Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

- syntax and semantics of LTL
- automata-based LTL model checking ⟵
- complexity of LTL model checking

Computation-Tree Logic

Equivalences and Abstraction

LTL model checking problem

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given: finite transition system $\mathcal{T}$ over *AP*
(without terminal states)
LTL-formula φ over *AP*

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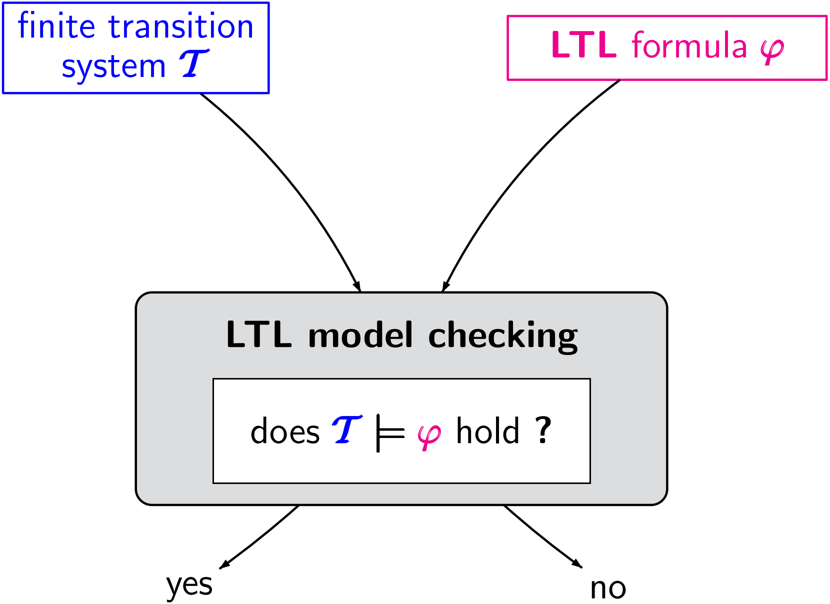
↑

construct the product-TS $\mathcal{T} \otimes \mathcal{A}$
search a path in the product that meets
the acceptance condition of $\mathcal{A}$

Automata-based LTL model checking



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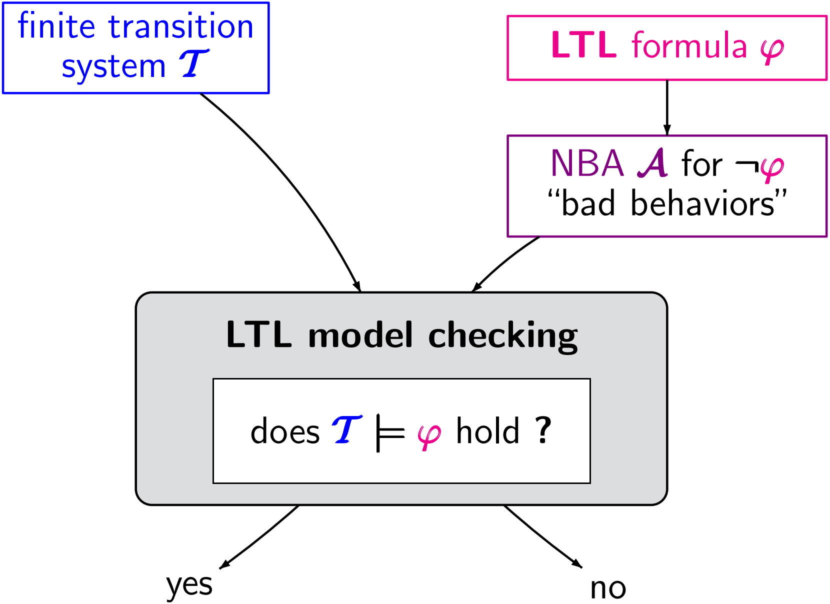

