

Traces

$$\{a\} \{a\} (\{a,b\} \{a\})^\omega$$

$$\uparrow$$

$$S_0 S_2 (S_3 S_2)^\omega$$

$$\parallel$$

$$\{a\} (\{a\} \{a,b\})^\omega$$

$$S_0 (S_2 S_3)^\omega$$

$$S_0 S_2 (S_3 S_2)^\omega \Rightarrow \{a\} \{a\} (\{a,b\} \{a\})^\omega$$

$$\Pi_0 S_0 (S_2 | S_2) S_3 (S_3^+ | (S_2 S_3)^+)^+ S_3^\omega$$

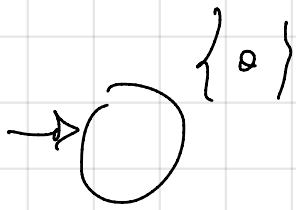
$$\Pi_1 S_0 (S_2 | S_2) S_3 (S^+ | (S_2 S_3)^+)^+ (S_2 S_3)^\omega$$

$$\Pi_2 S_0 (S_2 | S_2) S_3 (S^+ | (S_2 S_3)^+)^+ S_3^\omega$$

$$\text{Traces} = \left\{ \{a\} (\{a\} | \{a\}) \{a,b\} (\{a,b\}^+ | (\{a\} \{a,b\})^+)^+ \{a,b\}^\omega, \{a\} (\{a\} | \{a\}) \{a,b\} (\{a,b\}^+ | (\{a\} \{a,b\})^+)^+ \{a,b\}^\omega, \dots \right\}$$

1) initially  $a$  holds and  $b$  does not hold  $AP = \{a, b, c\}$

$$E = \{ A_0 A_2 A_2 \dots \in (2^{AP})^\omega \mid a \in A_0 \wedge b \notin A_0 \}$$



SAFETY KimBod Prefixes =  $\{ \{ \}, \{b\}, \{a, b\}, \{c\}, \{b, c\}, \{a, b, c\} \}$

Correct "prefixes" =  $\{ \{a\}, \{a, c\} \}$

$$2^{AP} = \{ \{s\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \}$$

2)  $c$  holds only finitely many times

$$E = \{ A_0 A_2 A_2 \dots \in (2^{AP})^\omega \mid \forall i \in \mathbb{N} \ c \notin A_i \}$$

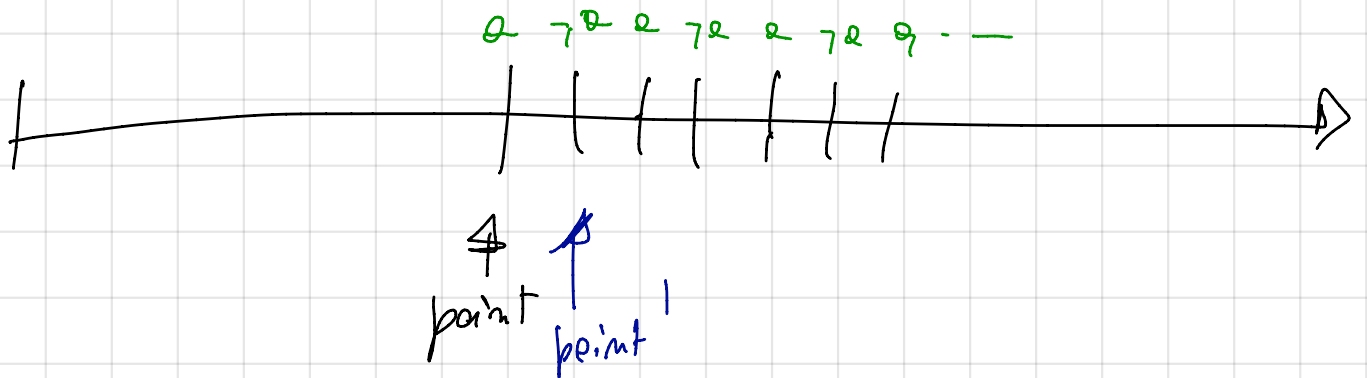
|||

$$\exists k \in \mathbb{N} : \forall i \in \mathbb{N}. i > k \Rightarrow c \notin A_i$$

