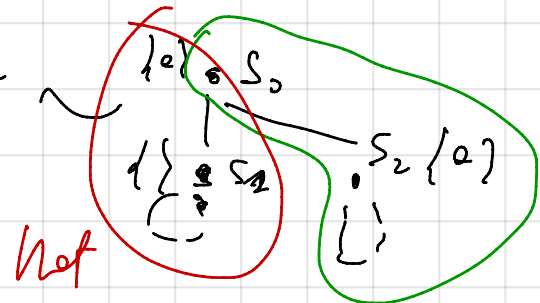


$$\pi = s_0 s_2^{\omega} \Rightarrow s_0 \models \exists \square a$$

$$\pi' = s_0 s_1^{\omega} \Rightarrow s_0 \not\models \forall \square a$$

$$\downarrow$$

$$\{a\} \{a\}^{\omega}$$



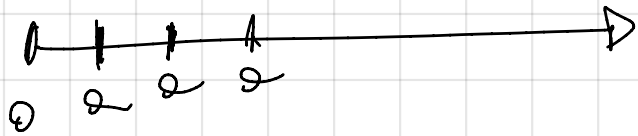
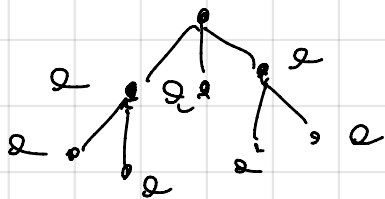
$$\text{if } s \models \forall \square \alpha$$

$$\Rightarrow \forall \pi \in \text{Path}(s) : \forall j \in \mathbb{N} \pi_j \models \alpha$$

$$\Rightarrow \exists \pi \in \text{Path}(s) : \curvearrowright$$

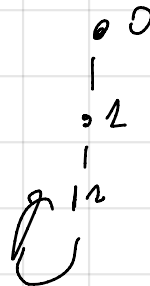
because we consider Transition systems that are not blocked  $\approx$  For each state  $s$  there is at least an outgoing transition

$$\Phi_{CTL} = \forall \square \forall \square \alpha \stackrel{?}{=} \square \square \alpha \quad \text{EX [6.14]}$$

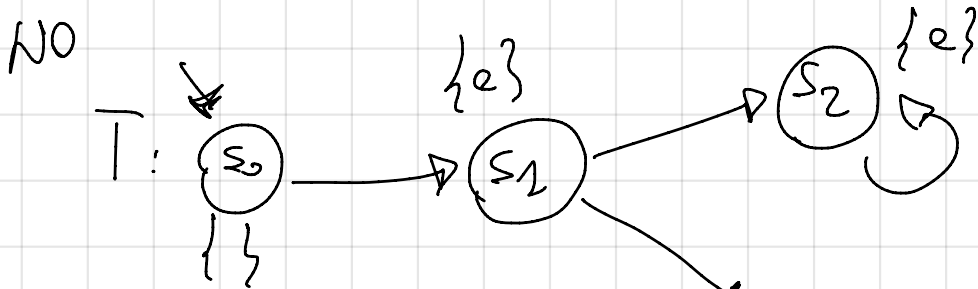


CTL LTL

$$\Phi \equiv \psi \text{ iff } \forall \text{ Transition system } T : T \models_{CTL} \Phi \Leftrightarrow T \models_{LTL} \psi$$



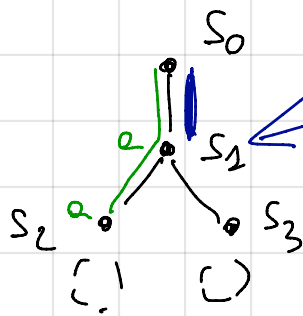
c)  $\forall \diamond (a \wedge \exists \bigcirc a) \equiv \diamond (a \wedge \bigcirc a)$   
 ?



$\not\models_{LTL} \diamond (a \wedge \bigcirc a)$

because  $s_0 s_1 s_3^\omega \not\models a \wedge \bigcirc a$

$\models_{CTL} \forall \diamond (a \wedge \exists \bigcirc a)$   
 yes



this state here satisfies  $a \wedge \exists \bigcirc a$

EX (6.7)

$$\forall O \left( \exists ( \neg x ) \wedge ( b \vee \neg c ) \right) \vee \exists O \forall O x$$
$$\equiv \{ \forall O \psi \equiv \neg \exists O \neg \psi \}$$

$$\forall O \left( \exists ( \neg x ) \wedge ( b \vee \neg c ) \right) \vee \exists O \neg \exists O \neg x$$
$$\equiv \{ \# \}$$

$$\neg \exists O \neg \left( \exists ( \neg x ) \wedge ( b \vee \neg c ) \right) \vee \exists O \neg \exists O \neg x$$
$$\equiv \{ \text{De Morgan} \}$$

$$\neg \exists O \left( \neg \exists ( \neg x ) \wedge ( b \vee \neg c ) \right) \wedge \neg \exists O \neg \exists O \neg x$$

this is in existential normal form