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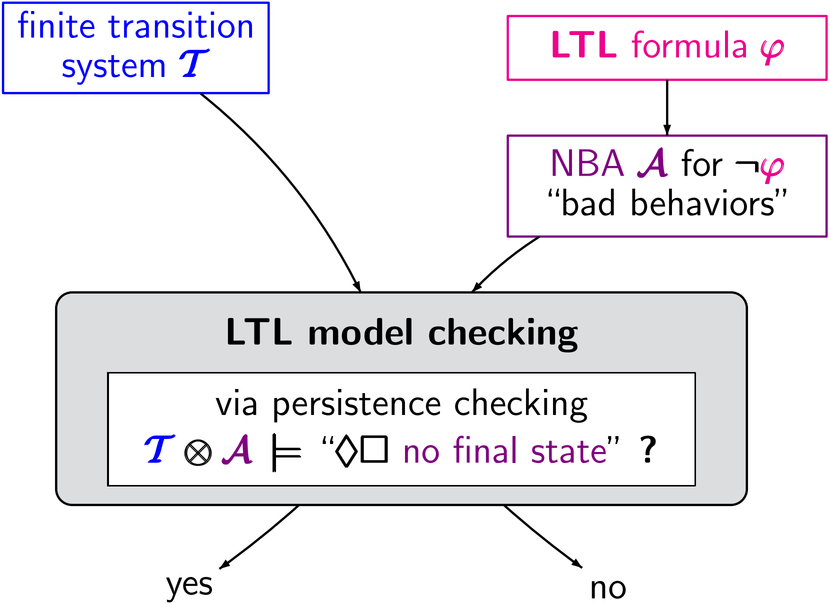

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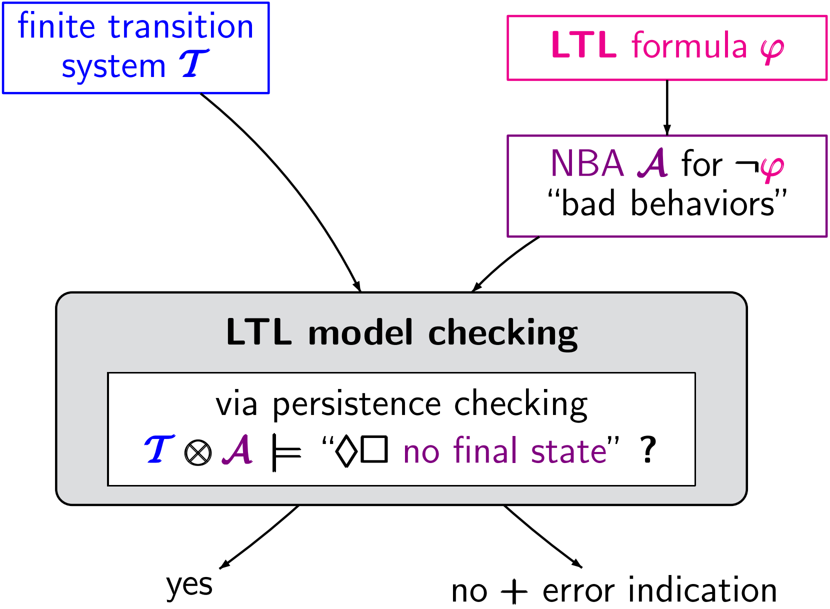

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Complexity of LTL model checking

main steps of automata-based LTL model checking:

construction of an NBA $\mathcal{A}$ for $\neg\varphi$

persistence checking in the product $\mathcal{T} \otimes \mathcal{A}$

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> The **LTL** model checking problem is **PSPACE**-complete

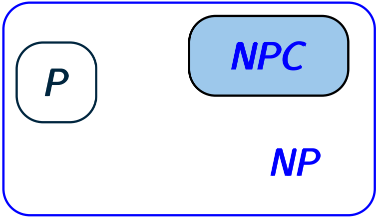
Recall: complexity classes

Complexity classes *P*, *NP*

P = class of decision problem solvable in deterministic polynomial time

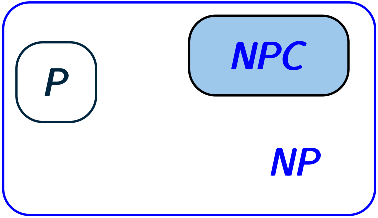
NP = class of decision problem solvable in nondeterministic polynomial time