LIVENESS

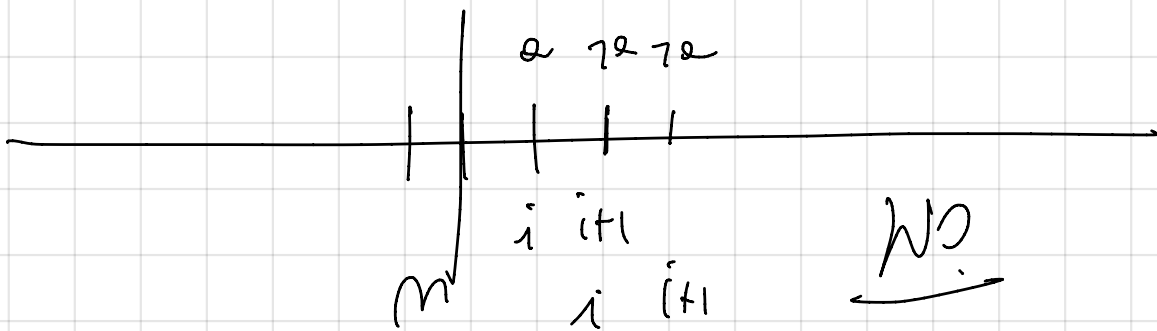
ex  $\{a\} \{c\} \{b\} \{ \}$  can be completed to  $\{a\} \{c\} \{b\} \{s\} (\{a\})^\omega$

3) from some point on the truth value of  $a$  alternates between true and false



$$E = \{ A_0 A_1 \dots \in (2^{AP})^\omega \mid \exists m \in \mathbb{N} : \forall i \in \mathbb{N}.$$

$$i > m \Rightarrow (a \in A_i \Rightarrow a \notin A_{i+1})$$



$$E = \{ A_0 A_1 \dots \in (2^{AP})^\omega \mid \exists m \in \mathbb{N} : \forall i \in \mathbb{N}.$$

$$i > m \Rightarrow ((a \in A_i \Rightarrow a \notin A_{i+1}) \wedge$$

$$(a \notin A_i \Rightarrow a \in A_{i+1})) \}$$

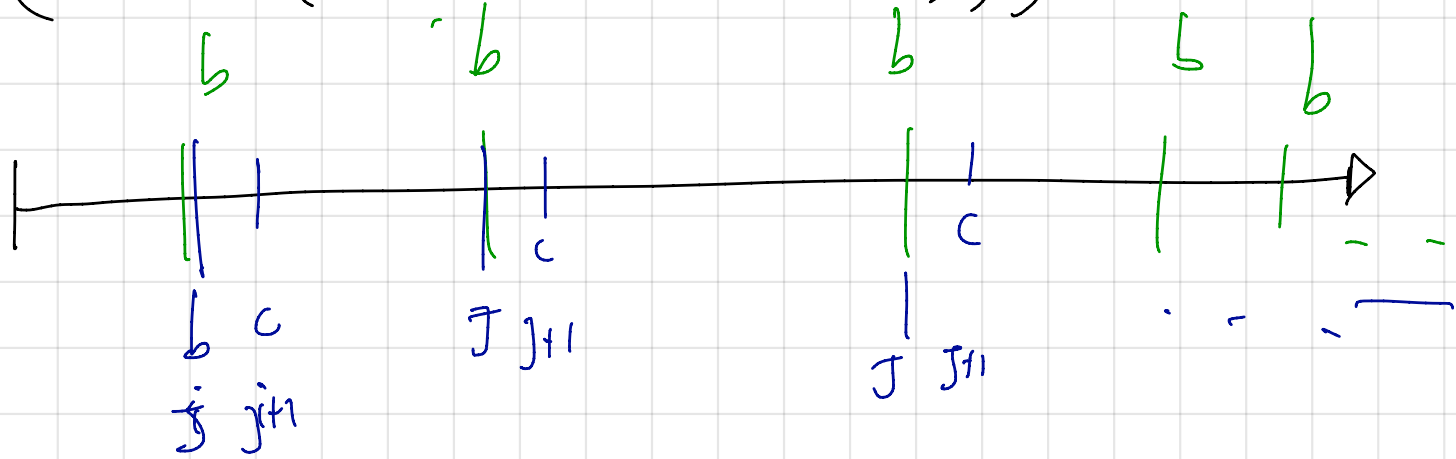
LIVENESS

$A_0 \dots A_m$   $\leftarrow$  extend to  $\nearrow$

a)  $b$  holds infinitely many times and whenever  $b$  holds then  $c$  holds immediately afterwards

$$E = \{ A_0 A_1 \dots \in (\Sigma^{AP})^\omega \mid (\exists i \in \mathbb{N} : b \in A_i) \}$$

$$\left( \forall j \in \mathbb{N} . (b \in A_j \Rightarrow c \in A_{j+1}) \right)$$



$$\text{MIXED} = \text{SAFE} \wedge \text{LIVE}$$

$$\text{LIVE} = \{ A_0 \dots \mid \exists i \in \mathbb{N} : b \in A_i \}$$

$$\text{SAFE} = \{ A_0 \dots \mid \forall j \in \mathbb{N} . b \in A_j \Rightarrow c \in A_{j+1} \}$$