Complexity classes *P*, *NP*



NPC = class of ***NP***-complete problems

Complexity classes P, NP



NPC = class of NP-complete problems

(1) $L \in NP$

(2) L is NP-hard, i.e., $K \leq_{poly} L$ for all $K \in NP$

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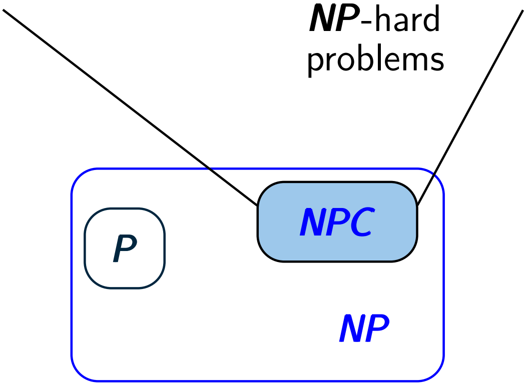

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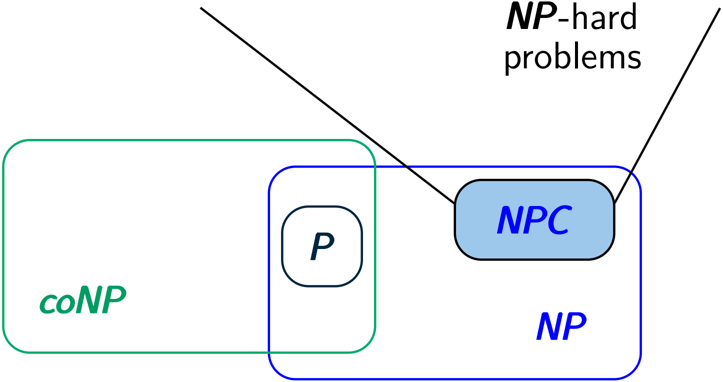

$$coNP \;=\; \{\, \overline{L} : L \in NP \,\}$$

↑ complement of L

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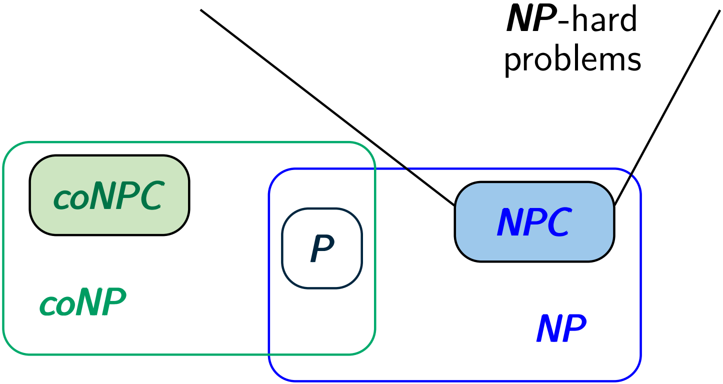

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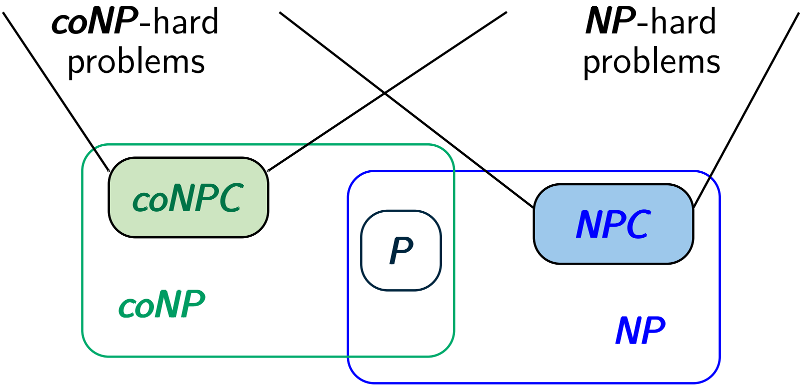

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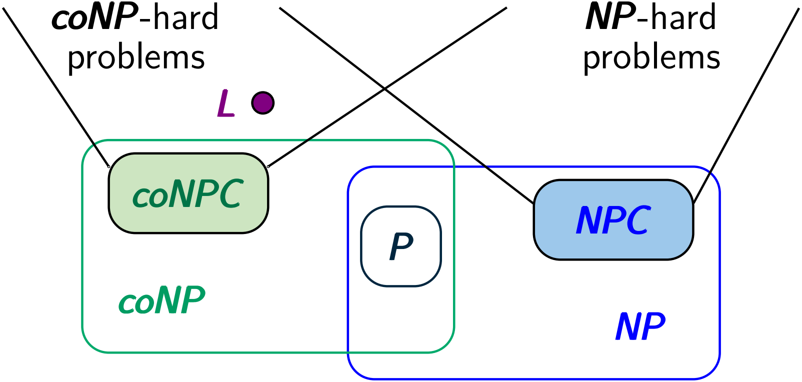

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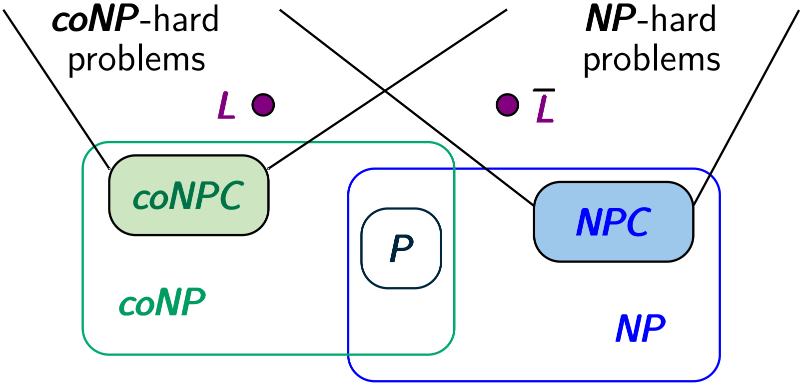

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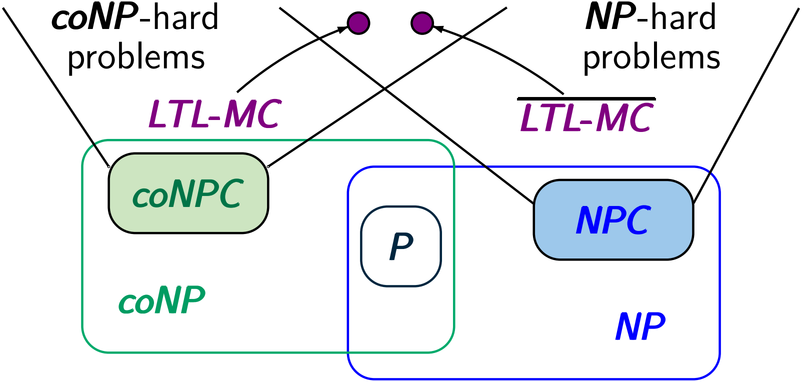

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***coNP*-hardness**

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proof by a polynomial reduction

$$HP \leq_{poly} \overline{LTL\text{-}MC}$$

Hamilton path problem ↗ (HP)

complement of the **LTL** model checking problem ↖ ($\overline{LTL\text{-}MC}$)

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HP: Hamilton path problem

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Complexity of LTL model checking

We just saw:

> The **LTL** model checking problem is ***coNP***-hard

We now prove:

> The **LTL** model checking problem is ***PSPACE***-complete

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  > **DFS**-based analysis of the computation tree of an ***NP***-algorithm

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  DFS-based analysis of the computation tree of an ***NP***-algorithm

  space requirements:

  recursion depth $\widehat{=}$ height of computation tree

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To prove $L \in$ ***PSPACE*** it suffices to provide a nondeterministic polynomially space-bounded algorithm for the complement $\overline{L}$ of L

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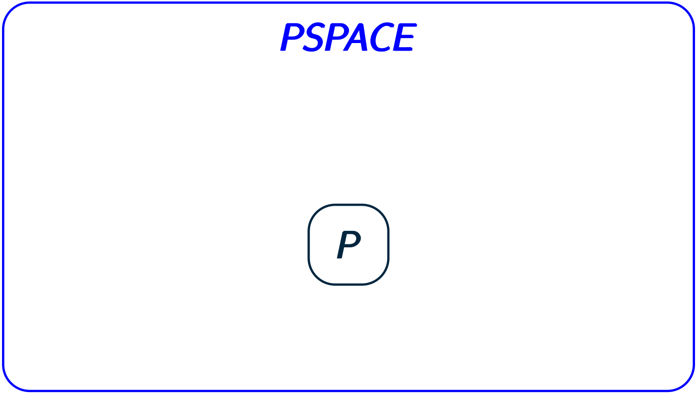

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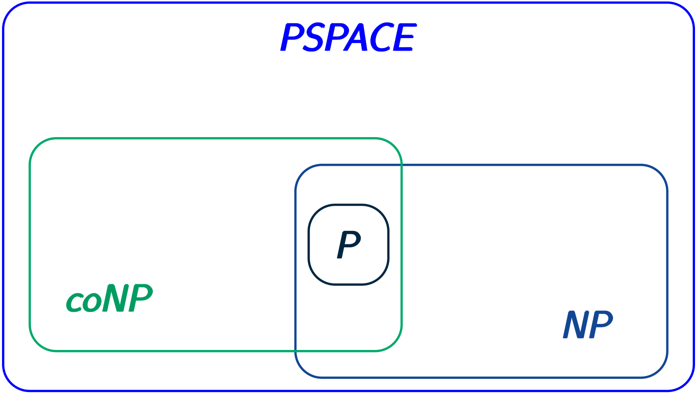

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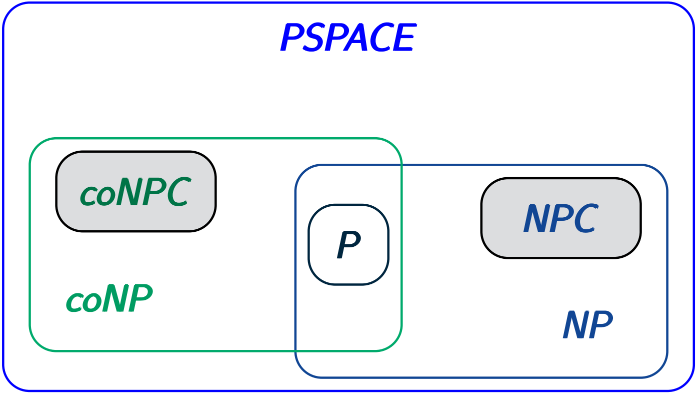

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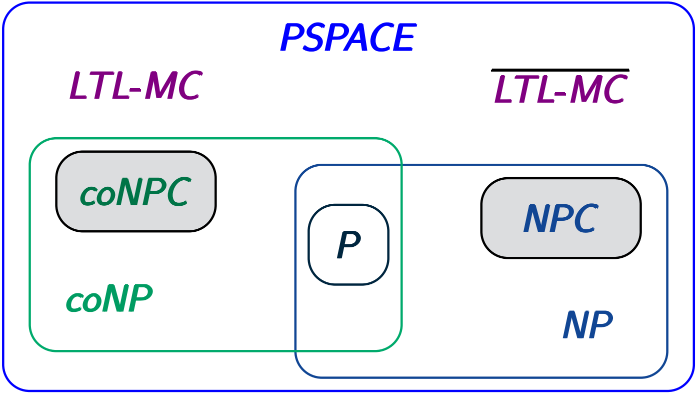

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